

Lecture 6

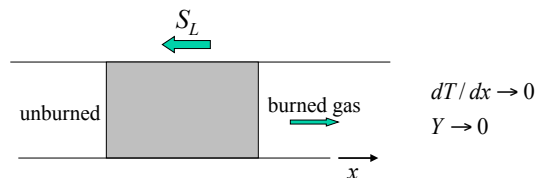
The Structure of a Planar Laminar Flame

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laboratory frame

$$\rho = \rho_u; \quad T = T_u$$

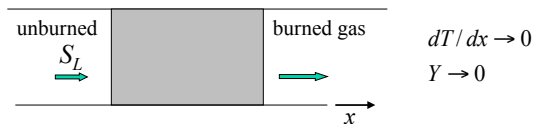
$$Y_F = Y_{F_u}; \quad Y_O = Y_{O_u}; \quad u = 0$$



in a frame moving with the flame

$$\rho = \rho_u; \quad T = T_u$$

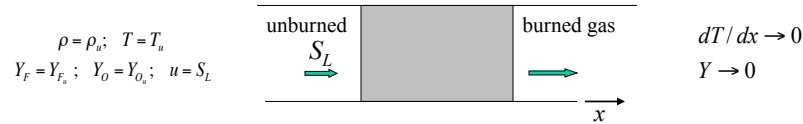
$$Y_F = Y_{F_u}; \quad Y_O = Y_{O_u}; \quad u = S_L$$



one-step, irreversible, global reaction



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Steady, one-dimensional equations

We will use a “tilde” for the dimensional quantities

$$\begin{aligned}\frac{d}{dx}(\tilde{\rho}\tilde{u}) &= 0 \Rightarrow \tilde{\rho}\tilde{u} = \tilde{m} \\ \tilde{\rho}\tilde{u}c_p \frac{d\tilde{T}}{dx} - \lambda \frac{d^2\tilde{T}}{dx^2} &= Q\tilde{\omega} \\ \tilde{\rho}\tilde{u} \frac{dY_F}{dx} - \rho\mathcal{D}_F \frac{d^2Y_F}{dx^2} &= -\nu_F W_F \tilde{\omega} \\ \tilde{\rho}\tilde{u} \frac{dY_O}{dx} - \rho\mathcal{D}_O \frac{d^2Y_O}{dx^2} &= -\nu_O W_O \tilde{\omega} \\ \tilde{\omega} &= B \left(\frac{\tilde{\rho}Y_O}{W_O} \right)^{n_O} \left(\frac{\tilde{\rho}Y_F}{W_F} \right)^{n_F} \exp(-E/R\tilde{T})\end{aligned}$$

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$$\begin{aligned}\tilde{\rho}\tilde{u} &= \text{const.} \\ \tilde{\rho}\tilde{u}c_p \frac{d\tilde{T}}{dx} - \lambda \frac{d^2\tilde{T}}{dx^2} &= Q\tilde{\omega} \\ \tilde{\rho}\tilde{u} \frac{dY_F}{dx} - \rho\mathcal{D}_F \frac{d^2Y_F}{dx^2} &= -\nu_F W_F \tilde{\omega}\end{aligned}$$

Assume the mixture is lean; this implies that the fuel is completely depleted

$$\frac{d}{dx} \left[\tilde{\rho}\tilde{u} \left(c_p \tilde{T} + \frac{QY_F}{\nu_F W_F} \right) \right] - \frac{d}{dx} \left[\lambda \frac{d\tilde{T}}{dx} + \frac{\rho\mathcal{D}_F Q}{\nu_F W_F} \frac{dY_F}{dx} \right] = 0$$

Integrating from $-\infty$ to $+\infty$ and using the BCs, yields $\tilde{\rho}\tilde{u} \left(c_p \tilde{T} + \frac{QY_F}{\nu_F W_F} \right) \Big|_{-\infty}^{\infty} = 0$

Similarly for a rich mixture. Use the equation for Y_O instead of that of Y_F and assume now that all the oxidizer is depleted.

$$\text{Adiabatic flame temperature} \quad \tilde{T}_a = \begin{cases} \tilde{T}_u + \frac{(Q/c_p)Y_{F_u}}{\nu_F W_F} & \phi < 1 \\ \tilde{T}_u + \frac{(Q/c_p)Y_{O_u}}{\nu_O W_O} & \phi > 1 \end{cases}$$

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The residual fuel/oxidizer can be computed from

$$\begin{aligned}\tilde{\rho}\tilde{u}\frac{dY_F}{dx} - \rho\mathcal{D}_F\frac{d^2Y_F}{dx^2} &= -\nu_F W_F \tilde{\omega} \\ \tilde{\rho}\tilde{u}\frac{dY_O}{dx} - \rho\mathcal{D}_O\frac{d^2Y_O}{dx^2} &= -\nu_O W_O \tilde{\omega}\end{aligned}$$

$$\frac{d}{dx}\left[\tilde{\rho}\tilde{u}\left(\frac{Y_O}{\nu_O W_O} - \frac{Y_F}{\nu_F W_F}\right)\right] - \frac{d}{dx}\left[\frac{\rho\mathcal{D}_O}{\nu_O W_O}\frac{dY_O}{dx} - \frac{\rho\mathcal{D}_F}{\nu_F W_F}\frac{dY_F}{dx}\right] = 0$$

Integrating from $-\infty$ to $+\infty$ and using the BCs, yields

$$\begin{aligned}Y_F &= 0, & Y_O &= Y_{O_u}(1-\phi) & \text{for } \phi < 1 \\ Y_O &= 0, & Y_F &= Y_{F_u}(1-\phi^{-1}) & \text{for } \phi > 1\end{aligned}$$

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$$\begin{aligned}\tilde{\rho}\tilde{u}c_p\frac{d\tilde{T}}{dx} - \lambda\frac{d^2\tilde{T}}{dx^2} &= Q\tilde{\omega} \\ \tilde{\rho}\tilde{u}\frac{dY_F}{dx} - \rho\mathcal{D}_F\frac{d^2Y_F}{dx^2} &= -\nu_F W_F \tilde{\omega} \\ \tilde{\rho}\tilde{u}\frac{dY_O}{dx} - \rho\mathcal{D}_O\frac{d^2Y_O}{dx^2} &= -\nu_O W_O \tilde{\omega}\end{aligned}$$

$$\tilde{\omega} = B\left(\frac{\tilde{\rho}Y_O}{W_O}\right)^{n_O}\left(\frac{\tilde{\rho}Y_F}{W_F}\right)^{n_F}\exp(-E/RT)$$

$$\tilde{\rho} = \rho_u; \quad \tilde{T} = T_u; \quad d\tilde{T}/dx = 0; \quad Y_F = Y_{F_u}; \quad Y_O = Y_{O_u}; \quad \tilde{u} = S_L \quad \text{as } x \rightarrow -\infty$$

$$d\tilde{T}/dx = dY_F/dx = dY_O/dx = 0 \quad \text{as } x \rightarrow +\infty$$

The problem is mathematically ill-posed because the reaction rate does not vanish at the far left (the cold boundary difficulty). The difficulty is due to the idealization (infinite domain) and the nature of the Arrhenius law.

Can be resolved

(i) if the mixture is introduced at a finite location (i.e. a burner)

(ii) by introducing a switch-on temperature; i.e. multiply the RHS by $H(T - T_i)$.

These resolutions are not needed when the asymptotic solution for large activation energy is sought.

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non-dimensionalization

length $l_f = \mathcal{D}_{th}/S_L$ (where $\mathcal{D}_{th} = \lambda/\rho_u c_p$ is the mixture thermal diffusivity),
 speed $u = \tilde{u}/S_L$, temperature $T = \tilde{T}/T_u$ density $\rho = \tilde{\rho}/\rho_u$

Governing equations

$$\begin{aligned}\rho u &= 1 \\ \frac{dT}{dx} - \frac{d^2 T}{dx^2} &= q\omega \\ \frac{dY_F}{dx} - Le_F^{-1} \frac{d^2 Y_F}{dx^2} &= -\omega \\ \frac{dY_O}{dx} - Le_O^{-1} \frac{d^2 Y_O}{dx^2} &= -\nu\omega \\ \omega &= D \rho^n Y_O^{n_O} Y_F^{n_F} \exp\left(\frac{\beta_0}{T_a} - \frac{\beta_0}{T}\right)\end{aligned}$$

Parameters

$$\begin{aligned}q &= \frac{Q/v_F W_F}{c_p T_u} = \text{heat release parameter} \\ \beta_0 &= \frac{E}{RT_u} = \text{activation energy parameter} \\ Le_i &= \frac{\mathcal{D}_{th}}{\mathcal{D}_i} = \frac{\text{thermal diffusivity (of the mixture)}}{\text{mass diffusivity (of species i)}} \\ \phi &= \nu Y_{F_i}/Y_{O_i} = \text{equivalence ratio} \\ T_a &= \begin{cases} 1+qY_{F_u} & \text{for } \phi < 1 \\ 1+q\nu^{-1}Y_{O_u} & \text{for } \phi > 1 \end{cases}\end{aligned}$$



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The Damköhler number

$$D = \frac{(l_f / S_L) \exp(-E / RT_a)}{[(\rho_u / W_F)^{n_F-1} (\rho_u / W_O)^{n_O} \nu_F B]^{-1}} = \frac{l_f / S_L}{[\tilde{B} \exp(-E / RT_a)]^{-1}} = \frac{\text{flow time}}{\text{reaction time}}$$

Damkohler number

$$\tilde{B} = (\rho_u / W_F)^{n_F-1} (\rho_u / W_O)^{n_O} \nu_F B \quad \text{has units of } 1/\text{s}$$

$$D = \frac{(\lambda / \rho_u c_p) / S_L^2}{[\tilde{B} \exp(-E / RT_a)]^{-1}}$$

D contains the unknown flame speed, S_L , and is often referred to as the burning-rate eigenvalue.

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A note about the Arrhenius exponential

$$E/R \approx 20\,000K - 30\,000K, \quad T_u \approx 300K, \quad T_a \approx 2\,000 - 3\,000K$$

$$\exp(-E/RT_u) \sim 10^{-28} - 10^{-43}; \quad \exp(-E/RT_a) \sim 10^{-3} - 10^{-6}$$

$$\tilde{B} \approx 10^{12} - 10^{17}$$

For example, for methane oxidation
 $\text{CH}_4 + 2 \text{O}_2 \rightarrow \text{CO}_2 + 2 \text{H}_2\text{O}$

$$dY/dt = BYe^{-E/RT}$$

$$E \approx 160,000 \text{ J/mol} \quad E/R \sim 20,000 \quad \text{and} \quad B = 10^{10} \text{ s}$$

$$E/RT_u \sim 65 \quad \exp(-E/RT_u) \times 10^{-28}$$

$$t_{\text{reaction}} \sim [B \exp(-E/RT_u)]^{-1} \sim 10^{18} \text{ s} \sim 10^{11} \text{ years}$$

$$E/RT_a \sim 10 \quad \exp(-E/RT_a) \sim 2 \times 10^{-4}$$

$$t_{\text{reaction}} \sim [B \exp(-E/RT_a)]^{-1} \sim 2 \times 10^{-6} \text{ s} \sim 2\mu\text{s}$$

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For simplicity of presentation we will consider first a mixture deficient in one component; i.e either lean (fuel deficient) or rich (oxidizer deficient), for which the excess component remains nearly unchanged¹. We thus remove all subscripts, and denote by Y the mass fraction of the deficient component. Results for the general case will be presented later.

$$\begin{aligned} \frac{dT}{dx} - \frac{d^2T}{dx^2} &= q\omega & dT/dx = 0, \quad T = 1, \quad Y = Y_u & \text{as } x \rightarrow -\infty \\ \frac{dY}{dx} - Le^{-1} \frac{d^2Y}{dx^2} &= -\omega & T = T_a, \quad Y = 0 & \text{as } x \rightarrow \infty \\ \omega &= D\rho^n Y^n \exp\left(\frac{\beta_0}{T_a} - \frac{\beta_0}{T}\right) \end{aligned}$$

nonlinear eigenvalue problem for the determination of T, Y and D

$$Le = \frac{\mathcal{D}_{th}}{\mathcal{D}_m} = \frac{\text{thermal diffusivity (of the mixture)}}{\text{mass diffusivity (of deficient reactant)}}$$

in lean mixtures: $\mathcal{D}_m = \mathcal{D}_{\text{fuel}}$
in rich mixtures $\mathcal{D}_m = \mathcal{D}_{\text{oxidizer}}$

¹ We are effectively considering the case $R \rightarrow P$, for which the reaction is of order n , or $n_F = n$, $n_O = 0$, and $\nu_F = 1$.

Activation energy asymptotics

$$\omega = D\rho Y^n \exp\left(\frac{\beta_0}{T_a} - \frac{\beta_0}{T}\right) = D\rho Y^n \exp\left[\frac{\beta_0(T - T_a)}{TT_a}\right]$$

For $\beta_0 \gg 1$, we see that ω is exponentially small, unless $T - T_a \approx \beta_0^{-1}$

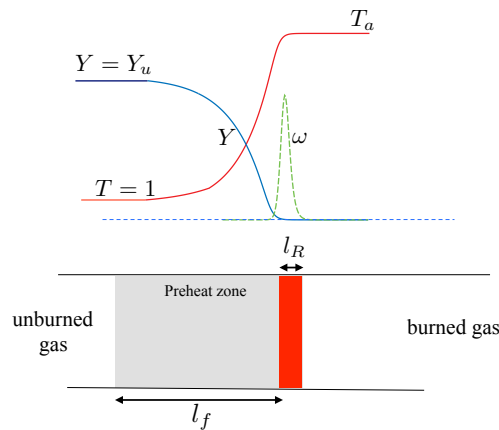
In the preheat zone the reaction rate is exponentially small because the exponential term $\rightarrow 0$ very rapidly. The reaction term becomes appreciable only in the “reaction zone” where T is close to T_a . Beyond the reaction zone, $Y \equiv 0$ and the reaction rate vanishes identically.

The more appropriate expansion parameter is the reciprocal Zel’dovich number β , rather than β_0 , where

$$\beta = \frac{E(\tilde{T}_a - \tilde{T}_u)}{\mathcal{R}\tilde{T}_a^2} \quad \text{typically, } \beta \sim 10$$

$$\beta = \beta_0 \frac{(T_a - 1)}{T_a^2} \Rightarrow \omega = D\rho^n Y^n \exp\left[\frac{\beta T_a}{T_a - 1} \frac{T - T_a}{T}\right]$$

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$$l_f = \mathcal{D}_{th}/S_L$$

typically, $l_f \sim 1 \text{ mm}$

$$l_R = \mathcal{D}_{th}/\beta S_L \ll l_f$$

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Introduce $T = T^{(0)} + \beta^{-1}T^{(1)} + \dots$
 $Y = Y^{(0)} + \beta^{-1}Y^{(1)} + \dots$

to all orders in β^{-1}

$$\frac{dT}{dx} - \frac{d^2T}{dx^2} \sim 0$$

$$\frac{dY}{dx} - Le^{-1}\frac{d^2Y}{dx^2} \sim 0$$

\Rightarrow

$$T \sim \begin{cases} 1 + (T_a - 1)e^x & (x < 0) \\ T_a & (x > 0) \end{cases}$$

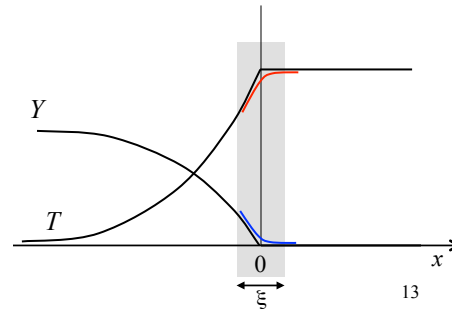
$$Y \sim \begin{cases} Y_u(1 - e^{Le^x}) & (x < 0) \\ 0 & (x > 0) \end{cases}$$

where continuity at $x = 0$ has been applied

the solution is not valid near
 $x = 0$ (the reaction zone)
 where ω is no longer negligible),
 and must be reexamined
 by rescaling the coordinate

$$x = \beta^{-1}\xi$$

with $-\infty < \xi < \infty$



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Reaction zone solution

introducing a stretching transformation, $x = \beta^{-1}\xi$, into the governing equations,
 we seek solutions of the form

$$T = T_a + \beta^{-1}\phi(\xi) + \dots$$

$$Y = \beta^{-1}\psi(\xi) + \dots$$

with the “eigenvalue” expanded as $D = \beta^{n+1}\Lambda + \dots$

$$\beta^{-1}\frac{d\phi}{d\xi} - \frac{d^2\phi}{d\xi^2} + \dots = q\Lambda\rho_b^n\psi^n e^{\phi/(T_a-1)}$$

$$\beta^{-1}\frac{d\psi}{d\xi} - Le^{-1}\frac{d^2\psi}{d\xi^2} + \dots = -\Lambda\rho_b^n\psi^n e^{\phi/(T_a-1)}$$

the reaction zone is a diffusive-reactive zone,
 with convection playing a secondary role

here ρ_b is the (dimensionless) density of the burned gas $= 1/T_a$

the expansion of the exponential yields

$$\exp\left[\frac{\beta T_a}{T_a - 1} \frac{T - T_a}{T}\right] = \exp\left[\frac{\beta}{T_a - 1} (\beta^{-1}\phi + \dots)\right] \sim \exp[\phi/(T_a - 1)]$$

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$$\frac{d^2\phi}{d\xi^2} = -q\Lambda\rho_b^n \psi^n e^{\phi/(T_a-1)}$$

$$Le^{-1} \frac{d^2\psi}{d\xi^2} = \Lambda\rho_b^n \psi^n e^{\phi/(T_a-1)}$$

The solution for ϕ and ψ must match T and Y as $\xi \rightarrow \mp\infty$

$$\begin{aligned} \frac{d\phi}{d\xi} &\sim (T_a - 1), & \frac{d\psi}{d\xi} &\sim -Y_u Le & \text{as } \xi \rightarrow -\infty \\ \phi &\sim \psi \sim 0, & & & \text{as } \xi \rightarrow +\infty \end{aligned}$$

using

$$T \sim \begin{cases} 1 + (T_a - 1)e^x & (x < 0) \\ T_a & (x > 0) \end{cases} \quad Y \sim \begin{cases} Y_u(1 - e^{Le x}) & (x < 0) \\ 0 & (x > 0) \end{cases}$$

the behavior of T, Y as $x \rightarrow 0^\mp$ or,
for $x = \beta^{-1}\xi$ as $\xi \rightarrow \mp\infty$

$$\Rightarrow \begin{aligned} T &\sim \begin{cases} T_a + \beta^{-1}(T_a - 1)\xi & \text{as } \xi \rightarrow -\infty \\ T_a & \text{as } \xi \rightarrow +\infty \end{cases} \\ Y &\sim \begin{cases} -\beta^{-1}Y_u Le\xi & \text{as } \xi \rightarrow -\infty \\ 0 & \text{as } \xi \rightarrow +\infty \end{cases} \end{aligned}$$

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$$Le \frac{d^2\phi}{d\xi^2} + q \frac{d^2\psi}{d\xi^2} = 0 \quad \Rightarrow \quad Le\phi + q\psi = 0$$

after use was made of the matching conditions

$$\begin{aligned} \frac{d^2\psi}{d\xi^2} &= \Lambda Le \rho_b^n \psi^n e^{-q\psi/Le(T_a-1)} \\ \frac{d\psi}{d\xi} &\sim -Y_u Le \text{ as } \xi \rightarrow -\infty, \quad \psi \sim 0 \text{ as } \xi \rightarrow +\infty \end{aligned}$$

multiply by $d\psi/d\xi$ and integrate from $\xi = -\infty$ to ∞ (or from $\psi = \infty$ to 0)

$$\begin{aligned} \frac{1}{2} \int_{-\infty}^{\infty} \frac{d}{d\xi} \left(\frac{d\psi}{d\xi} \right)^2 d\xi &= \Lambda Le \rho_b^n \int_{-\infty}^0 \psi^n e^{-q\psi/Le(T_a-1)} d\psi \\ \frac{1}{2} Y_u^2 Le^2 &= \Lambda Le^{n+2} Y_u^{n+1} \rho_b^n \underbrace{\int_0^{\infty} Z^n e^{-Z} dZ}_{\Gamma(n+1)} \\ &= n! \text{ for } n \text{ integer} \end{aligned}$$

$$D = \frac{Le^{-n} \beta^{n+1}}{2\Gamma(n+1)\rho_b^n Y_u^{n-1}}$$

a second integration is needed to obtain the actual profiles, but this need to be done numerically 16

For $n = 1$

$$S_L = \sqrt{\frac{\tilde{\rho}_b}{\tilde{\rho}_u^2} \frac{2(\lambda/\tilde{c}_p)B}{\beta^2 L e^{-1}}} e^{-E/2\mathcal{R}\tilde{T}_a}$$

Arbitrary reaction order n

$$S_L = \sqrt{\frac{\tilde{\rho}_b^n}{\tilde{\rho}_u^2} \frac{2\Gamma(n+1)(\lambda/\tilde{c}_p)BY_u^{n-1}}{W^{n-1}\beta^{n+1}Le^{-n}}} e^{-E/2\mathcal{R}\tilde{T}_a}$$

$$\text{Adiabatic flame temperature} \quad \tilde{T}_a = \tilde{T}_u + (Q/c_p)Y_u$$

$$\text{Flame thickness (thermal)} \quad l_f = \lambda/\tilde{\rho}_u c_p S_L$$

Recall that we have been considering here the very lean (or very rich) case; i.e., the reaction is of order n in the deficient component and is independent of the abundant reactant.

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Comments:

- Most of the reaction occurs, in a narrow temperature interval near T_a which is on the order $\beta^{-1}l_f \equiv l_R$
- The preheat zone is the region where the temperature rises from T_u to approximately T_a . Most of this increase (more than 99%) occurs in 5 units of length, so that the actual width of the preheat zone is $\sim 5l_f$
- The analysis could be easily extended to account for temperature-dependent transport since this will only affect the preheat zone. The resulting profiles (with λ measured with respect to its value in the unburned gas) are

$$T = 1 + (T_a - 1)e^{-\int_x^0 \frac{1}{\lambda} dx'} \quad Y = Y_u[1 - e^{-\int_x^0 \frac{1}{\lambda} dx'}]$$

The consequence on the flame speed is that λ needs to be evaluated at T_a .

- To complete the solution we note that $\rho = 1/T$ and $u = T$; as a result of gas expansion the density decreases and the gas speeds up. The pressure drop across the flame, calculated from the momentum equation, is

$$p \sim \begin{cases} -(1 - \frac{4}{3}\text{Pr})(T_a - 1)e^x & (x < 0) \\ -(T_a - 1) & (x > 0) \end{cases}$$

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- It is possible to find the next correction to the burning rate eigenvalue. For $n = 1$ if

$$D = \beta^2(\Lambda_0 + \beta^{-1}\Lambda_1 + \dots)$$

one finds

$$S_L = \sqrt{\frac{\tilde{\rho}_b}{\tilde{\rho}_u^2} \frac{2(\lambda/\tilde{c}_p)B}{\beta^2 Le^{-1}}} e^{-E/2\mathcal{R}\tilde{T}_a} \left[1 + \beta^{-1} \left(1.344 - 3 \frac{\tilde{T}_a - \tilde{T}_u}{\tilde{T}_a} \right) \right]$$

- The strong dependence of the Arrhenius rate, and hence of the flame speed on temperature suggests that a small temperature variation cause a significant change in speed; more precisely $O(\beta^{-1})$ changes in T_a cause $O(1)$ changes in S_L .

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- The effect of stoichiometry is found by considering a two-reactants system. For $n = 2$, with $n_F = n_O = 1$, the flame speed generalizes to

$$S_L = \sqrt{\frac{\tilde{\rho}_b^2}{\tilde{\rho}_u^2} \frac{2B(\lambda/c_p)}{\beta^2}} \mathcal{S} e^{-E/2\mathcal{R}\tilde{T}_a}$$

$$\text{where } \mathcal{S} \sim \begin{cases} Le_F \frac{\nu_F Y_{O_u}}{W_O} [(1 - \phi) + 2\beta^{-1} Le_O \phi] & (\phi < 1) \\ Le_O \frac{\nu_O Y_{F_u}}{W_F} [(1 - \phi^{-1}) + 2\beta^{-1} Le_F \phi^{-1}] & (\phi > 1) \end{cases}$$

with $\phi = \nu Y_{F_u}/Y_{O_u}$ the equivalence ratio and $\nu = \nu_O W_O/\nu_F W_F$.

- Note that at stoichiometry ($\phi = 1$), the two expressions are identical and reduce to

$$S_L = \sqrt{\frac{\tilde{\rho}_b^2}{\tilde{\rho}_u^2} \frac{4B(\lambda/c_p)}{\beta^3 Le_F^{-1} Le_O^{-1}}} \frac{\nu_F Y_{O_u}}{W_O} e^{-E/2\mathcal{R}\tilde{T}_a}$$

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Some observations from the derived expressions

- The adiabatic flame temperature T_a exerts the strongest influence on S_L , so that reactions with larger values of heat release Q propagate faster flames.
- The flame speed S_L increases with increasing T_u .
- The flame gets thinner when increasing Q or T_u .
- The adiabatic flame temperature peaks at stoichiometry. The dependence $S_L \sim \sqrt{Le_F Le_O}$, however, suggests that there will be a shift in the peaking of the flame speed towards the lean or rich side, depending on the fuel. For hydrogen, for example, the peaking is towards the rich side because of the relatively low value of Le_F that is dominant under lean conditions.
- The dependence of S_L , and hence of l_f , on pressure depends on the reaction order;

$$S_L \sim \frac{\tilde{\rho}_b^{n/2}}{\tilde{\rho}_u} \sim P^{\frac{n}{2}-1} \quad l_f = \lambda / \rho_u c_p S_L \sim P^{-\frac{n}{2}}$$

generally $n < 2$ and S_L decreases slightly with pressure, but it is independent of pressure for a second order reaction

The flame thickness, however, always decreases with increasing pressure

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The flame thickness, however, always decreases with increasing pressure

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The strong dependence of S_L on T_a (the Lewis number for methane is near one)

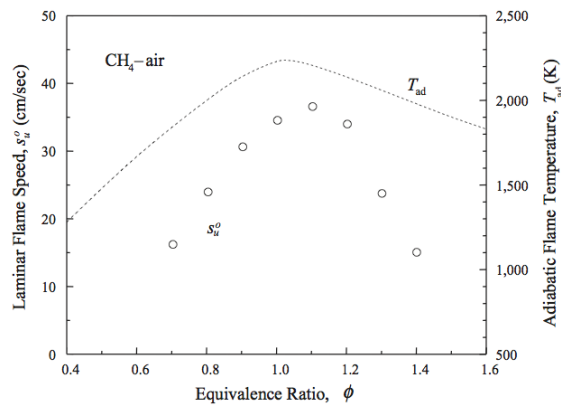
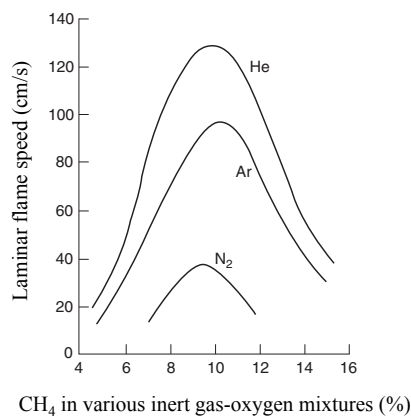


Figure 7.7.1. Calculated adiabatic flame temperatures and measured laminar flame speeds of atmospheric methane-air mixtures.

Law, 2006

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Clingman et al. 1953

All mixtures have the same heat release. Ar and N_2 have the same thermal diffusivity, but T_a is larger for the Ar mixture because Ar has a lower c_p (being a monatomic gas).

The flame temperature of the He mixture, also a monatomic gas, is nearly that of Ar, but the higher speed is due to the higher thermal diffusivity of He (higher λ and lower ρ).

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The dependence of $S_L \sim \sqrt{Le}$ is evident here because of the wide variations in Le , from 0.3 for lean to 2.3 for rich conditions.

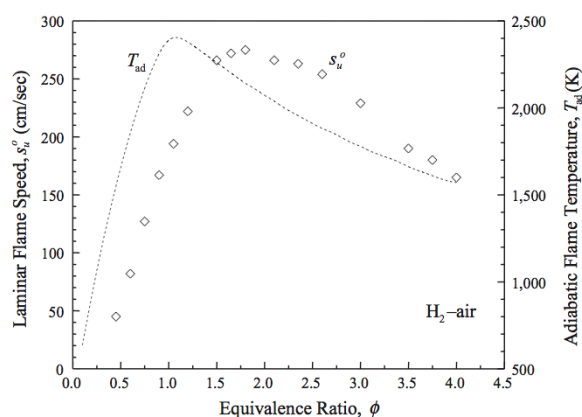


Figure 7.7.2. Calculated adiabatic flame temperatures and measured laminar flame speeds of atmospheric hydrogen-air mixtures.
Law, 2006

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The flame speed increases with increasing Q (through T_a)

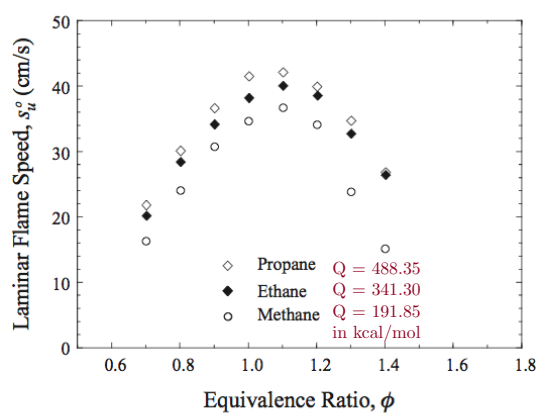


Figure 7.7.3. Measured laminar flame speeds of methane, ethane, and propane in air.
Law, 2006

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indistinguishable effect because of the relatively small variations in Q

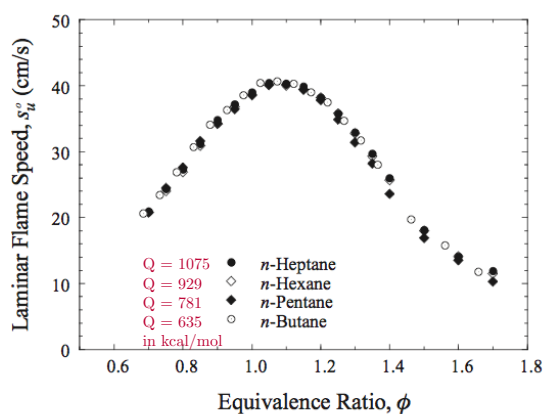
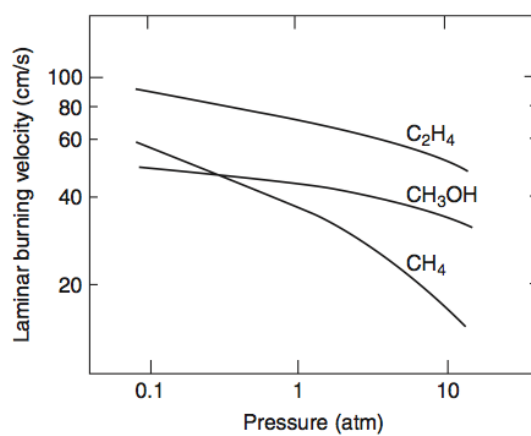


Figure 7.7.4. Measured laminar flame speeds of *n*-butane, *n*-pentane, *n*-hexane, and *n*-heptane in air. Law, 2006

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Westbrook and Dryer, 1984

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Summary and Generalization

The asymptotic treatment shows that the reaction zone, as seen from the relatively larger scale of the flame l_f , is a discontinuity across which the variables remain continuous but their derivatives jump. Furthermore, it states that the flux of reactants to the sheet (determined from the internal structure of the reaction zone) is an Arrhenius function of the flame temperature T_f .

In dimensional form (for reaction of order one)

$$\begin{aligned} [Y] &= [T] = 0 \\ Q \left[\rho \mathcal{D} \frac{\partial Y}{\partial n} \right] + \left[\lambda \frac{\partial T}{\partial n} \right] &= 0 \\ -\rho \mathcal{D} \frac{\partial Y}{\partial n} \Big|_{n=0^-} &= \frac{\lambda/c_p}{\sqrt{\rho \mathcal{D}}} \sqrt{2\rho_b B/\beta^2} Y_u e^{-E/2\mathcal{R}T_f} \end{aligned}$$

$[\cdot]$ denotes the jump across the reaction sheet located at $n = 0$,

$\partial/\partial n$ is the directional derivative along the normal to the sheet.

