Course Outline

A) Introduction and Outlook
B) Flame Aerodynamics and Flashback
C) Flame Stretch, Edge Flames, and Flame Stabilization Concepts
D) Disturbance Propagation and Generation in Reacting Flows

E) Flame Response to Harmonic Excitation

- Governing Equations
- Premixed Flame Dynamics
  - General characteristics of excited flames
  - Wrinkle convection and flame relaxation processes
  - Excitation of wrinkles
  - Interference processes
  - Destruction of wrinkles
- Non Premixed Flame Dynamics
- Global heat release response and Flame Transfer Functions
Flame Response to Harmonic Disturbances

- Combustion instabilities manifest themselves as narrowband oscillations at natural acoustic modes of combustion chamber.
Basic Problem

• Wave Equation:
  \[ p''_t - c^2 p''_{xx} = (\gamma - 1) q'_t \]

• Key issue – combustion response
  - How to relate \( q' \) to variables \( p', u' \), and etc., in order to solve problem
  - Focus of this talk is on sensitivity of heat release to flow disturbances
Response of Global Heat Release to Flow Perturbations

What factors affect slope of this curve (gain relationship)?

Why does this saturate? Why at this amplitude?

\[ \dot{Q}(t) = \int_{\text{flame}} \dot{m}_F^\prime \cdot h_R \, dA \]
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Analytical Tools/Governing Equations

- Work within fast chemistry, flamelet approximation and use G- and Z- equations to describe flame dynamics
Analytical Tools – Z Equation

- Key assumptions
  - Le=1 assumption
  - flame sheet at $Z=Z_{st}$ surface

- Imposed flow field
- Equal diffusivities

\[
\rho \frac{D Y_F}{Dt} - \nabla \cdot (\rho \mathcal{D}_F \nabla Y_F) = \dot{\omega}_F
\]

\[
\rho \frac{D(Y_{Pr}/(\nu + 1))}{Dt} - \nabla \cdot (\rho \mathcal{D}_{Pr} \nabla (Y_{Pr}/(\nu + 1))) = \frac{\dot{\omega}_F}{\nu + 1}
\]

Add these species equations:

\[
\rho \frac{D(Y_F + Y_{Pr}/(\nu + 1))}{Dt} - \nabla \cdot (\rho \mathcal{D} \nabla (Y_F + Y_{Pr}/(\nu + 1))) = 0
\]

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Analytical Tools – Z Equation

• Recall the definition of mixture fraction:

\[ Z = Y_F + \frac{1}{(\nu + 1)} Y_{pr} \]

• Yields:

\[ \rho \frac{DZ}{Dt} - \nabla \cdot (\rho \nabla Z) = 0 \]

\[ \frac{\partial Z}{\partial t} + \bar{u} \cdot \nabla Z = \nabla \cdot (\bar{u} \nabla Z) \]
Premixed Flame Sheets: G-Equation

Flame fixed (Lagrangian) coordinate system:

\[ \frac{D}{Dt} G(\bar{x}, t) \bigg|_{\text{at the flame front}} = 0 \]

Coordinate fixed (Eulerian) coordinate system:

\[ \frac{\partial G}{\partial t} + \vec{v}_F \cdot \nabla G = 0 \]
\[ \vec{n} = \nabla G / |\nabla G| \]

\[ \frac{\partial G}{\partial t} + \left( \vec{u} - s_d \frac{\nabla G}{|\nabla G|} \right) \cdot \nabla G = 0 \]

\[ \frac{\partial G}{\partial t} + \vec{u} \cdot \nabla G = s_d |\nabla G| \]
G-equation for single valued flame front

Two-dimensional flame front

Position is single valued function, \( \xi \), of the coordinate \( y \).

Define and substitute \( G(x, y, t) \equiv x - \xi(y, t) \)

\[
\frac{\partial \xi}{\partial t} - u_x + u_y \frac{\partial \xi}{\partial y} = -s_d \sqrt{1 + \left( \frac{\partial \xi}{\partial y} \right)^2}
\]
Governing Equations

• Left side:
  – Same convection operator
  – Wrinkles created on surface by fluctuations normal to iso- $G$ or $Z$ surfaces

• Right side:
  – Non-premixed flame – diffusion operator, linear
  – Premixed flame – flame propagation, nonlinear
  – Right side of both equations becomes negligible in $Pe = \frac{uL}{\mathcal{D}} > 1$ or $u/s_d > 1$ limits

\[
\frac{\partial Z}{\partial t} + \vec{u} \cdot \nabla Z = \nabla \cdot (\mathcal{D} \nabla Z)
\]

\[
\frac{\partial G}{\partial t} + \vec{u} \cdot \nabla G = s_d |\nabla G|
\]
• G-equation only physically meaningful at the flame surface, $G=0$
  – Can make the substitution,

$$G(x, y, z, t) = x - \xi(y, z, t)$$

• Z-equation physically meaningful everywhere
  – Cannot make analogous substitution
Governing Equations

- Reflects fundamental difference in problem physics
  - Premixed flame sheet only influenced by flow velocity at flame
  - Non-premixed flame sheet influenced by flow disturbances everywhere
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Excited Bluff Body Flames
(Mie Scattering)

\[ \frac{u'}{U_o} = 0 \]

\[ \frac{u'}{U_o} = 0.01 \]

\[ \frac{u'}{U_o} = 0.02 \]

\[ \frac{u'}{U_o} = 0.1 \]

Increased Amplitude of Forcing
Excited Swirl Flame (OH PLIF)
Excited Bluff Body Flames
(Line of sight luminosity)

18 m/s 294K

38 m/s 644K

127 m/s 644K

170 m/s 866K
Overlay of Instantaneous Flame Edges

18 m/s  294K
38 m/s  644K
127 m/s 644K
170 m/s 866K
Quantifying Flame Edge Response

$L'(x, f_0)$

Power Spectrum

Time Series

$L(x,t)$
Convective wavelength:

\[ \lambda_c = \frac{U_0}{f_0} \]

- distance a disturbance propagates at mean flow speed in one excitation period

Spatial Behavior of Flame Response

- Strong response at forcing frequency
  - Non-monotonic spatial dependence
1. Low amplitude flame fluctuation near attachment point, with subsequent growth downstream

2. Peak in amplitude of fluctuation, $L' = L'_{\text{peak}}$

3. Decay in amplitude of flame response farther downstream

4. Approximately linear phase-frequency dependence
Typical Results – Other Flames

- Magnitude can oscillate with downstream distance

50 m/s, 644K

1.8m/s, 150hz

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Analysis of Flame Dynamics

1. Wrinkle convection and flame relaxation processes
2. Excitation of wrinkles
3. Interference processes
4. Destruction of wrinkles
Level Set Equation for Flame Position

\[ S_L \cdot \vec{n}(x,t) = \vec{u}_f(x,t) \]

\[ L(x,t) \]

G-equation:

\[ \frac{\partial L}{\partial t} + \left( u_f \frac{\partial L}{\partial x} - v_f \right) = S_L \sqrt{1 + \left( \frac{\partial L}{\partial x} \right)^2} \]
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Wrinkle Convection

Model problem: Step change in axial velocity over the entire domain from $u_a$ to $u_b$, both of which exceed $s_d$:

$$u = \begin{cases} 
  u_a & t < 0 \\
  u_b & t \geq 0 
\end{cases}$$
Wrinkle Convection

- Flame relaxation process consists of a “wave” that propagates along the flame in the flow direction.

\[
\theta = \sin^{-1}\left(\frac{s_d}{u_a}\right)
\]

\[
\theta = \sin^{-1}\left(\frac{s_d}{u_b}\right)
\]
Harmonically Oscillating Bluff Body

Phase Characteristics of Flame Wrinkle

Convection speed of Flame wrinkle, $u_{c,f}$

$\neq$

Mean flow velocity, $u_0$

$\neq$

Disturbance Velocity, $u_{c,v}$

Harmonically Oscillating Bluff Body

- Linearized, constant burning velocity formulation:
  - Excite flame wrinkle with spatially constant amplitude
  - Phase: linearly varies

- Wrinkle convection is controlling process responsible for low pass filter character of global flame response
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Excitation of Wrinkles on Anchored Flames

\[ \frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t} \int_{0}^{x} \frac{\partial u_n'(t - \frac{x - x'}{u_t})}{\partial x} \, dx' + \frac{1}{u_t} \cdot u_n'(x = 0, t = t - \frac{x}{u_t}) \]

- Linearized solution of G Equation, assume anchored flame
- Wrinkle convection can be seen from delay term
Excitation of Flame Wrinkles – Spatially Uniform Disturbance Field

\[
\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t} \int_0^x \frac{\partial u'_n}{\partial x}(x',t - \frac{x - x'}{u_t}) \, dx' + \frac{1}{u_t} u'_n(x = 0, t = t - \frac{x}{u_t})
\]

- Wave generated at attachment point \((x=0)\), convects downstream
- If excitation velocity is spatially uniform, flame response exclusively controlled by flame anchoring “boundary condition”
  - Kinetic /diffusive/heat loss effects, though not explicitly shown here, are very important!
Near Field Behavior- Predictions

• Can derive analytical formula for nearfield slope for arbitrary velocity field:

\[
\frac{\partial |L'|}{\partial x} = \frac{1}{\cos^2 \theta} \frac{|u'_n|}{\overline{u}_t}
\]
Comparisons With Data

\[
\frac{1}{\cos^2 \theta} \frac{u'_n}{\overline{u}_t} = \frac{\partial |L'|}{\partial x}
\]

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Near Field Behavior

- Flame starts with **small amplitude** fluctuations because of attachment
  \[ L'(x=0, t) = 0 \]

- Nearfield dynamics are essentially **linear** in amplitude

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Excitation of Flame Wrinkles – Spatially Varying Disturbance Field

\[
\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t} \int_0^x \frac{\partial u'_n(x',t - \frac{x-x'}{u_t})}{\partial x} \, dx' + \frac{1}{u_t} \cdot u'_n(x=0, t = t - \frac{x}{u_t})
\]

- Flame wrinkles generated at all points where disturbance velocity is non-uniform, \( du'/dx \neq 0 \)
  - Flame disturbance at location \( x \) is convolution of disturbances at upstream locations and previous times

- Convecting vortex is continuously disturbing flame
  - Vortex convecting at speed of \( u_{c,v} \)
  - Flame wrinkle that is excited convects at speed of \( u_t \)

Model Problem: Attached Flame Excited by a Harmonically Oscillating, Convecting Disturbance

- Model problem: flame excited by convecting velocity field,

\[ \frac{u_n'}{u_{t,0}} = \varepsilon_n \cos(2\pi f (t - x / u_{c,v})) \]

- Linearized solution:

\[ \frac{\xi_1}{u_{t,0}/f} = \text{Real} \left\{ \frac{-i \cdot \varepsilon_n / \sin \theta}{2\pi \left( u_{t,0} \cos \theta / u_{c,v} - 1 \right)} \times \left[ e^{i2\pi f \left( y/(u_{c,v} \tan \theta) - t \right)} - e^{i2\pi f \left( y/(u_{t,0} \sin \theta) - t \right)} \right] \right\} \]
Solution Characteristics

• Note interference pattern on flame wrinkling

• Interference length scale:

\[
\lambda_{\text{int}} \left/ \left( \lambda_{\text{v}} \sin \theta \right) \right. = \frac{1}{|u_{\text{v}}/u_{\text{c,v}} - 1|}
\]
Interference Patterns

Comparison with Data

- Result emphasizes “wave-like”, non-local nature of flame response
- Can get multiple maxima/minima if excitation field persists far enough downstream

\[
x_{peak} / \lambda_c = \frac{\cos^2 \theta}{2 \cdot \frac{u_0}{u_{c,v}} \cos^2 \theta - 1}
\]

Aside: Randomly Oscillating, Convecting Disturbances

- Space/time coherence of disturbances key to interference patterns
- Example: convecting random disturbances to simulate turbulent flow disturbances
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Flame Wrinkle Destruction Processes: Kinematic Restoration

- Flame propagation normal to itself smoothes out flame wrinkles
- Typical manifestation: vortex rollup of flame
- Process is amplitude dependent and strongly nonlinear
  - Large amplitude and/or short length scale corrugations smooth out faster

Kinematic Restoration Effects: Oscillating Flame Holder Problem

\[ u_0 \varepsilon \sin(\omega_0 t) \]


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Kinematic Restoration Effects

- Leads to nonlinear farfield flame dynamics
- Decay rate is amplitude dependent

Numerical Calculation

Experimental Result

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Multi-Zone Behavior of Kinematic Restoration

- Near flame holder
  - Higher amplitudes and shorter wavelengths decay faster

- Farther downstream
  - Flame position independent of wrinkling magnitude
  - Flame position only a function of wrinkling wavelength
  - is determined by the leading points

Sung et al., Combustion and Flame, 1996

Flame Wrinkle Destruction Processes:
Kinematic Restoration

Flame Wrinkle Destruction Processes: Flame Stretch in Thermodydiffusively Stable Flames

|\tilde{\xi}(\tilde{\xi}, \omega_0)| \approx \exp(-\tilde{\sigma}\tilde{s}_{L,0}\tilde{\xi}) \approx \tilde{\sigma} \cdot \text{Normalized Markstein length}

Linear in amplitude wrinkle destruction process
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**Flame Geometry**

- **Conditions**
  - Over ventilated flame
  - Fuel & oxidizer forced by spatially uniform flow oscillations
  - Will show illustrative solution in $Pe >> 1$ (i.e., $W_{II}u_0 >> \mathcal{D}$) limit


Solution characteristics of $Z$ field

$$Z_1 = \sum_{n=1}^{\infty} \left[ \frac{i \varepsilon (A_n)^2 (2/n\pi) \sin(A_n)}{2\pi S_{tW} Pe} \right] \cos \left( A_n \frac{\nu}{W_{II}} \right) \exp \left( -A_n^2 \frac{x}{Pe W_{II}} \right) \left\{ 1 - \exp \left( 2\pi i S_{tW} \frac{x}{W_{II}} \right) \right\} \exp(-i\omega t)$$

$Z(x, \xi, t) = Z_{st}$
Solution characteristics of $\mathcal{Z}$ field

$$Z_1 = \sum_{n=1}^{\infty} \left[ \frac{i \varepsilon (A_n)^2 (2/n\pi) \sin(A_n)}{2\pi St_w Pe} \right] \cos \left( A_n \frac{y}{W_{II}} \right) \exp \left( -A_n^2 \frac{x}{Pe W_{II}} \right) \left\{ 1 - \exp \left( 2\pi i St_w \frac{x}{W_{II}} \right) \right\} \exp(-i\omega t)$$
Solution: Space-Time Dynamics of $\mathcal{Z}_{st}$ Surface

\[ \xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f} \sin\theta_0(x) \left\{ 1 - \exp\left(i2\pi f \frac{x}{u_{x,0}} \right) \right\} \exp[-i2\pi f \cdot t] \]

Flame wrinkling only occurs through velocity fluctuations normal to flame

Low pass filter characteristic

Flame wrinkles propagate with axial flow (cause interference)
Illustrative Result of Flame Front Dynamics

\[
\frac{\text{Convective wavelength } \left( \frac{u_{x,0}}{f} \right)}{\text{Flame length } \left( L_f \right)} = 3.3
\]
Illustrative Result of Flame Front Dynamics

\[
\frac{\text{Convective wavelength} \ (u_{x,0} / f)}{\text{Flame length} \ (L_f)} = 3.3
\]

Oxidizer

\[\rightarrow\]

Fuel

\[\rightarrow\]

\[L_f\]
Illustrative Result of Flame Front Dynamics

\[ \frac{\text{Convective wavelength} \left( \frac{u_{x,0}}{f} \right)}{\text{Flame length} \left( L_f \right)} = 0.5 \]

Oxidizer

\[ L_f \]

Fuel
Illustrative Result of Flame Front Dynamics

\[
\frac{\text{Convective wavelength} \left( \frac{u_{x,0}}{f} \right)}{\text{Flame length} \left( L_f \right)} = 0.5
\]
Comparison - similarities

• Non-premixed

\[ \xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f} \sin \theta(x) \left[ 1 - \exp \left( i2\pi f \frac{x}{u_{x,0}} \right) \right] \exp[-i2\pi ft] \]

• Premixed

\[ \xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f} \sin \theta \cdot \left[ 1 - \exp \left( i2\pi f \frac{x}{u_{x,0} \cos \theta} \right) \right] \exp[-i2\pi ft] \]

Similarities between space/time dynamics of premixed and non-premixed flames responding to bulk flow perturbations

> Magnitude
> Flame Angle
> Wave Form
Comparison - difference

- Non-premixed

\[ \xi_{1,n}(x,t) = \frac{i \varepsilon u_{x,0}}{2 \pi f} \sin \theta(x) \left\{ 1 - \exp \left( i2\pi f \frac{x}{u_{x,0}} \right) \right\} \exp[-i2\pi ft] \]

- Premixed

Convective wave speeds

\[ \xi_{1,n}(x,t) = \frac{i \varepsilon u_{x,0}}{2 \pi f} \sin \theta \cdot \left\{ 1 - \exp \left( i2\pi f \frac{x}{u_{x,0} \cos \theta} \right) \right\} \exp[-i2\pi ft] \]
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Spatially Integrated Heat Release

• Unsteady heat release

\[ \dot{Q}(t) = \int_{\text{flame}} \dot{m}_F' \kappa_R \, dA \]

  – Flame surface area (Weighted Area)
  – Mass burning rate (MBR)
  – We’ll assume constant composition

• Flame describing function:

\[ \mathcal{F} \equiv \frac{\ddot{Q}_1 / \dot{Q}_0}{\tilde{u}_{x,1} / u_{x,0}} = \mathcal{F}_{WA} + \mathcal{F}_{MBR} \]

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Premixed Flames

- Spatially integrated heat release:

\[ \dot{\dot{Q}}(t) = \int_{\text{flame}} \rho^u s_c^u h_R dA \]

- Linearized for constant flame speed, heat of reaction, and density:

\[ \frac{\dot{Q}_1(t)}{\dot{Q}_0} = \int_{\text{flame}} \frac{dA}{A_0} \quad \text{Proportional to flame area} \]

\[ \frac{A(t)}{A_o} = \sin \theta \int_{\text{flame}} W(y) \sqrt{1 + \left( \frac{\partial \xi}{\partial y} \right)^2} \, dy \]
• $W(y)$ is a geometry dependent weighting factor:

\[
W(y) = \frac{1}{W_f} \quad 2\pi \left( W_f - y \right) / \left( \pi W_f^2 \right) \quad 2\pi y / \left( \pi W_f^2 \right)
\]

where: $W_f = L_F \tan \theta$
Premixed Flame TF Gain – Bulk Flow Excitation

- $St << 1$: $\mathcal{F} = 1$
- $St >> 1$: $\mathcal{F} \sim 1/\text{St}$
Why the 1/St Rolloff?

• Flame position ~1/St
  \[ \xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f} \sin \theta \left\{ 1 - \exp \left( i2\pi St_f \frac{x}{L_{f,0}} \right) \right\} \exp[-i2\pi ft] \]
  Low pass filter characteristic!

• Flame area/unit axial distance:
  \[ dA = \sqrt{1 + \left( \frac{\partial \xi}{\partial x} \right)^2} \, dx \]

• Linearized:
  \[ \frac{dA}{dx} = \sqrt{1 + \left( \frac{\partial \xi_0}{\partial x} \right)^2 + \frac{\partial \xi_0}{\partial x} \frac{\partial \xi_1}{\partial x}} \propto \varepsilon \sin \theta \exp \left( i2\pi St_f \frac{x}{L_{f,0}} \right) \exp[-i2\pi ft] \]
Why the 1/St Rolloff?

- Consider spatial integral of traveling wave disturbance:

\[
\int_{x=0}^{L_F} \cos \left[ \omega \left( t - \frac{x}{u} \right) \right] dx = -\frac{u}{\omega} \left\{ \sin \left[ \omega \left( t - \frac{L_F}{u} \right) \right] - \sin[\omega t] \right\}
\]

  Traveling Wave

  1/St due to interference effects associated with tangential convection of wrinkles

- 1/St comes from the integration!
Premixed Flame Response - Phase

- Phase rolls off linearly with St (for low St values)
  - Time delayed behavior
- 180° phase jumps at nodal locations in the gain
Premixed Flame Response - Phase

Flame area-velocity relationship for convectively compact flame (low St values):

\[
\frac{A_1(t)}{A_0} = n \frac{u_1(t - \tau)}{u_0}
\]

\[\tau = C \frac{L_f}{u_0}\]

Axi-symmetric Wedge: \[C = \frac{2(1 + k_c^{-1})}{3 \cos^2 \theta}\]

Axi-symmetric Cone: \[C = \frac{2(k_c + 1)}{3k_c \cos^2 \theta}\]

Two-dimensional: \[C = \frac{(k_c + 1)}{2k_c \cos^2 \theta}\]
Nonpremixed Flames-Bulk Flow Excitation

• Returning to spatially integrated heat release:

\[
\dot{Q}(t) = \int \dot{m}_F'' h_R^{' \prime} dA
\]

• Linearize the MBR and area terms:

\[
\frac{\dot{Q}(t)}{h_R^{' \prime}} = \int \dot{m}_F'' \, dA_0 + \int \dot{m}_F'' \, dA_1 + \int \dot{m}_F'' \, dA_0
\]

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Non-Premixed Flames: Role of Area Fluctuations

\[ F_{WA} = \frac{\int_{\text{flame}} (m''_F)_0 \, dA_1}{\int_{\text{flame}} (m''_F)_0 \, dA_0} \]

Very strong function of \( x \)!

For the higher velocity,
- Area increases \( \Rightarrow \) Premixed
- Weighted area decreases \( \Rightarrow \) Non-premixed
Weighted Area cont’d

At low frequencies

- Non-premixed
  - Weighted Area

- Premixed
  - Area (as weighting is constant)

At low frequencies, area and weighted area are out of phase
Mass Burning Rate

\[ MBR = \frac{\int (m''_F)_1 \, dA_0}{\int (m''_F)_0 \, dA_0} \]

- Non-premixed: \( (m''_F)_1 \sim \frac{1}{\cos \theta} \frac{\partial Z_1}{\partial y} \)
  - Fluctuations in spatial gradients of the mixture fraction

- Premixed: \( (m''_F)_1 \sim \frac{\partial s_L}{\partial \phi} \)
  - Stretch sensitivity of the burning velocity

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Significant differences in dominant processes controlling heat release oscillations

- Non-premixed: Mass burning rate
- Premixed: Area

Comparisons of Gain and Phase of FTF

**Gain**

\[ \frac{1}{\sqrt{St}} \]

\[ F_{\text{Non-premixed}} \]

\[ 1/ St \]

\[ F_{\text{Premixed}} \]

(weak flame stretch)

**Phase**

\[ \angle F_{\text{Non-premixed}} \]

\[ \angle F_{\text{Premixed}} \]

(weak flame stretch)

\[ St \ll 1 \quad : \sim 1 \]

\[ St \gg 1 \quad : \text{Non-premixed flames } \sim 1/St \]

\[ St \sim O(1) \quad : \text{Non-premixed flame } \sim 1/St^{1/2} \quad > \quad \text{Premixed } \sim 1/St \]

- At \( St \sim 0(1) \), non-premixed flames are more sensitive to flow perturbations

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Premixed Flame TF’s: More Complex Disturbance Fields

Disturbance convecting axially at velocity of $U_c$ & $k_c = U_o / U_c$

Axisymmetric wedge flame:

$$\frac{v_{F,n1}(x,y,t)}{u_{x,0}} \bigg|_{x=\xi(y,t)} = \varepsilon_n \cos(2\pi f(t - x / u_c)) \bigg|_{x=\xi(y,t)}$$

- Gain
  - $\text{fcn}(\text{St}_2, k_c)$
  - Unity at low $\text{St}_2$
  - Gain increases greater than unity
  - "Nodes" of zero heat release response
Closing Remarks

• Flame response exhibits “wavelike”, non-local behavior due to wrinkle convection, leading to:
  
  • maxima/minima in gain curves, interference phenomenon, etc.
  • 1/f behavior in transfer functions

• Premixed flame wrinkles controlled by different processes in different regions

• Role of area, weighted area, mass burning rate are quite different for premixed and non-premixed flames
Summary

• Many great fundamental problems in gas turbine combustion
  – Edge flames
  – Reacting swirl flows
  – Differential diffusion effects on turbulent flames
  – Turbulent flames in highly preheated flows
  – Response of flames to harmonic disturbances
  – …and many more!!