**Combustion dynamics**

**Lecture 5a**

S. Candel, D. Durox, T. Schuller

CentraleSupélec
Université Paris-Saclay, EM2C lab, CNRS

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**Intrinsic flame instabilities will not be considered**

- Darrieus-Landau instabilities
- Thermo-diffusive instabilities

- Markstein (1964)
- Clavin et al. (1990)
- Buckmaster and Ludford (1982)
- Sivashinski (1976…)
- Law (2008)
- Clanet & Searby (1998)
- Searby et al. (2001)

Growth rates are relatively weak: in many applications (but not all!) other mechanisms dominate.

Exceptions: Oxy-fuel welding torch is notably influenced by D-L instability.
First example

During unstable regime, walls are “breathing” highlighting large pressure oscillations within the combustion

Second example

Stable regime: the combustion zone (luminous zone) features small stochastic fluctuations around its mean location (effects of turbulence). Radiated noise is weak and broad band: “combustion roar”.

Unstable regime: Large synchronized motion featuring peak noise emission. Intensification of luminosity near the wall: higher heat fluxes to the boundary. Induces flame flashback.

Azimuthally coupled instabilities in an annular combustor equipped with 16 matrix burners

Third example

Standing mode @ $f=380$ Hz

Spinning mode @ $f=498$ Hz


Acoustically induced combustion Instabilities *(thermo-acoustic instabilities)*

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Flowrate disturbances

Experiments with a fixed mixture composition when the flame is submitted to harmonic flowrate modulations.

\[ \frac{\dot{Q}_1}{\dot{Q}_0} = \int_A dA_1(\mathbf{K} \cdot \mathbf{v}) \]

Flame surface wrinkles produce heat release rate disturbances

Methane/air

\[ \phi = 0.95, \frac{u}{u'} \sim 0.3, \ u \sim 1 \text{ m/s} \]

Equivalence ratio inhomogeneities

\[ \tau_i + \tau_{conv} + \tau_c = \frac{(2n - 1)T}{2} \]

Combustion dynamics of flames interacting with equivalence ratio perturbations

Conical flame perturbed by equivalence ratio modulations

Velocity field induced on the upstream side of the flame by interactions with equivalence ratio modulations

Inverted flame perturbed by equivalence ratio oscillations

Premixed flow with equivalence ratio perturbations

$$\phi(t) = \phi_0 + \phi_1 \sin \omega t$$  $\phi_0 = 0.8$ and $\phi_1 = 0.1$

Harmonic mixture composition oscillations are convected and wrinkle the flame (no acoustic forcing):
(1) Fluctuations in the burning rate
(2) Flame surface area disturbances
(3) Feedback on the flow field

Result in large heat release rate oscillations (nonlinear)

Heat release rate fluctuations

Volumetric heat release rate controlled by the fuel supply
Only lean flames are considered

\[
\dot{Q} = \int_A Y_F \rho S_d dA(\phi, v)(-\Delta h_f^0)
\]

\[
\dot{m}_f = Y_F \rho S_d dA(\Phi, v)
\]

- Fuel heating value (J/kg) is constant
- equivalence ratio
- stretch effects

\[
\frac{\dot{Q}_1}{\dot{Q}_0} = \frac{\int \dot{m}_f_1(\phi, \epsilon)dA_0}{\dot{m}_f_0 \int dA_0} + \frac{\int dA_1(\Phi, v)}{\int dA_0}
\]

Heat release rate fluctuations
Mass burning rate fluctuations
averaged over the flame surface area

Fuel mass burning rate oscillations

Mixture composition disturbances lead to fuel mass burning rate perturbations

\[
\frac{\int \dot{m}_f_1(\phi, \epsilon)dA_0}{\dot{m}_f_0 \int dA_0} = \frac{m(\phi_0) \int \phi_1 dA_0}{\phi_0 \int dA_0}
\]

\[
\dot{m}_f = Y_F \rho S_d
\]

\[
m(\phi_0) = \left[ \frac{\partial(\rho/\rho_0)}{\partial(\phi/\phi_0)} + \frac{\partial(Y_f/Y_{f0})}{\partial(\phi/\phi_0)} + \frac{\partial(S_L/S_{L0})}{\partial(\phi/\phi_0)} \right]_{\phi=\phi_0}
\]

\[
m(\phi_0) \approx 1 + a
\]

Example: CH$_4$/air

\[
\phi_0 = 0.8 \quad a = \frac{\partial(S_L/S_{L0})}{\partial(\phi/\phi_0)} = 2.30
\]

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Fluctuations of the flame displacement speed

Mixture composition oscillations

\[ \phi = \phi_0 + \phi_1(t) \]
\[ \phi_1(t) = \Phi \exp(-i\omega t) \]

The burning velocity describes (twisted) cycles around steady conditions for increasing modulation frequencies.

Lauvergne & Egolfopoulos (2000)

Response of lean premixed flames to mixture composition disturbances

Heat release rate fluctuations

\[ \frac{\dot{Q}_1}{\dot{Q}_0} = \frac{m(\phi_0)}{\phi_0} \int \frac{\phi_1}{dA_0} \int dA_0 \]

Mass burning rate fluctuations averaged over the flame surface area

\[ m(\phi_0) \simeq 1 + a \]

Flame surface area fluctuations

Heat release rate fluctuations result from fuel mass burning rate (function of \( \phi \) and mean flame surface area) and flame surface area fluctuations (function of \( \phi \), \( \nu \) and mean flame surface area).
Flame surface area fluctuations

Kinematic description of the flame sheet (G=0) separating the burnt gases (G>0) from the fresh reactants (G<0)

\[ \frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d |\nabla G| \]

\[ S_d(\phi, \epsilon, \kappa, t) \simeq S_L(\phi) \]

<table>
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<th>Flame type</th>
<th>Some references</th>
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<td>V-flames</td>
<td>Boyer &amp; Quinard (1990), Dowling (1999), Schuller et al. (2003)</td>
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<td>Swirling flames</td>
<td>Palies et al. (2011), Acharya et al. (2012)</td>
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The G-equation

The flame is described by a level set. One level corresponds to the flame position

\[ G(x, t) = G_0 \]

This expression may be differentiated with respect to time

\[ \frac{dG(x, t)}{dt} = \frac{\partial G}{\partial t} + \mathbf{w} \cdot \nabla G = 0 \]

\[ \mathbf{n} = -\nabla G / |\nabla G| \]
Using the previous expressions for the normal and the absolute flame velocity one obtains

\[
\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d |\nabla G|
\]

This expression can now linearized by introducing small perturbations around the mean value

\[
G = G_0 + G_1 \\
\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1 \\
S_d = S_{d0} + S_{d1}
\]

Retaining only terms up to first order one finds that

\[
\left| \nabla G_0 + \nabla G_1 \right| = \left| \nabla G_0 \right| + \mathbf{n}_0 \cdot \nabla G_1
\]

\[
\frac{\partial G_1}{\partial t} + \mathbf{v}_0 \cdot \nabla G_0 + \mathbf{v}_1 \cdot \nabla G_0 + \mathbf{v}_0 \cdot \nabla G_1 = (S_{d0} + S_{d1})(|\nabla G_0| + \mathbf{n}_0 \cdot \nabla G_1)
\]

(1) - Transport equation for the mean \( G_0 \) field

\[
\mathbf{n}_0 = -\frac{\nabla G}{|\nabla G|} \\
\mathbf{v}_0 \cdot \nabla G_0 = S_{d0}|\nabla G_0|
\]

\[
S_{d0} - \mathbf{v}_0 \cdot \mathbf{n}_0 = 0
\]
(2) -Transport equation for the perturbed $G_1$ field

$$\frac{\partial G_1}{\partial t} + v_1 \cdot \nabla G_0 + v_0 \cdot \nabla G_1 = S_{d0} n_0 \cdot \nabla G_1 + S_{d1} |\nabla G_0|$$

which may be rearranged in the form

$$\frac{\partial G_1}{\partial t} + (v_0 - S_{d0} n_0) \cdot \nabla G_1 = -v_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0|$$

Making use of the result obtained at zero-th order one may write

$$v_0 - S_{d0} n_0 = v_0 - (v_0 \cdot n_0)n_0 = v_{0t}$$

This is the velocity vector projected on the plane tangent to the flame

$$\frac{\partial G_1}{\partial t} + v_{0t} \cdot \nabla G_1 = -v_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0|$$

Recalling that $S_{d0} = v_0 \cdot n_0$

The right hand side of the previous equation may be written in the form

$$-v_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0| = (v_1 - \frac{S_{d1}}{S_{d0}} v_0) \cdot n_0 |\nabla G_0|$$

One obtains in this way the following equation

$$\frac{\partial G_1}{\partial t} + v_{0t} \cdot \nabla G_1 = (v_1 - \frac{S_{d1}}{S_{d0}} v_0) \cdot n_0 |\nabla G_0|$$

Burning velocity and velocity perturbations generate disturbances of the flame position in the normal direction, which are then convected along the flame front by the component of the mean local flow velocity $v_{0t}$ parallel to the mean flame front.
Equivalence ratio perturbations

Equivalence ratio perturbations induce changes in the local displacement velocity such that

\[ S_d = S_{d0} + S_{d1} \]

\[ S_{d1} = S_{d0} (1 + a \frac{\phi_1}{\phi_0}) \quad \text{where} \quad a = \frac{\phi_0}{S_{d0}} \left( \frac{\partial S_d}{\partial \phi} \right)_{\phi=\phi_0} \]

(1) - Transport equation for the mean \( G_0 \) field

\[ n_0 = -\frac{\nabla G_0}{|\nabla G_0|} \quad \mathbf{v}_0 \cdot \nabla G_0 = S_{d0} |\nabla G_0| \]

\[ S_{d0} - \mathbf{v}_0 \cdot n_0 = 0 \]

Perturbed flame position

(2) - Transport equation for the perturbed \( G_1 \) field

\[ \frac{\partial G_1}{\partial t} + \left[ \mathbf{v}_0 - S_{d0} \frac{\nabla G_0}{|\nabla G_0|} \right] \cdot \nabla G_1 = -\mathbf{v}_1 \cdot \nabla G_0 + aS_{d0} |\nabla G_0| \frac{\phi_1}{\phi_0} \]

\[ \frac{\partial G_1}{\partial t} + \mathbf{v}_0^t \cdot \nabla G_1 = \left( \mathbf{v}_1 - a \frac{\phi_1}{\phi_0} \mathbf{v}_0 \right) \cdot n_0 |\nabla G_0| \]

\[ \mathbf{v}_0^t = \mathbf{v}_0 - (\mathbf{v}_0 \cdot n_0) n_0 \quad \text{mean flow velocity parallel to the mean flame front}. \]

Composition and velocity disturb the flame position in the normal direction. These disturbances are convected along the flame front by the component of the mean local flow velocity \( \mathbf{v}_{dl} \) parallel to the mean flame front.
Inclined flames

Many flames are stabilized in a flow featuring a principal direction. The flame sheet then forms an angle with this direction.

![Image of inclined flames](image)

Application to inclined flames

The flame motion is easier to analyze in a reference frame attached to the flame front

\[ G(X, Y; t) = Y - \xi(X; t) = 0 \]

\[ \frac{\partial \xi}{\partial t} + U_0 \frac{\partial \xi}{\partial X} = V_1(X; t) \]

\[ \xi(0, t) = \xi_0(t) \]

- \( \xi \): normal flame front displacement with respect to the mean position
- \( U_0 \): mean flow velocity along the flame front
- \( V_1 \): velocity fluctuation normal to the flame front
- \( \xi_0 \): normal flame front displacement at the flame base

Normal flame displacement

Solution for normal flame sheet displacement

\[ \xi(X, t) = \frac{1}{U_0} \int_0^X V_1 \left( X', t - \frac{X - X'}{U_0} \right) dX' + \xi_0 \left( t - \frac{X}{U_0} \right) \]

Perturbed velocity field contribution  
Anchoring point dynamics

Interference integral for flame front perturbations  
Boyer & Quinard (1990), Schuller et al. (2003),  
Lee & Lieuwen (2009), Borghesi et al. (2009)

Wrinkles (i.e. normal flame displacement \( \xi(X, t) \) ) appear as convected by the mean flow with a wavelength \( \lambda = U_0/f \).

This convective wave is modulated by a complex amplitude given by the integral term.

Anchoring point dynamics

Ring modulation and acoustic waves produce the same type of wrinkles along the flame front

Vibrating rod  
Acoustic modulation  
Ring modulation  
Acoustic modulation

Petersen & Emmons (1961)  
Boyer & Quinard (1996)  
Kornilov et al. (2007)
Impact of flow perturbation model

Compact flame submitted to low frequency acoustic forcing

\[ \lambda = \frac{c}{f} \gg L \]

Forcing conditions:
\( f = 62.5 \text{ Hz}, \frac{\nu'}{\nu_0} = 0.20 \)

Operating conditions
\( \Phi = 1.05, \nu_0 = 0.97 \text{ m/s} \)

--- Predictions with a uniform flow modulation
--- Predictions with a convective wave perturbation

\( \Phi = 0.85, \nu_0 = 5.2 \text{ m/s}, \nu' = 1.35 \text{ m/s}, f = 500 \text{ Hz} \)

Incident flow perturbations determine the time lag of the flame response

\( kL \sim 1 \quad k = \frac{\omega}{u_0} \)

\[ u_1 = \bar{u}_1 \cos(\omega t - ky) \]

\[ \nabla \cdot \mathbf{v}_1 = 0 \]

Flow perturbations are convected by the mean flow and are of hydrodynamic type (incompressible)

De Soete (1964), Baillot et al. (1992), Schuller et al. (2002), Birbaud et al. (2006), Noiray et al. (2006)
The Flame Transfer Function (FTF) describes the flame frequency response in terms of heat release rate disturbances due to the acoustic forcing. The objective is to decouple the analysis of flow and combustion dynamics (nonlinear problem) from the analysis of the combustor acoustics (linear problem).

The FTF or Flame Impulse Response (FIR) are determined from DNS, LES, low-order models or experiments.

\[ \frac{\dot{Q}_1}{Q_0} = F_v \frac{v_1}{v_0} + F_\phi \frac{\phi_1}{\phi_0} \]

- DNS, LES @ single frequency & level
- Simulations with S.I. tools
- Low order models
- Experiments
Premixed conical flame FTF submitted to harmonic incoming velocity perturbations in a CH4/air mixture

Experimental FTF determination

Mixture kept at constant equivalence ratio
Modulation level kept constant

\[ \Phi = 0.95, \quad v_0 = 1.20 \text{ m/s}, \quad v_{1\text{rms}} = 0.19 \text{ m/s}, \quad v_{1\text{rms}}/v_0 = 0.16 \]

The velocity input is harmonic and the flame response (heat release rate fluctuation) remains also harmonic at these two forcing frequencies
FTF determination

The FTF is deduced from cross-spectral power analysis of the input and output signals

\[ F_v(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} \]

- Harmonic velocity disturbances @ \( \omega \)
- Heat release rate disturbances
- Cross-power spectral density of \( x(t) \) and \( y(t) \) examined at the forcing frequency
- Power spectral density of \( x(t) \) examined at the forcing frequency

A periodogram method helps to improve the signal to noise ratio. Statistical convergence requires a large number of periods (typically more than 100).

Conical Flame Transfer Function

\[ \frac{\dot{Q}_1}{Q_0} \approx \frac{I_1}{I_0} = F_v \frac{v_1}{v_0} \]

- Gain:
  - relative fluctuation amplitude
  - G>1 amplification
  - G<1 attenuation
  - Low pass filter

- Phase:
  - time lag \( \varphi = \omega T \)
  - convective at low frequencies
  - saturation at high frequencies

\( \Phi = 1.05, \ v_0 = 1.20 \text{ m/s}, \ v_{1\text{rms}} = 0.19 \text{ m/s} \)
Acoustic forcing synchronizes large vortices generated in shear layers that are responsible of rapid flame surface destruction when impacting the flame periphery.

Durox et al. (2005)

The flame front motion is controlled by the shear layer dynamics. The time lag corresponds to the travel time taken by a vortex to impinge the flame front (convected at about 0.5 $V_{\text{max}}$). This time lag is barely affected by the input level.
Vortex / flame interactions

$f = 70 \text{ Hz}$

Vortex / flame interactions

$f = 150 \text{ Hz}$
The perturbation level modifies the response of the flame operated at the same conditions $v_{rms} = 0.14 \text{ m/s}$ and $v_{rms} = 0.38 \text{ m/s}$.

When the perturbation level increases saturation occurs: energy is transferred to higher harmonics and the gain examined at the forcing frequency drops.

\[ \frac{Q_1}{Q_0} \approx \frac{I_1}{I_0} = F_v \frac{v_1}{v_0} \]

\[ F_v = G(\omega) \exp(i\varphi) \]

**Gain:**
- relative fluctuation amplitude
- Large overshoot $G>1$
- Gain reduces with increasing $v_1$
- Low pass filter

**Phase:**
- time lag $\varphi = \omega \tau$
- convective time lag independent of the input level

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Durox et al. (2005)