Self-induced instability of a premixed jet flame impinging on a plate

The Rayleigh criterion is often satisfied

\[ \int_T p'(t)q'(t)dt > 0 \]

Match of frequencies and proper phase difference

Instabilities
Instabilities are also observed in the case of unconfined flames

The coupling between acoustics and combustion differs from the case of confined flames and it is not well understood.

**Experimental set-up**

Outlet diameter : 22 mm \(L = 100, 164\) or 228 mm  
\(\text{CH}_4\) - air. Equivalence ratio : 0.95  
Mean flow velocities : 1.20, 1.44 or 1.68 m/s
Zhang & Bray classification (1999)

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<th>Cool central core flame</th>
<th>Envelope flame</th>
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Sound pressure level (dB) vs. Burner to plate distance (mm)

- L = 100 mm
- $\bar{V}_1 = 1.44 \text{ m/s}$

Steady operation
Unsteady operation

Sound Pressure level (dB)

$L = 100 \text{ mm}$

$\frac{v_1}{\bar{v}} = 1.44 \text{ m/s}$

$Z = 8.6 \text{ mm Medium tube}$

Power Spectrum (dB)

$Z = 8.6 \text{ mm Medium tube}$

Unsteady operation
Evolution of the fundamental frequency sawtooth pattern.

In this case, the frequency is around 200Hz when the sound pressure level is highest.
Velocity on the axis, at 1.5 mm above the nozzle exit and heat release detected by the PM.
At \( z = 8.6 \) mm, the instability is strong.

\[
\text{Acoustic response of the burner}
\]

\[
\omega_0^2 = \left( \frac{c^2 S_1}{L_{\text{eff}}} \right) / (V L_{\text{eff}})
\]

\[
L_{\text{eff}} = \int_{\text{in}}^{\text{out}} \frac{S_1}{S(z)} \, dz + \delta_e
\]

Helmholtz resonator
bulk oscillation
inside the burner

\( f_o = 202 \) Hz
Instability model: driven Helmholtz resonator

\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + kv_1' = -S_1 \frac{dp_1'}{dt} \]

- \( M = \bar{\rho} S_1 L_{eff} \) Effective mass of air in the pipe
- \( R = \bar{\rho} S_1 v_1 \) System damping
- \( k = \bar{\rho} c^2 S_1^2 / V \) Gas volume stiffness

The resonator is driven by external pressure fluctuations \( p_1' \) at the burner outlet

Instability model: origin of external pressure fluctuations - Model of Price et al. (1969)

\[ p(r, t) = \frac{\rho_0}{4\pi r} \left( \frac{\rho_u}{\rho_b} - 1 \right) \left[ \frac{d\dot{Q}}{dt} \right] t - \tau_a \]

\[ \dot{Q} \propto I \text{ or } A \]

- \( I \) is the light intensity emitted by free radicals
- \( A \) is the flame surface area
- \( \tau_a \) time delay between the source and the measurement point
Schuller et al. (2001)

Impinging flame with acoustic forcing of the flow

\[ p'(r, t) = K(r) \left[ \frac{dA'}{dt} \right]_{t=\tau_a} \]

The acoustic pressure and the time derivative of the heat release are similar, confirming that the source is suitably identified.

Z = 8.6 mm, \( L = 100 \text{mm}, \overline{V}_1 = 1.68 \text{ m/s} \)
Velocity perturbations are convected by the mean flow along the flame front.

Fluctuations of the flame surface $A(t)$ are induced by these velocity perturbations after a convective delay $\tau_c$.

$\tau_c$ is of the order of $z/v_1$.

$$A'(t) = n(v'_1)t - \tau_c$$

$n$ characterizes the coupling between the flame surface fluctuation and the velocity perturbation.

The phase difference between the velocity and the heat release yields a mean convective time. It is of the order of $z/\bar{v}_1$. 

Transfer function PM - LDV

Z = 8.6 mm

Amp Syy/Sxx

Phase (deg)

Freq (Hz)

0 100 200 300 400 500

-180 -90 0 90 180

-180 -90 0 90 180
\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + kv_1' = -S_1 \frac{dp_1'}{dt} \]

\[ p'(r, t) = K(r) \left[ \frac{dA'}{dt} \right]_{t-\tau_a} \]

\[ A'(t) = n(v_1')_{t-\tau_c} \]

\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + kv_1' = -S_1 K(r_{21}) n \left[ \frac{d^2 v_1'}{dt^2} \right]_{t-\tau} \]

with \( \tau = \tau_a + \tau_c \)

\[ \frac{d^2 v_1'}{dt^2} + 2\delta \omega_0 \frac{dv_1'}{dt} + \omega_0^2 v_1' = -N \left[ \frac{d^2 v_1'}{dt^2} \right]_{t-\tau} \]

where \( \delta \omega_0 = R/(2M) \quad N = S_1 K(r_{21}) n/M \)

\[ \tau_a \ll \tau_c \quad \Rightarrow \quad \tau \approx \tau_c \]

This equation has a solution at the resonant frequency \( f_0 \) if:

\[ \omega_0 \tau = (4m - 1) \pi / 2 \quad \text{where} \ m = 1, 2, ... \]

This imposes a condition on the convective delay:

\[ \omega_0 \tau_c \approx \omega_0 \tau = 3\pi / 2 \quad \text{(modulo } 2\pi) \]
\[ \omega_0 \tau_c = \frac{3\pi}{2} \pmod{2\pi} \Rightarrow f_0 \tau = \frac{3\pi}{4}, \frac{7\pi}{4}, \]
Conclusions

- Strong instabilities may be induced when a premixed flame anchored on a burner rim impacts on a plate facing the burner exhaust.
- In this study the burner behaves like a Helmholtz resonator.
- The frequency of oscillation evolves with the burner to plate separation around the fundamental resonance frequency.
- Sudden annihilation of flame surface area produces an intense source of sound.
- Flame wall interactions could play a role in the development of combustion instabilities.
- Even without a plate, if flame surface variations are important and fast, and if the sound influences the flow velocity, then an instability can be triggered.

Phase difference between the heat release and the velocity.

When the sound level is the highest, the phase difference is close to $\pi/2$. Then the heat release lags the velocity with a delay corresponding to $3\pi/2$. 