Structure and Propagation of Turbulent Premixed Flames

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Outline

i. Introduction

ii. Regime Diagrams

iii. Evolution of Flame Surfaces in Moderate Turbulence

iv. Local Flame Speed and Structure in Moderate and Intense Turbulence

v. Turbulent Flame Speed

vi. Turbulence-DL Instability Interaction
Three faces of a flame
Turbulent Premixed Flames in Engineering Devices

Turbulent premixed combustion

- SI engines
- Gas turbine engines: aircrafts (partially premixed) and stationary power systems
- Industrial gas burners
- Vapor cloud explosions
- Supernova Ia

DNS to study interaction of turbulence with freely propagating premixed flame

[2] Video Courtesy: Prof. Hong Im
Comparison of characteristic length-scales and time-scales in turbulent flow with the corresponding scales of chemical reaction and laminar flame

Comparison of scales helps in assessing whether a laminar flame structure can exist in a turbulent flow

Length-scales and time-scales in laminar flame:
\[ \ell_L = \frac{\alpha}{S_L}, \quad \tau_L = \frac{\alpha}{(S_L)^2} \]

In turbulent flow field, length-scales and time-scales can correspond to Integral scales or Kolmogorov micro-scales
Regime Diagrams

Assumptions:
- $Sc = 1, Pr = 1$
- The regimes so defined are only tentative and are not be taken as strict demarcations

Some important non-dimensional numbers are obtained on comparison of scales between turbulence and laminar flame, which help in making the regime diagram

$\nu = D = \alpha$, which leads to $\nu = \ell_L S_L$ and therefore

$$Re_o = \frac{u_o' \ell_o}{\ell_L S_L}$$  \hspace{1cm} (1)
Regime Diagrams: invoking Kolmogorov’s similarity hypotheses

**Kolmogorov’s first similarity hypothesis.** In every turbulent flow at sufficiently high Reynolds number, the statistics of the small-scale motions ($\ell < \ell_{E1}$) have a universal form that is uniquely determined by $\nu$ and $\varepsilon$.

**Kolmogorov’s second similarity hypothesis.** In every turbulent flow at sufficiently high Reynolds number, the statistics of the motions of scale $\ell$ in the range $\ell_0 \gg \ell \gg \eta$ have a universal form that is uniquely determined by $\varepsilon$, independent of $\nu$.

Regime Diagrams: invoking Kolmogorov’s 1st similarity hypothesis

Turbulent $Ka = \tau_L / \tau_\eta$
Using $Re_\eta = 1$ and $\nu = \alpha = D$, we get $Ka = (\ell_L / \eta)^2$; $Ka_R = (\ell_R / \eta)^2$
Using $\eta / \ell_o = Re_o^{-3/4}$ and $Re_o = u_0' \ell_o / \ell o S_L$, we get

$$Ka^2 = \left(\frac{\ell_o}{\ell_L}\right)^{-1} \left(\frac{u_0'}{S_L}\right)^3 \quad (2)$$

Turbulent $Da = \tau_o / \tau_L$
Using $\tau_o = \ell_o / u_0'$ and $\tau_L = \ell_L / S_L$, we get

$$Da = \left(\frac{\ell_o}{\ell_L}\right) \left(\frac{u_0'}{S_L}\right)^{-1} \quad (3)$$
Interpretation of $K_a$ and $D_a$

- **$K_a$ helps in assessing interaction of small scales** (of the order of Kolmogorov micro-scales) of turbulence with the flame

- **$K_a \ll 1 \Rightarrow$ flame time-scales are smaller than Kolmogorov time-scales in turbulence and it is difficult for small scales to disturb flame structure**

- **$D_a \gg 1 \Rightarrow$ flame time-scales are smaller than large time-scales in turbulence and it is difficult for large scales to disturb flame structure**

- **$D_a$ helps in assessing interaction of large scales** (of the order of Integral scales) of turbulence with the flame
Regime Diagrams

The regime diagram shown above is one example. Other regime diagrams also exist.
Regime Diagrams

**Wrinkled flamelet regime:** \((Re > 1, Ka_L < 1, u_0'/S_L < 1)\)
- \(\ell_L < \eta \Rightarrow \) flame element retains laminar flame structure within turbulent flow field
- \(u_0' < S_L \Rightarrow \) flamelet surface is only slightly wrinkled

**Corrugated flamelet regime:** \((Re > 1, Ka_L < 1, u_0'/S_L > 1)\)
- \(\ell_L < \eta \Rightarrow \) flame element retains laminar flame structure
- \(u_0' > S_L \Rightarrow \) flamelet surface is highly convoluted
Regime Diagrams

**Reaction-sheet regime:** 
\( (Re > 1, Ka_L < 1, Ka_R < 1) \)
- Eddies smaller than \( \ell_L \) penetrate the preheat zone; for large eddies flame is still a flamelet
- Reaction sheet thickness \( \ell_R < \eta \) \( \Rightarrow \) reaction sheet is only wrinkled

![Diagram showing reaction sheet regime]

**Well-stirred reactor regime:** 
\( (Re > 1, Ka_R > 1) \)
- Entire flow field behaves like a well-stirred reactor without any distinct local flame structure

![Diagram showing well-stirred reactor regime]
Genesis and Evolution of Premixed Flames in Moderate Turbulence
Laminar vs. Turbulent Premixed Flames

(a) Standard Premixed Flame

H₂-Air, φ=1.0, p=1 atm

Fractional Heat Release

Heat Release Rate/10 (J/cm²·sec)

Heat Release Rate/10

Temperature (K) or

0.00
0.05
0.10
0.15

Standard Premixed Flame[1]

Turbulent Premixed Flame

[2] Video courtesy: Prof. Hong Im
Objectives

- Turbulent flame surfaces are continuously generated and annihilated.

- Exact locations on a surface that generate complete new surfaces are not known a priori – need to look back in time.

Objectives

- Where do the fully developed, complete turbulent premixed flame surfaces evolve from? What are their special features?
- How do the flame surfaces generate and annihilate?
- What implication does generation and annihilation hold for local flame speed $S_d$?

Video courtesy: Prof. Hong Im
Zeldovich’s Theory of “Pilot Points”

“The pilot point in a non-stationary flame is the most forward-lying point of the flame front in the direction of combustion propagation. The igniting ‘impulse’ is transmitted from it to adjacent portions of the flame, and so on, until the flame front encompasses the entire mixture volume...[pilot points] establish the relationship between an integral characteristic of the process (the surface area of the flame) and a local quantity (the maximum velocity of the gas along the tube).”

Flame Particle Tracking

\[ \frac{\partial x^F (X^F (\psi_0), t)}{\partial t} = v^F (X^F (\psi_0), t) = \mathbf{u}^F + S^F_{\text{tang}} \mathbf{n}^F \]


Computational Details
DNS-Backward Flame Particle Tracking Methodology

DNS of statistically planar flames → Snapshots of DNS saved at fine time interval → Snapshots are fed to the BFPT algorithm in reverse order

What are flame particles?

- Flame particles\(^1\) are a class of surface points\(^2\) that co-move with an iso-scalar surface within the flame
- Provide spatio-temporal details of specific regions of a flame
- Ensemble of flame particles forms a flame surface and ensemble of flame surfaces forms a premixed flame

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Computational Methods
Direct Numerical Simulations (DNS)

- DNS Configuration
  - Statistically planar flames of H₂-air mixture with $\phi = 0.81$ and $P = 1$ atm
  - Detailed H₂-air reaction mechanism\(^{[1]}\) of 9 species and 21 reactions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case-1</th>
<th>Case-2</th>
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<td>$T_u$, K</td>
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<td>$L_x$, cm</td>
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<td>$Ka$</td>
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Both cases belong to the thin-reaction zone regime

Results & Discussions

1. Where do the flame surfaces evolve from?

Case-1

\( T_0 = 350K \) isotherm

Evolution of leading flame particles in time.

Taylor-scale Reynolds number \( Re_a = 86 \)

Multiple clusters in leading regions of flame surface

Uniform distribution of flame particles over flame surface

Progress of BFPT

Arrow of time

\( t_i \) \hspace{1cm} \( t_f \)

Results & Discussion
1. Features of the Multiple Clusters of Flame Particles – Leading Points

Clusters of flame particles are leading towards fresh reactants

Low fluid flow velocity
Results & Discussion
1. Features of the Multiple Clusters of Flame Particles – Leading Points

Clusters are positively curved (convex towards fresh reactants)

Stretch-rate
\[ K = \frac{1}{\delta A} \frac{d(\delta A)}{dt} = a_T + 2S_d \kappa_m \]

Clusters are positively stretched
Contribution of \(2S_d^F \kappa_m^F\) changes in time and limits stretch-rate

Strong resemblance between leading clusters of flame particles and concept of leading points by Zeldovich & co-workers
Results & Discussion
Mechanism of flame surface generation

Leading points at $t_i$ are clustered predominantly in the positively curved regions. The surface element is first stretched due to $2S_{d\kappa_m}$ along $\hat{e}_1$ since $|\kappa_1|>|\kappa_2|$, leading to a principal strain rate. The non-local, most extensive principal strain rate aligns the surface tangent $\hat{e}_2$ along $\hat{l}_1$ which results in strong tangential straining $a_T$ along $\hat{e}_2$.

$a_T$ and $2S_{d\kappa_m}$ repeat to generate a much larger surface. Several such surfaces join to form the full surface.
Results & Discussion
Flame Particle Dispersion Statistics

- The dispersion of flame particles for both sets follow modified Batchelor’s dispersion law\[^1\], where $C_2$ depends on isotherm

$$\langle |\Delta^F - \Delta^E_0|^2 \rangle = \frac{11}{3} C_2 (\Delta^F_0 \epsilon)^2 t^2$$

Interim Summary

- Fully developed turbulent flame surfaces evolve from multiple leading points
- Leading locations stretch due to
  - $2S_d \kappa_m$ along direction of maximum curvature $\hat{e}_1$
  - $a_T$ along direction of minimum curvature $\hat{e}_2$
- Relationship is developed between $S_T(t_f)$ and the local $S_d(t_c)$
- Flame particles disperse as per modified Batchelor’s law
- Dispersion is due to
  - Flame propagation up to the Gibson scale $S_L^3/\langle \epsilon \rangle$
  - Turbulence beyond the Gibson scale

Case-1

$T_0 = 350K$ isotherm
Evolution of Set-G and Set-D flame particles in time.
Taylor-scale Reynolds number $Re_L = 86$
Evolution of Local Flame Displacement Speeds in Moderate and Intense Turbulence

Flame displacement speed ($S_d$): Speed with which a flame surface propagates, locally, relative to the local flow velocity in the direction of the local surface normal vector.
Local Flame Speeds

Flame Consumption Speed:

\[ S_c = \frac{\int_{\Omega} \omega_F \, d\Omega}{(\rho Y_F)_{\text{react}} A_{\text{ref}}} \]

Flame Displacement Speed:

\[ S_d = (v_F - u) \cdot n \]
G-equation

Equation governing a propagating surface in a flow
\[ G(x, t) = 0 \] as the geometry of surface
\[ G < 0 \] as reactants
\[ G > 0 \] as products
\[ S_d \] is local flame displacement speed along \( n \)
\( n \) is local normal to surface, where \( n = -\nabla G / |\nabla G| \)

unburned gas
\( G < 0 \)

\( u(x, t) \)

Burned gas
\( G > 0 \)

\( G(x, t) = 0 \)
Derivation of G-equation

Using multivariate Taylor’s expansion,

\[ G(x + \Delta x, t + \Delta t) = G(x, t) + \frac{\partial G}{\partial t} \Delta t + \Delta x \cdot \nabla G + H. O. T \]

Since, \( G(x + \Delta x, t + \Delta t) = G(x, t) = G_0 \) = constant and taking a limit \( \Delta t \to 0 \)

\[ \frac{\partial G}{\partial t} + (\frac{dx}{dt}) \cdot \nabla G = 0 \]

Since, \( \frac{dx}{dt} = u + S_d n \), and \( n = -\nabla G / |\nabla G| \)

\[ \frac{\partial G}{\partial t} + u \cdot \nabla G = S_d |\nabla G| \]

The G-equation is a Hamilton-Jacobi equation similar to ones found in level-set methods

Introduction

Motivation

- Flame displacement speed $S_d$
  - manifestation of chemical reactions and diffusion processes within a premixed flame
  - crucial parameter in flame-front tracking methods\(^{[1-3]}\)

\[
\frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = S_d |\nabla G| \\
\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\mathbf{u} + S_d \mathbf{n}) \Sigma = \langle K \rangle \Sigma
\]

- related to turbulent flame speed $S_T$

\[
S_T = \frac{1}{A_L} \int_A \bar{S}_d dA
\]

- How do flames respond to strain and curvature effects in turbulence?

Turbulent flame is wrinkled & stretched at many length- and time-scales

Ensemble of perturbed flamelets

Perturbed flamelets are modeled using stretched laminar flame models

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\[2\] Poinsot, T.J. and Veynante, D., 2005. Theoretical and Numerical Combustion, Philadelphia, PA, USA
Introduction
Modeling Efforts Over Last Eight Decades for Laminar Flames

1940s

1960s
Markstein[3]

1980s

2000 - present
Current understanding[8,9]

$S_d = S_L$ at each point on the flame
Stability of planar premixed flame

$S_d = S_L - C\kappa$
Effect of curvature
$C$ is phenomenological coefficient

$S_d = S_L - \mathcal{L}K$
Effect of strain and curvature
$\mathcal{L}$ is Markstein length and $K$ stretch-rate

$\overline{S_d} = S_L - \mathcal{L}_K K - \mathcal{L}_K (S_L\kappa)$
Two-parameter Markstein length model

$\mathcal{L}_K$ - stretch Markstein length
$\mathcal{L}_K$ - curvature Markstein length

Local flame speed in turbulence

Turbulent Premixed Flame

Introduction
State of the art prior to 2020

- Chen and Im (1998, 2000)[1,2] observed a wide distribution of $S_d$ in turbulence.
- Hawkes and Chen (2005)[3] stated that “… steady and/or small curvature models are unlikely to be successful for modelling the stretch response of premixed flame”
- Chakraborty et. al. (2007)[4] stated that “there remains a need to model the curvature response of the combined reaction and normal diffusion components of $S_d$ to account properly for curvature stretch effects”
- Recently, Im et. al. (2016)[5] have highlighted on the need to understand the greater excursions of local $S_d$, questioning the validity of $S_d − K$ relations based on laminar flame theory

Computational Methods
Direct Numerical Simulations (DNS)

- DNS Configuration
  - Statistically planar flames of H₂-air mixture with \( \phi = 0.81 \) and \( P = 1 \text{atm} \)
  - Detailed H₂-air reaction mechanism\(^{[1]}\) of 9 species and 21 reactions

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<td>( Da )</td>
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<tr>
<td>( Ka )</td>
<td>13</td>
<td>18</td>
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Computational Methods
Generating a Manifold of All Possible States

Evolution of Flame Particles from Generation to Annihilation

\( T_0 = 665K \) isotherm

Turbulent Reynolds Number \( Re_0=1261 \)

\( P = 1 \text{atm} \)

\[
\frac{dx^F}{dt} = u^F + S_d^F n^F
\]

Clusters of flame particles in leading regions

Uniform distribution of flame particles over entire surface

Flame particles in trailing regions

\( t_i \) \( t_f \) \( t_e \)

BFPT\(^{[1]}\)

FFPT\(^{[2]}\)

Any flame state must lie between \( t_i \) and \( t_e \) of some flame particle

\( t_i < t_f < t_e \)

\( \therefore \) If we study a large number of flame particles over their complete lifetime: generation to annihilation, and record all their states, a manifold of states is created which can represent any possible state realizable from the turbulent flame with same inlet and ambient conditions.


Results and Discussion
Identifying the Two Phases of Flame Particles

Lifetime of Flame Particles
$0 \leq t/\tau_{F,L} \leq 1$

Phase-I\([1]\)
$0 \leq t/\tau_{F,L} \leq 0.8$
Variations in $\hat{S}_d^F$ are mild and gradual

Phase-II\([1]\)
$0.8 < t/\tau_{F,L} \leq 1.0$
Variations in $\hat{S}_d^F$ are large and drastic

Results and Discussion

Phase-I: Application of the Two-parameter Markstein Length Model

- Two-parameter Markstein length model\[1\]
  \[
  \tilde{S}_d(\theta) = S_L - L_K(\theta)K - L_K(\theta)(S_L\kappa) \tag{1}
  \]

- Different definitions for stretch-rate exists
  - Full stretch-rate: \( K = \nabla \cdot \mathbf{u} - n n : \nabla \mathbf{u} + S_d\kappa \)
  \[
  \frac{\tilde{S}_d^F}{S_L} = \frac{1 - L_K a_T^F / S_L - L_K \kappa F}{1 + \theta L_K \kappa F} \tag{2}
  \]
  - Asymptotic stretch-rate: \( K^* = -n n : \nabla \mathbf{u} + S_L\kappa \)
  \[
  \frac{\tilde{S}_d^F}{S_L} = 1 - \frac{L_K K^* F}{S_L} - L_K \kappa F \tag{3}
  \]
  - New stretch-rate: \( K' = \nabla \cdot \mathbf{u} - n n : \nabla \mathbf{u} + S_L \left( \frac{T_0}{T_u} \right) \kappa \)
  \[
  \frac{\tilde{S}_d^F}{S_L} = 1 - \frac{L_K}{S_L} a_T - \left( L_K \frac{T_0}{T_u} + L_K \right) \kappa F \tag{4}
  \]

Stretch Markstein length

\[
L_K = \alpha - \int_1^\theta \frac{\lambda(x)}{x} \, dx - \int_\theta^\sigma \frac{\lambda(x)}{x - 1} \, dx
\]

\[
\alpha = \frac{\sigma}{\sigma - 1} \left[ \int_1^\sigma \frac{\lambda(x)}{x} \, dx + \beta (L e_{\text{eff}} - 1) \int_1^\sigma \ln \left( \sigma - 1 \right) \frac{\lambda(x)}{x} \, dx \right]
\]

Curvature Markstein length

\[
L_\kappa = \int_\theta^\sigma \frac{\lambda(x)}{x - 1} \, dx
\]

Global reaction model quantities from Sun et. al.\[2\]

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<thead>
<tr>
<th>( T_0 (K) )</th>
<th>( L_K (\text{cm}) )</th>
<th>( L_\kappa (\text{cm}) )</th>
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<td>1321</td>
<td>+3.62\times10^{-4}</td>
<td>+4.24\times10^{-3}</td>
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Results and Discussion
Comparison of Theory with DNS

\[
\frac{\overline{S_d^F}}{S_L} = \frac{1 - \mathcal{L}_K a_T}{1 + \theta \mathcal{L}_K \kappa^F} \quad (2)
\]

\[
\frac{\overline{S_d^F}}{S_L} = 1 - \frac{\mathcal{L}_K K^F}{S_L} - \mathcal{L}_K \kappa^F \quad (3)
\]

\[
\frac{\overline{S_d^F}}{S_L} = 1 - \frac{\mathcal{L}_K}{S_L} a_T - \left( \mathcal{L}_K \frac{T_0}{T_u} + \mathcal{L}_K \right) \kappa^F \quad (4)
\]
Results and Discussion

Error Estimates

Phase-I

\[
\frac{S_d^F}{S_L} = 1 - \frac{\mathcal{L}_K \alpha_T / S_L - \mathcal{L}_K \kappa^F}{1 + \theta \mathcal{L}_K \kappa^F}
\]

Phase-II

\[
\frac{S_d^F}{S_L} = 1 - \frac{\mathcal{L}_K \kappa^F}{S_L} - \mathcal{L}_K \kappa^F
\]

\[
\frac{S_d^F}{S_L} = 1 - \frac{\mathcal{L}_K}{S_L} \alpha_T - \left( \frac{\mathcal{T}_0}{\mathcal{T}_u} + \mathcal{L}_K \right) \kappa^F
\]
Results and Discussion
Markstein Length Model – Applicability and Issues

One-time Joint Probability Density Function

Large stretch-rates are mostly found in regions in large negative curvature

\[ T_0 = 350K \]  \[ T_0 = 665K \]  \[ T_0 = 1321K \]

\[ \bar{S}_d = S_L - \mathcal{L}_K K - \mathcal{L}_K (S_L \kappa) \]  

(1)
Flame structure in moderate turbulence

Results and Discussion
Problem: Unsteady Inwardly Propagating Cylindrical Premixed Flame

- **Governing Equations**

\[
\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda r \frac{\partial T}{\partial r} \right) + q_w \\
\rho \frac{\partial Y}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda r \frac{\partial Y}{\partial r} \right) - w
\]

- **Boundary Conditions**

At \( r = 0 \), \( \frac{\partial T}{\partial r} = 0 \) and \( \frac{\partial Y}{\partial r} = 0 \) for \( t \geq 0 \)

At \( r = +\infty \), \( T = T_b \) and \( Y = 0 \) for \( t \geq 0 \)

- **Initial Condition**: When flame is sufficiently far from center \( r = 0 \), stationary cylindrical flame solution is applicable

- In the preheat zone, unsteady and diffusion processes dominate and balance
Results and Discussion
Problem-2: Unsteady Inwardly Propagating Cylindrical Premixed Flame

\[ \rho C_p \frac{\partial T}{\partial t} = \lambda \frac{d^2 T}{dr^2} + \frac{\lambda dT}{r dr} \]

- Using a stretched coordinate \( \xi = r/r_f \), and \( \theta = (T_0 - T)/(T_0 - T_u) \), \( T_0 \) is the isotherm at the boundary of preheat & reaction zone

\[
\frac{\partial \theta}{\partial t} - \frac{\xi r_f \partial \theta}{r_f \partial \xi} = \frac{\alpha_u}{r_f^2} \left[ \frac{1}{\xi} \frac{\partial \theta}{\partial \xi} + \frac{\partial^2 \theta}{\partial \xi^2} \right]
\]

- Integrating above equation for \( \xi = 0 \) to 1, and simplifying using

\[
\dot{\theta}_\xi(1) + \frac{\dot{r}_f}{r_f} \theta_\xi(1) = -\frac{\alpha_u}{r_f^2} \theta_\xi(1)
\]

- To estimate \( \dot{\theta}_\xi \) and \( \theta_\xi \), we have to analyze the reaction zone

- When \( r_f \gg \delta_R \), reaction zone is quasi-planar and quasi-steady, and diffusion and reaction terms dominate and balance
Results and Discussion

Problem: Unsteady Inwardly Propagating Cylindrical Premixed Flame

\[ \lambda \frac{\partial^2 T}{\partial r^2} = -qw \]
\[ \rho D \frac{\partial^2 Y}{\partial r^2} = w \]

- It can be shown\(^1\), that \( \frac{\partial T}{\partial x}|_{r=r_f} = (2LeDa^0/Ze^2)\exp(-T_b^0/T_b)\left(\frac{T_b}{T_b^0}\right)^4 \)
- Using \( T_b/T_b^0 = 1 + \bar{K}(1/Le - 1)\[^1\] \), we can write \( \theta(1) = Ar_f + B\dot{r}_f \), where A and B are constants
- Therefore,

\[ Br_f^2\ddot{r}_f + \dot{r}_f (3Ar_f + 2Br_f\dot{r}_f + 6\alpha_uB) + 6\alpha_uAr_f = 0 \]

- For \( Le = 1 \),

\[ \dot{r}_f = -\frac{2\alpha_u}{r_f} \quad \dot{r}_f = -S_d, \text{ since gas-mixture is at rest} \]

- Since, \( \kappa = -\frac{1}{r_f} \Rightarrow S_d = -2\alpha_u\kappa \)

\[ A = \frac{f^0_q Y_u}{\lambda} \left[ \frac{2Da^0 Le}{Ze^2} \right]^{1/2} (T_{ig} - T_u) \]
\[ B = \frac{A}{2} \left( \frac{1}{Le} - 1 \right) \left( 4 + \frac{T_a}{T_b^0} \right) \]

Results and Discussion
Verification of the Interaction Model (Black Line)

One-time Joint Probability Density Function

\[
\frac{\bar{S}_d}{S_L} = 1 - \mathcal{L}_{\kappa} \kappa
\]  
(1)

\[
\frac{\bar{S}_d}{S_L} = 1 - \frac{\alpha T_u}{T_0 \delta_L S_L} (1 + C) \kappa \delta_L
\]  
(2)

\[
\frac{\bar{S}_d}{S_L} = 1 - \frac{L_{\kappa}}{S_L} K - \mathcal{L}_{\kappa} \kappa
\]  
(3)

Refer to these recent works for topology of pocket formation:


Trivedi, S., Griffiths R.A.C., Kolla H., Chen J.H., Cant R. S., 37 (2019) 2616-2626

Interim Summary

Crucial insights from spatio-temporal analysis of turbulent premixed flame

Local regions of a turbulent premixed flame

Non-interacting regions

Interacting regions

Flame particle tracking

- Perturbed by \textit{turbulent flow-field}
- Local flame structure is close to the standard premixed flame
- Variations in $\overline{S_d}$ are mild and gradual and close to $S_L$
- Two-parameter Markstein length model is applicable

\[ \overline{S_d}(\theta) = S_L - L_K(\theta)K - L_\kappa(\theta)(S_L\kappa) \]

- Perturbed by \textit{flame-flame interactions}
- Local flame structure is drastically different than the standard premixed flame
- Variations in $\overline{S_d}$ are large and drastic and much away from $S_L$
- Interaction model is applicable

\[ \overline{S_d}(\theta) = S_L - (1 + C) \frac{\alpha T_u}{T_0} \kappa \]
Extreme turbulence: high Karlovitz number flame propagation and structure
Regime Diagrams

The regime diagram shown above is one example. Other regime diagrams also exist.

High Ka-flames in Michigan Hi-Pilot burner


Recent developments: Regime diagram from piloted Bunsen flames


\[ \text{Re}_T = 58,200, 22,300, 99,600; \frac{u'}{S_L} = 80, 77, 123; K_a_{TP} = 57, 85, 103, \text{ for (a, b, c)} \]
High $Ka$ flame DNS
Flame displacement speeds in intense turbulence

Yuvraj, Song, W., Dave, H.L., Im, H.G. and Chaudhuri, S., https://arxiv.org/abs/2106.08407
\[
\tilde{S}_d = S_L - 2\alpha_0 \kappa
\]

\[
\tilde{S}_d = S_L - \alpha_0 (1 + C) \kappa \frac{T_u}{T_0}
\]

<table>
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<tr>
<th>Case/T_o</th>
<th>385 K</th>
<th>641 K</th>
<th>983 K</th>
<th>1325 K</th>
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Yuvraj, Song, W., Dave, H.L., Im, H.G. and Chaudhuri, S., https://arxiv.org/abs/2106.08407
Turbulent Flame Speed
Turbulent Premixed Flame
Turbulent Flame Speed

\[ \overrightarrow{v_f} = \overrightarrow{u} + S_d \hat{n} \]

\[ \therefore \overrightarrow{v_r} = \overrightarrow{u} - \overrightarrow{v_f} = -S_d \hat{n} \]

Mass flow rate of premixed reactants into a flame surface

\[ \dot{m} = - \int_{A_T} \rho (\overrightarrow{v_r} \cdot \hat{n}) dA \]

\[ \therefore \dot{m} = \int_{A_T} \rho (S_d \hat{n} \cdot \hat{n}) dA \]

Defining

\[ \rho u S_{T,c_0} A = \int_{A_{T,c_0}} \rho S_d dA \]

\[ S_{T,c_0} = \frac{1}{A} \int_{A_{T,c_0}} \overline{S_d} dA \]
Damköhler (1940) discussed two limiting cases:
- Wrinkled flamelet regime
- Thin reaction zone regime

Turbulent flame propagation mode are fundamentally different in the two limiting cases

<table>
<thead>
<tr>
<th>Turbulence Scales</th>
<th>Flame Thickness</th>
<th>Turbulence Effect</th>
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<tbody>
<tr>
<td>Larger</td>
<td>Than</td>
<td>Increases surface area</td>
</tr>
<tr>
<td>Smaller</td>
<td>Than</td>
<td>Modifies the transport process</td>
</tr>
</tbody>
</table>
Turbulent Flame Speed

Wrinkled flamelet regime

- $A_T$ is instantaneous flame area
- $A$ is projected area

Since, mass-flux is constant $\dot{m} = \rho u S_T A = \rho u S_L A_T$

Thus, turbulence increases area $A_T > A$ which causes $S_T > S_L$ to the leading order

Using simple geometric arguments it can be shown for weak turbulence:

$$\frac{S_T}{S_L} = \sqrt{1 + \left(\frac{u_0'}{S_L}\right)^2}$$
Turbulent Flame Speed: literature

“... one of the most important unresolved problems in premixed turbulent combustion is determining the turbulent burning velocity”, Turbulent Combustion, Norbert Peters, Cambridge University Press, 2000.

Experiments:

Theories/models/computations:

It is of interest to seek a solution:

\[ \frac{S_T}{S_L} = f(u'_0, S_L, l_0, l_L,...) \]
Turbulent Flame Speed: Damkohler’s Derivation for Small Scale Turbulence

\[ S_L \sim \sqrt[\frac{\alpha}{\tau_c}] \]

By analogy in thin reaction zone regime:

\[ S_T \sim \sqrt[\frac{\alpha_T}{\tau_c}] \Rightarrow \frac{S_T}{S_L} \sim \sqrt[\frac{\alpha_T}{\alpha}] \]

Using \( \alpha_T \sim u'_o \ell_o \) and \( \alpha = \ell L S_L \)

\[ \frac{S_T}{S_L} \sim \sqrt[\frac{u'_o \ell_o}{S_L \ell L}] \Rightarrow S_T \sim \sqrt{u'_o \ell_o} \]
Experiments
Turbulent Bunsen Flames

Fig. 2. Instantaneous schlieren photographs of the turbulent flames
(a) $P = 0.1$ MPa; (b) $P = 1.0$ MPa; (flow velocity $U = 2.0$ m/s, hole diameter $d = 2.0$ mm).
Extract the mean flame cone

Fig. 3. Procedure of image processing to determine the mean flame cone.
$S_T = U \sin(\theta / 2)$

(a) 

(b) 

Fig. 4. Effects of pressure and turbulence intensity on the shape of mean flame cone (a) $P = 0.1$ MPa, $U = 2.0$ m/s, $u' = 0.1$ m/s, $d = 1$ mm; (b) $P = 3.0$ MPa, $U = 2.4$ m/s, $u' = 0.5$ m/s, $d = 3$ mm.

Turbulent Flame Speed

**Figure 6.** Relationship between $S_f/S_L$ and $u'/S_L$ for various ambient pressures.

Dual Chamber High Pressure Turbulent Combustion Vessel at Princeton
Experimental Setup

Dual-chamber design: Constant pressure (up to 25atm)
Fan-generated nearly isotropic homogeneous turbulence
High speed Schlieren imaging
<R> = 2.74mm, time = 0.24 ms
Emergence of Fine Scales with Pressure

CH$_4$-air, ($\phi=0.9$, $Le=1$)

High Speed Mie Images and Vector Fields

kHz rate PIV on Constant Pressure Expanding Turbulent Flame
Experiments:

- Experiments conducted at $u_{rms} \sim 1-6 \text{ m/s}$ and at pressures $p = 1, 2, 3, 5 \text{ atm}$.
- High Speed Schlieren Imaging characterizes flame propagation.
- High Speed Particle Image Velocimetry characterizes non-reacting turbulence parameters.

\[ \langle R \rangle = \sqrt{A/\pi} \text{ where } A \text{ is the area enclosed by the Schlieren edge} \]
\[ S_L = \frac{\alpha}{\tau_c} \]

By analogy in thin reaction zone regime:

\[ S_T = \sqrt{\frac{\alpha_T}{\tau_c}} \Rightarrow \frac{S_T}{S_L} = \sqrt{\frac{\alpha_T}{\alpha}} \]

Using \( \alpha_T \sim u'_{\langle R \rangle} \langle R \rangle \) and \( \alpha = \ell_L S_L \)

\[ \frac{S_T}{S_L} \sim \sqrt{\frac{u'_{\langle R \rangle} \langle R \rangle}{S_L \ell_L}} \Rightarrow S_T \sim \sqrt{u'_{\langle R \rangle} \langle R \rangle} \]
Turbulent Flame Propagation Rate

Larger set: larger range of fuel, turbulence intensity
Scaling with Flame Thickness

Symbols: $\text{C}_2\text{H}_4$-15% $\text{O}_2$-85% $\text{N}_2$, $\phi$=1.3; $\text{CH}_4$-air $\phi$=0.9, $\text{C}_2\text{H}_4$-air, $\phi$=1.3, $\text{n-C}_4\text{H}_{10}$-air, $\phi$=0.8 and $\text{C}_2\text{H}_6\text{O}$-air $\phi$=1.0; Pressure 1-5atm. Lines: Leeds data (Lawes et. al. CNF 159, 2012) for the iso-$\text{C}_8\text{H}_{18}$ $\phi$=0.8-1.2 Pressure 1-10atm
Comprehensive ½-Power Scaling: Present \((C_0, C_1, C_2, C_4)\) Fuels and Leeds \(C_8\) Data

Symbols: \(C_2H_4-15\%\) \(O_2-85\%\) \(N_2\), \(\phi=1.3\); \(CH_4\)-air \(\phi=0.9\), \(C_2H_4\)-air, \(\phi=1.3\), \(n-C_4H_{10}\)-air, \(\phi=0.8\) and \(C_2H_6O\)-air \(\phi=1.0\): Pressure 1-5atm. Lines: Leeds data (Lawes et. al. CNF 159, 2012) for the iso-\(C_8H_{18}\) \(\phi=0.8\)-1.2 Pressure 1-10atm

\[
I_0 = 1 - \frac{\langle R \rangle}{\delta_{M,b}}
\]

Turbulent Flame Speed: Surface Fitting

\[ \frac{d \langle R \rangle}{dt} \propto \left( \frac{u_{\text{eff}}}{S_L} \right)^m \left( \frac{\langle R \rangle}{\delta_M} \right)^n \]

\[ m = 0.43, \ n = 0.45 \]

[\approx 0.5]

**C<sub>4</sub>-C<sub>8</sub> n-alkanes**

![Diagram showing regimes of experimental conditions on Borghi diagram.](image)

Fig. 2. Regimes of the present experimental conditions on Borghi diagram. $S_L$ and $\delta_L$ of only n-hexane/air are used since other C<sub>4</sub>–C<sub>8</sub> n-alkanes have almost the same values.

Scaling for C₄-C₈ n-alkanes

CH4-air data from National Central University, Taiwan

H₂ blend – air data from Xian Jiaotong University, China

Data from Georgia Tech, USA

Experiments on Instability Turbulence Interaction

**Laminar Flame with Cellular Instability**

- Cellular instability:
  - Increase the surface area
  - Flame acceleration

**Turbulent Flame**

- Turbulence:
  - Wrinkle the surface & enhanced mixing
  - Flame acceleration
Methodology

Cellular instability
- Flame acceleration due to cellular instability ($Le = 1$ and $Le < 1$)
- Scaling analysis ($Le = 1$)
- Flame acceleration due to turbulence

Identify conditions

Experimental investigation ($Le = 1$ and $Le < 1$)
Propagation of Turbulent Flames

Turbulent flame speed scaling\(^{[1]-[4]}\)

\[
\overline{S_T} \sim \text{Re}_{T,f}^{1/2}
\]

\[
\text{Re}_{T,f} = \frac{U_{\text{rms}} \langle R \rangle}{S_L \delta_L}
\]

In our case,

\[
U_{\text{rms}} \propto \langle R \rangle^{0.33}
\]

So, in turbulent flame propagation

\[
\overline{S_T} \sim \left[ \langle R \rangle / \delta_L \right]^{\beta_T} = P e^{\beta_T}, \quad \beta_T \approx 0.67
\]

Experimental Setup

Dual-chamber design: Constant pressure (up to 25 atm)
Fan-generated nearly isotropic homogeneous turbulence
High speed Schlieren imaging
Propagation of Laminar Cellular Flames ($Le = 1$)

DL instability $\rightarrow$ Three-stage behavior$^{[1]}$

- Smooth expansion
- Transition
- Saturation

$H_2/O_2/N_2, \phi = 1, T_f = 2400K (Le \approx 1)$

$S_{L,b} \sim Pe^\beta_c, \beta_c \approx 0.3$

acceleration exponent

Propagation of Laminar Cellular Flames \((Le < 1)\)

- For \(Le < 1\), diffusional-thermal instability also appears

\[
H_2/O_2/\text{He}/\text{Ar}, \phi = 0.4, T_f = 2110K
\]

- Three-stage behavior still exists

- The smaller the \(Le\), the larger the \(S_{L,b}\)

- In saturation stage, \(S_{L,b} \sim Pe^{\beta_c}, \beta_c \approx 0.33\)
Acceleration Exponent

Laminar flame with Cellular instability

\[ \overline{S_{L,b}} \sim Pe^{0.3} \]

Turbulent cellularly-stable flame

\[ \overline{S_T} \sim Pe^{0.67} \]
Turbulence-DL Instability Interaction
Turbulence-Instability Interaction

Conjecture: DL instability develops in turbulence if

\[ \sigma_{DL}^{(k)} >> \omega_{turb}^{(k)} \]

DL growth rate Turbulent eddy frequency

\[ \sigma_{DL}^{(k)} = X(\theta)S_L k (1 - k/k_c) \]

\[ X(\theta) = \frac{\theta}{\theta + 1} [(\theta + 1 - \theta^{-1})^{1/2} - 1] \]

\[ k_c/L = h_b + \frac{3\theta - 1}{\theta - 1} M k - \frac{2\theta}{\theta - 1} \int_1^\theta \frac{h(\vartheta)}{\vartheta} d\vartheta + (2 Pr - 1) \left( h_b - \frac{\int_1^\theta h(\vartheta) d\vartheta}{\theta - 1} \right)^{-1} \]

\[ \tau_{turb}^{(k)} = \frac{2 \pi}{\omega_{turb}^{(k)}} \]

\[ \tau_{turb}^{(k)} = \frac{u_{(k)}^2}{\varepsilon} = \left( \frac{l_0}{u'_0} \right) \left( \frac{k}{k_0} \right)^{-2/3} \]

Turbulence-DL Instability Regime Diagram

\[ \beta = \min_{k_c > k > k_0} \left\{ \frac{U_{rms}}{XS_L} \left( \frac{k_0}{k} \right)^{1/3} \left( 1 - \frac{k}{k_c} \right)^{-1} \right\} \]

Three Cases

Case I: $S_{L0} > U_{rms}$
- $\omega_{DL}/\omega_T = S_{L0}/u'_k > 1$
- DL instability dominates
- Wrinkled Flamelet Regime

Case II: $u'_\eta < S_{L0} < U_{rms}$

Case III: $S_{L0} < u'_\eta$
Case I: \( S_{L0} > U_{rms} \)

\[
Le = 1
\]

Saturation stage of DL

\[
H_2/O_2/N_2, \phi = 1, T_f = 2400K, P = 1 - 10\text{atm}
\]

Turbulent cellularly-stable flame

\[
\text{Darrieus-Landau instability dominates the flame propagation in Wrinkled Flamelet Regime!}
\]

- Acceleration exponent is close to the saturation stage of DL instability

\[
Le < 1
\]

H_2/O_2/He/Ar, \( \phi = 0.4, T_f = 2110K, P = 1\text{atm} \)

The acceleration exponent is independent of \( Le \), but \( \overline{S_T} \) decreases with \( Le \).

- Cellular instability dominates in this regime for \( Le < 1 \)
Regimes

Case I: $S_{L0} > U_{rms}$

- $\omega_{DL}/\omega_T = S_{L0}/u'_k > 1$
- DL instability dominates
- Wrinkled Flamelet Regime

Case II: $u'_\eta < S_{L0} < U_{rms}$

- $\omega_{DL}/\omega_T = S_{L0}/u'_k \sim o(1)$
- DL instability & turbulence interact
- Corrugated Flamelet Regime

Case III: $S_{L0} < u'_\eta$

- DL instability dominates
- Wrinkled Flamelet Regime
- Corrugated Flamelet Regime

\[\frac{U_{rms}}{S_{L0}} \quad \frac{U_{rms}}{U_{rms}}\]
Case II: $U_{rms} > S_{L,0} > u_\eta'$

- Acceleration exponent is between the saturation stage of DL instability and the turbulent cellularly-stable flame

$Le = 1$

Saturation stage of DL

$H_2/O_2/N_2, \phi = 1, T_f = 1800K, P = 1 - 10$ atm

Turbulent cellularly-stable flame

$Le < 1$

$H_2/O_2/He/Ar, \phi = 0.4, T_f = 2110K, P = 1$ atm

The acceleration exponent is independent of $Le$, but $S_T$ decreases with $Le$.

Darrieus-Landau instability and turbulence both contribute to flame propagation in Corrugated Flamelet Regime!
Regimes

**Case I:** $S_{L0} > U_{rms}$
- $\omega_{DL}/\omega_T = S_{L0}/u'_k > 1$
- DL instability dominates
- Wrinkled Flamelet Regime

**Case II:** $u'_\eta < S_{L0} < U_{rms}$
- $\omega_{DL}/\omega_T = S_{L0}/u'_k \sim o(1)$
- DL instability & turbulence interact
- Corrugated Flamelet Regime

**Case III:** $S_{L0} < u'_\eta$
- $\omega_{DL}/\omega_T = S_{L0}/u'_k < 1$
- Turbulence dominates
- Thickened Flamelet Regime
Case III: $u_\eta' > S_{L0}$

\begin{align*}
Le &= 1 \\
\text{Turbulent cellularly-stable flame} &\quad \text{Saturation stage of DL instability} \\
\text{C}_3\text{H}_8/\text{O}_2/\text{Ar}, \phi = 0.8, T_f = 2150K, P = 5 - 20\text{atm} &\quad \text{H}_2/\text{O}_2/\text{He}/\text{Ar}, \phi = 0.4, T_f = 1440K, P = 1\text{atm}
\end{align*}

\begin{align*}
Le &= 1 \\
\text{Turbulence dominates the flame propagation in Thickened Flamelet Regime!} &\quad \text{The acceleration exponent is independent of } Le, \text{ but } S_T \text{ decreases with } Le.
\end{align*}

\begin{itemize}
    \item Acceleration exponent is close to the turbulent cellularly-stable flame
    \item Turbulence dominates in this regime for $Le < 1$
\end{itemize}
Instability Turbulence Interaction

$S_T/S_L \sim Pe^\beta$

$\beta \approx 0.35 \pm 0.02 : Le \approx 1$
$\beta \approx 0.44 \pm 0.01 : Le \approx 1$
$\beta \approx 0.67 \pm 0.01 : Le \approx 1$
$\beta \approx 0.35 \pm 0.02 : Le < 1$
$\beta \approx 0.44 \pm 0.01 : Le < 1$
$\beta \approx 0.67 \pm 0.01 : Le < 1$
Summary

- Darrieus-Landau instability in turbulent flames:
  - revised regime diagram (with respect to DL instability)

- Diffusional-thermal instability in turbulent flames:
  - Influences the total burning rate, but does not influence the acceleration exponent.

Blowoff

Flame blowoff in the SR-71 during a high-acceleration turn, Campbell and Chambers
Characteristics of flows separated by bluff bodies:
Non-reacting flows
Characteristics of flows separated by bluff bodies:
Reacting flows

(a) 

(b) 

S Chaudhuri, PhD Thesis 2010
Early views on blowoff

- Longwell (1953) suggested: blowoff due to imbalance in rate of reactions in RZ

- Insufficient heat supply by RZ to fresh gases (Williams GC, Hottel H. et al. 1951)

- Insufficient contact time of the fresh mixture in the shear layer with the burnt product in RZ. (Zukoski 1954)

- Blowoff is preceded by two distinct stages of flame hole formation (Nair and Lieuwen, 2007)

- But these studies did not connect the early stages of blowoff dynamics with the final blowoff event and a complete mechanism was lacking.
Effects of exothermicity

Results indicate substantially reduced turbulence intensities and vorticity magnitudes in combusting flows relative to the non-reacting flow for e.g. by Soteriou, Ghoniem (1994).

Fureby and Lofstrom (1994): vorticity field strength was much weaker and “less structured” (1994) in the presence of combustion.

Fuji and Eguchi (1981) and Bill and Tarabanis (1986) noted that turbulence levels in the reacting flow were much lower than the non-reacting case, particularly in the vicinity of the recirculation zone boundary.
The Vorticity Transport Equation

\[
\frac{D\vec{\omega}}{Dt} = \left( \vec{\omega} \cdot \nabla \right) \vec{V} - \vec{\omega} \left( \nabla \cdot \vec{V} \right) - \frac{\nabla p \times \nabla \rho}{\rho} + \nabla \times \frac{\nabla \cdot \vec{S}}{\rho}
\]

The kinematic gas viscosity, in term 4 rapidly increases through the flame, due to its larger temperature sensitivity. This substantially enhances the rate of diffusion and damping of vorticity, an effect emphasized by Coats (1996).

Term 3, i.e. the Baroclinic vorticity production, originates from the pressure and density gradient mismatch.

Term 2, i.e. dilatation also acts as a vorticity sink.
Near Blowoff Dynamics in Bluff Body Stabilized Flames

• Many researchers observed that near blowoff flames are highly unsteady and unstable (Zukoski (1958), Williams (1966) H.M. Nicholson (1948))

• Nicholson and Field (1948) described large scale pulsations in rich bluff body flames as they were blowing off.

• Observations of large scale, sinuous oscillations of a flame near blowoff were presented by Thurston (1958).

• Hertzberg et al. (1991) measured velocity fluctuations in a bluff body wake, indicating a growing amplitude of a relatively narrowband oscillation as blowoff was approached that they attributed to vortex shedding.

• A number of more recent studies by Nair and Lieuwen (2007), Kiel et al. (2007) and Erickson et al. have also noted these dynamics (2007).
Early views on blowoff

- Longwell (1953) suggested: blowoff due to imbalance in rate of entrainment of reactants (a PSR RZ)
- Insufficient heat supply by RZ to fresh gases (Williams GC, Hottel H. et al. 1951)
- Insufficient contact time of the fresh mixture in the shear layer with the burnt product in RZ. (Zukoski 1954)
- Extinction of a strained flamelet (Yamaguchi 1985)
- But these studies did not connect the early stages of blowoff dynamics with the final blowoff event as complete mechanism was lacking.
Blowoff Correlation

\[ Da = bRe_D^{a-\log(Re_D)} = a \log(Re_D) + b \]

Summary of data used for data compilation reported. (Italized and non-italicized text under "Blowoff type" denotes asymmetric and centrally two-dimensional bluff body, respectively.)

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<thead>
<tr>
<th>Reference</th>
<th>Symbol</th>
<th>Symbol color</th>
<th>Ref. year</th>
<th>( U_\text{max} )</th>
<th>( \text{Re}_\text{D} )</th>
<th>( D_\text{ap} )</th>
<th>( T_\text{FSR} )</th>
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<th>Bluff body type</th>
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<td>Wieland et al. [78]</td>
<td>( \circ )</td>
<td>Red</td>
<td>1982</td>
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<td>Lienhard [44]</td>
<td>( \bullet )</td>
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<td>1982</td>
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<tr>
<td>Rasing [62]</td>
<td>( \circ )</td>
<td>Cyan</td>
<td>1998</td>
<td>10 - 100</td>
<td>4 - 20.5</td>
<td>1</td>
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<tr>
<td>Filipe [46]</td>
<td>( \bullet )</td>
<td>Magenta</td>
<td>1998</td>
<td>10 - 100</td>
<td>4 - 20.5</td>
<td>1</td>
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<tr>
<td>Nakajima et al. [45]</td>
<td>( \circ )</td>
<td>Yellow</td>
<td>1994</td>
<td>20 - 90</td>
<td>10 - 40</td>
<td>1</td>
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<tr>
<td>van Carverden et al. [42]</td>
<td>( \circ )</td>
<td>Green</td>
<td>1994</td>
<td>30 - 100</td>
<td>4 - 20</td>
<td>1</td>
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<td>Radul and Goldsmith [46]</td>
<td>( \bullet )</td>
<td>Blue</td>
<td>1998</td>
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<td>Batt [79]</td>
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<td>Blue</td>
<td>1990</td>
<td>190 - 1800</td>
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<tr>
<td>Rasing and Monin [95]</td>
<td>( \bullet )</td>
<td>Black</td>
<td>1995</td>
<td>0 - 10</td>
<td>0.0 - 100</td>
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<tr>
<td>Rasing and Monin [96]</td>
<td>( \bullet )</td>
<td>Red</td>
<td>1995</td>
<td>10 - 100</td>
<td>0.0 - 100</td>
<td>1</td>
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<td>Propane, circular</td>
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<tr>
<td>Rasing and Monin [96]</td>
<td>( \circ )</td>
<td>Blue</td>
<td>1995</td>
<td>10 - 100</td>
<td>0.0 - 100</td>
<td>1</td>
<td>250</td>
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<tr>
<td>Shibata and Nishiyama [97]</td>
<td>( \circ )</td>
<td>Yellow</td>
<td>1994</td>
<td>20 - 100</td>
<td>0.0 - 100</td>
<td>1</td>
<td>250</td>
<td>Propane, circular</td>
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<tr>
<td>Pyne and Nishiyama [98]</td>
<td>( \circ )</td>
<td>Blue</td>
<td>1995</td>
<td>200 - 1000</td>
<td>0.0 - 1000</td>
<td>1</td>
<td>250</td>
<td>Propane, circular</td>
<td></td>
</tr>
</tbody>
</table>

Shanbhogue et al. (2009)

Two stages of blowoff

Stage 1:

Initiation of flame hole, its convection downstream and healing. However the flame can persist indefinitely at this stage. This local extinction is hypothesized to be occurring at points where $k_{\text{local}} > k_{\text{extinct}}$.

Sequence of flame images, 10 ms apart, taken during the first preblowoff stage at $\phi = 0.65$. Note presence of flame holes in the images (flow direction is from bottom to top), Nair and Lieuwen (2007).

Computed reaction rate contours of a V-Cutter stabilized flame exhibiting localized extinction ($Re_D = 56,000$, $T_b/T_a = 2.9$) (Smith et al. 2007).

Stage 2:
Moments away from blowoff

Return to asymmetry near blowoff

Quasi Real Scenario: Prototypical afterburner emulating PW F119 of F22
The Afterburner Rig

Experimental Setup

Simultaneous PIV PLIF setup

Imaging setup

PIV Laser Nd:Yag 532nm
Laser beam from dye laser at 283nm
310nm filter MCT
PIV Camera
Stable Flame at $\phi = 0.85$

High speed chemiluminescence emission images for a stable flame very far from blowoff for $U_m = 18.3$ m/s at $\phi = 0.85$ at 500 frames per second and 100 $\mu$s exposure.
Extinction reignition and blowoff: movie

Extinction and Reignition
Laser Induced Fluorescence

Incident Laser

Simultaneous PIV and OH PLIF

Stable flame \( \phi = 0.85 \)
Near blowoff flame
$\phi = 0.60$

Extinction along shear layers and recirculation zone burn
Simultaneous PIV and OH PLIF in a small scale experiment

Stable flame at $\phi = 0.9$
Unstable flame at $\phi = 0.77$ near blowoff

Extinction along shear layers
Mean $U_y$ and $\omega_z$ superimposed with OH-PLIF

$\phi = 0.90$ : Far from blowoff

$\phi = 0.77$ : Near blowoff
Mean profiles of $U_x$, $\omega_z$ and OH

$\phi = 0.85$

$\phi = 0.65$

Left panels: Mean axial velocity from PIV superimposed with OH fluorescence signal from PLIF.
Right panels: Mean out of plane vorticity superimposed with OH fluorescence, both at axial locations of 30 mm for $\phi = 0.85$ (a,b) and for $\phi = 0.65$ (c,d).
Basics of Premixed Flame Extinction

1. Extinction by volumetric heat loss
2. Extinction by stretch
   a. $Le > 1$
   b. $Le < 1$

\[ \rho_u s_L \frac{dT}{dx} = \frac{d}{dx} \left( \frac{\lambda}{c_p} \frac{dT}{dx} \right) + \frac{Q}{c_p} \omega - \alpha(T - T_u). \]

\[ M = \frac{s_L}{s_{L,\text{ref}}}. \]

\[ 4 M^2 (1 - M^{1/2}) = 2 \pi Ze \]

\[ M^2 \ln(M^2) = -2 \pi Ze \]
Stretch Rate Pdfs

Probability density function of $|K_s|$ at (a) $\phi = 0.85$ (b) $\phi = 0.65$ and (c) Mean pdfs of $|K_s|$ at $\phi = 0.85$ and $\phi = 0.65$. 
Towards blowoff $\phi \downarrow$ and hence $S_L \downarrow$, so flame shifts from outside towards the shear layer vortices. Partial flame extinction along shear layers due to $\kappa_{\text{flame}} > \kappa_{\text{extinction}}$ by convecting vortices.

Non reacting unburnt mixture entrains into RZ and due to favorable flow time scales reacts within RZ. Hence OH and chemiluminescence

Reacting RZ reignites the shear layers to cause reignition

Reacting RZ fails to reignite the shear layers

More parts of the shear layers become “cold” Absolute instability: Asymmetric mode steps in to cause greater perturbations

Blowoff
Works at Cambridge: lean CH$_4$-air flames
Far from blowoff

Near blowoff

J. Kariuki, J. Dawson and E. Mastorakos, Combustion and Flame 2012
Blowoff in Vitiated Flows

Fig. 9. High-speed chemiluminescence images of a blowoff event at $\phi_f = 0.51$, $\phi_u = 0.15$, and 59 m/s, gathered at 500 frames.

Vitiated

Unvitiated

Fig. 10. High-speed chemiluminescence images of a blowoff event at $\phi = 0.65$, $\phi_l = 0.04$, and 18.5 m/s, gathered at 500 frames.

PIV–PLIF of near blowoff vitiated flames

Significant difference between vitiated and unvitiated blowoff
Forced blowoff mechanism

Flame images obtained by reversing the Mie scattering images obtained during the PIV experiments for the 10-mm-diameter disk-shaped bluff-body flame holder (arrows show the length scale $\lambda = U_m f$). (The values in black represent the ratio of the length of recirculation zone to $\lambda$.

A. Chapparo, B.M. Cetegen, Combustion and Flame 2006

Fig. 1. Ratio of blowoff equivalence ratio at a particular frequency of perturbation with blowoff equivalence ratio at no perturbation as a function of the ratio of $l_m/\lambda$ for $U_m = 5, 10$ and 15 m/s. A schematic of the burner is shown in the inset.
Mean flow and PLIF fields for $U_m = 10\text{m/s}, f = 200\text{Hz}$ $L_{RZ\text{mean}}/\lambda_{\text{mean}} = 0.4$

\begin{tabular}{|c|c|c|c|c|}
\hline
$\phi=0.85$ & 200Hz & OH PLIF, $U, \omega_z$ & $<U_x>$ & $<\omega_z>$ & $<\text{OH-PLIF}>$ \\
\hline
0ms & & & & & \\
\hline
2ms & & & & & \\
\hline
3ms & & & & & \\
\hline
\end{tabular}

S. Chaudhuri, S. Kostka, M. Renfro, B. Cetegen, Combustion and Flame 2012
Forced Blowoff: Forced Vortex Shedding

Forced Blowoff

Unforced Blowoff

S. Chaudhuri, S. Kostka, M. Renfro, B. Cetegen, Combustion and Flame 2012
Blowoff in Swirl Stabilized Flames (Ga.Tech)

Fig. 2: Time-series data of OH chemiluminescence signal for equivalence ratio $\phi = 0.865$ and 0.821 ($\phi_{LBO} = 0.802$). The expanded time series for the last case is also shown.

Fig. 5: Variation of average number of events per second as a function of equivalence ratio. $\phi_{LBO}$: LBO limit for these conditions.

Fig. 3: High-speed visualization images (inverted grayscale): case a, equivalence ratio $\phi = 0.79$, time between images 2 ms; case b, $\phi = 0.76$, time between images 16 ms showing nearly total loss of flame followed by reattachment ($\phi_{LBO} = 0.802$). The location of the combustor inlet is indicated in the first image of case b.

Blowoff in Swirl Stabilized Flames (DLR)

Experimental Setup

Time averaged OH* and streamlines

M. Stoehr, I. Box, C. Carter, W. Meier, Proceedings of the Combustion Institute 2011
Near and Final Blowoff

Consecutive images with PIV-PLIF near blowoff

Enlarged views

Final blowoff
Thank you!
Turbulent Flame Speed: Analytical Derivation

\[ \frac{\partial G}{\partial t} + \mathbf{V} \cdot \nabla G = S_d |\nabla G| \]

\[ S_d = S_L - S_L l_m K - l_m a_T \]

For a statistically planar and steady flame in isotropic turbulence, setting:

\[ G(x, y, z, t) = z + g(x, y, z, t) = 0; \quad \langle g(x, y, z, t) \rangle = 0 \]

\[ S_{T,0} / S_L = \left\langle |\nabla G| \right\rangle \]

\[ S_T / S_L \sim \left[ \frac{1}{Mk} \left( \frac{u_0}{S_L} \right) \left( \frac{l_0}{l_L} \right) \right]^{1/2} \]
Computational Details
BFPT Algorithm: Estimation Stage

Flame particle’s equation
\[
\frac{d\mathbf{x}^F}{dt} = \mathbf{v}^F = \mathbf{u}^F + S_d^F \mathbf{n}^F
\]

During backtracking, the discretized eq. is implicit\(^1,2\)
\[
\mathbf{x}^F(t - \Delta t) = \mathbf{x}^F(t) + (\mathbf{v}^F(t - \Delta t))\Delta t
\]

BFPT Algorithm

Estimation Stage
Correlation Stage

During estimation, \(\mathbf{v}^F(t - \Delta t) \approx \mathbf{v}^F(t)\)
\[
\mathbf{x}^F(t - \Delta t) = \mathbf{x}^F(t) + (\mathbf{v}^F(t))\Delta t
\]

Computational Details
BFPT Algorithm: Correlation Stage

Computational Details
BFPT Algorithm: Correlation Stage

\[ D_i = \frac{A_i \cdot B_i}{\max\{|A_i||B_i|\}^2} \]

- \( A_i \) - “available” velocity vector
- \( B_i \) - “required” velocity vector

DNS of statistically planar flames

Snapshots of DNS saved at fine time interval

Snapshots are fed to the BFPT algorithm in reverse order

Source location of turbulent flame surfaces
<table>
<thead>
<tr>
<th>$Re_t$</th>
<th>Ka</th>
<th>$Ka$</th>
</tr>
</thead>
<tbody>
<tr>
<td>686</td>
<td>23</td>
<td>1127</td>
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Yuvraj, Song, W., Dave, H.L., Im, H.G. and Chaudhuri, S., https://arxiv.org/abs/2106.08407

- DNS data of lean premixed H$_2$-air flame from Song, Perez, Tingas and Im (2020)
- Simulation Conditions: $P = 1$ atm, $T_u = 300$ K, $T_b = 2008.49$ K, $S_L = 135.619$ cm/s, $\phi = 0.7$