Lecture 13

The Turbulent Burning Velocity

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- One of the most important unresolved problems in premixed turbulent combustion is that of the turbulent burning velocity

- This statement implies that the turbulent burning velocity is a well-defined quantity that only depends on local mean quantities

- The mean turbulent flame front is expected to propagate with that burning velocity relative to the flow field

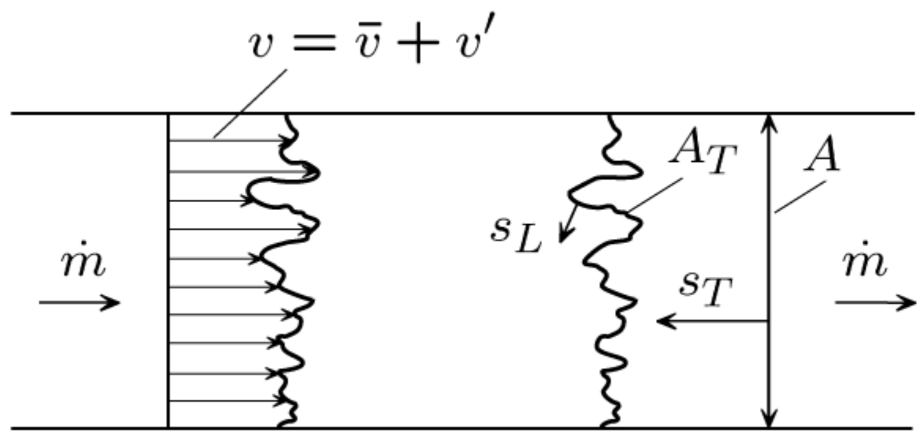
- Gas expansion effects induced at the mean front will change the surrounding flow field and may generate instabilities in a similar way as flame instabilities of the Darrieus-Landau type are generated by a laminar flame front (cf. Clavin, 1985)

Damköhler Theory for Turbulent Burning Velocity

- Damköhler (1940) was the first to present theoretical expressions for the turbulent burning velocity
- He identified two different regimes of premixed turbulent combustion, which he called large scale and small scale turbulence
- We will identify these two regimes with the corrugated flamelets regime and the thin reaction zones regime, respectively

- Damköhler equated mass flux through the instantaneous turbulent flame surface area A_T with the mass flux through the cross sectional area A, using the laminar burning velocity s_L for the mass flux through the instantaneous surface and the turbulent burning velocity s_T for the mass flux through the cross-sectional area A as

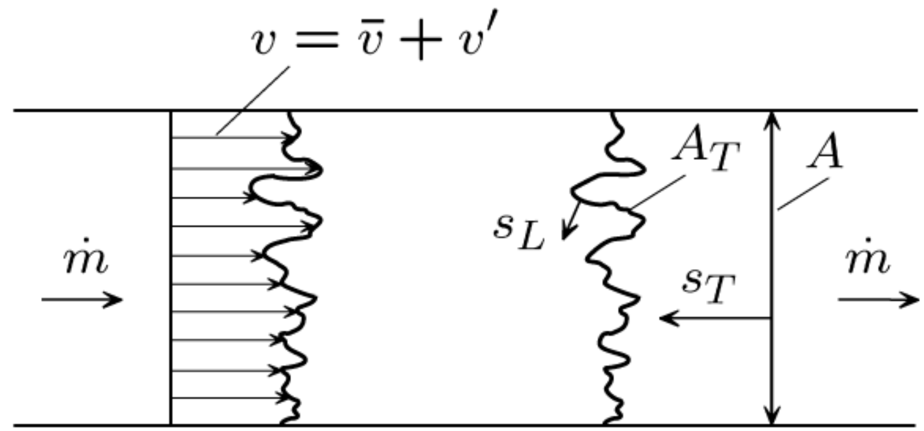
$$\dot{m} = \rho_u s_L A_T = \bar{\rho}_u s_T A$$



- The burning velocities s_L and s_T are defined with respect to the conditions in the unburnt mixture and the density ρ_u is assumed constant

- From that equation it follows

$$\frac{s_T}{s_L} = \frac{A_T}{A}$$



- Since only continuity is involved, averaging of the flame surface area can be performed at any length scale Δ within the inertial range

- If Δ is interpreted as a filter width one obtains a filtered flame surface area $\hat{A}_T$

$\dot{m} = \rho_u s_L A_T = \bar{\rho}_u s_T A$ then also implies $s_L A_T = \hat{s}_T \hat{A}_T = s_T A$

- This shows that the product $\hat{s}_T \hat{A}_T$ is inertial range invariant, similar to the dissipation in the inertial range of turbulence

- For large scale turbulence, Damköhler (1940) assumed that the interaction between a wrinkled flame front and the turbulent flow field is purely kinematic

- Using the geometrical analogy with a Bunsen flame, he related the area increase of the wrinkled flame surface area to the velocity fluctuation divided by the laminar burning velocity

- In a Bunsen flame, an increase in flow velocity leads to a proportional increase in flame area

- Hence the model

$$\frac{A_T}{A} \sim \frac{v'}{s_L}$$

- Combining

$$\frac{s_T}{s_L} = \frac{A_T}{A} \qquad \text{and} \qquad \frac{A_T}{A} \sim \frac{v'}{s_L}$$

leads to

$$s_T \sim v'$$

in the limit of large v'/s_L, which is a kinematic scaling

- We now want to show that this is consistent with the modeling assumption for the *G*-equation in the corrugated flamelets regime

- For small scale turbulence, which we will identify with the thin reaction zones regime, Damköhler (1940) argued that turbulence only modifies the transport between the reaction zone and the unburnt gas

- In analogy to the scaling relation for the laminar burning velocity

$$s_L \sim \left(\frac{D}{t_c}\right)^{1/2}$$

where t_c is the chemical time scale and *D* the molecular diffusivity, he proposes that the turbulent burning velocity can simply be obtained by replacing the laminar diffusivity *D* by the turbulent diffusivity D_t

$$s_T \sim \left(\frac{D_t}{t_c}\right)^{1/2}$$

while the chemical time scale remains the same

- Here, it is implicitly assumed that the chemical time scale is not affected by turbulence

- This assumption breaks down when Kolmogorov eddies penetrate into the thin reaction zone
- This implies that there is an upper limit for the thin reaction zones regime which was identified as the condition $Ka_\delta = 1$

- Combining

$$s_L \sim \left(\frac{D}{t_c}\right)^{1/2} \quad \text{and} \quad s_T \sim \left(\frac{D_t}{t_c}\right)^{1/2}$$

the ratio of the turbulent to the laminar burning velocity becomes

$$\frac{s_T}{s_L} \sim \left(\frac{D_T}{D}\right)^{1/2}$$

- Since the turbulent diffusivity D_T is proportional to the product $v'\ell$, and the laminar diffusivity is proportional to the product of the laminar burning velocity and the flame thickness ℓ_F one may write

$$\frac{s_T}{s_L} \sim \left(\frac{D_T}{D}\right)^{1/2}$$

as

$$\frac{s_T}{s_L} \sim \left(\frac{v'}{s_L}\frac{\ell}{\ell_F}\right)^{1/2}$$

showing that for small scale turbulence, the burning velocity ratio not only depends on the velocity ratio v'/s_L but also on the length scale ratio l/l_F

- There were many attempts to modify Damköhler's analysis and to derive expressions that would reproduce the large amount of experimental data on turbulent burning velocities

- By introducing an adjustable exponent n, where 0.5 < n < 1.0

$$s_T \sim v' \quad \text{and} \quad \frac{s_T}{s_L} \sim \left(\frac{v'}{s_L}\frac{\ell}{\ell_F}\right)^{1/2}$$

may be combined to obtain expressions of the form

$$\frac{s_T}{s_L} = 1 + C\left(\frac{v'}{s_L}\right)^n$$

- This includes the limit $v' \rightarrow 0$ for laminar flame propagation where $s_T = s_L$

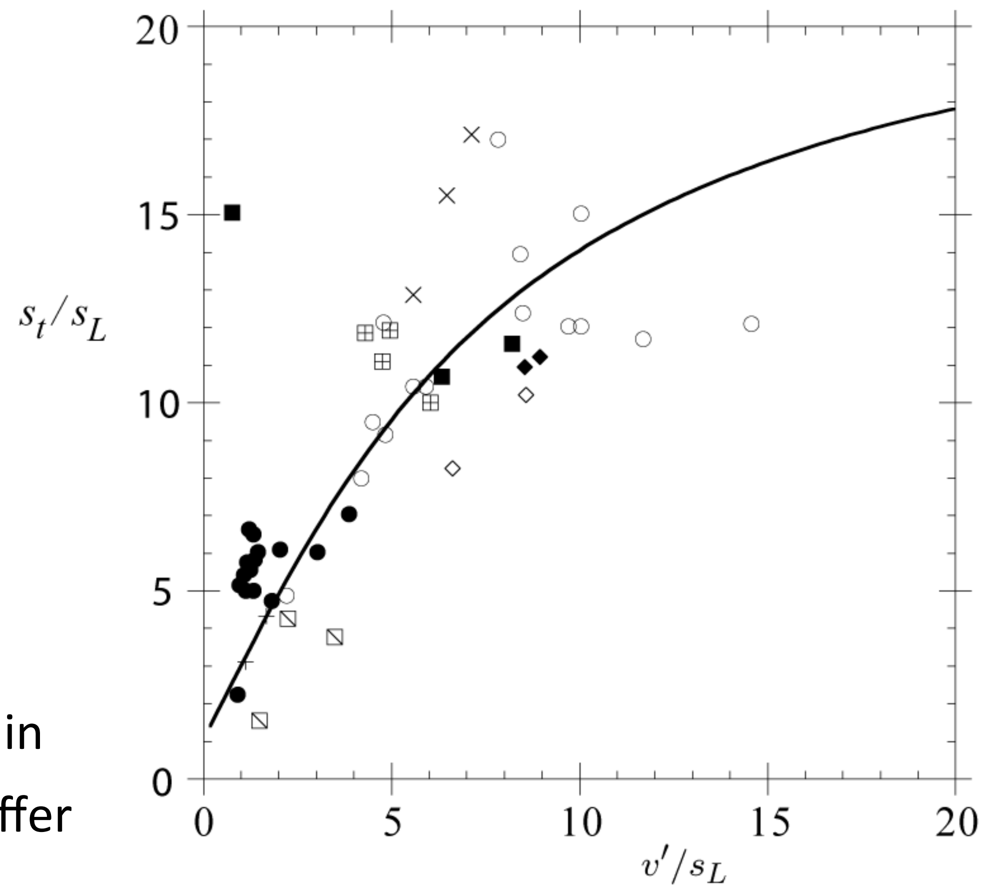
- The constant C is expected to depend on the length scale ratio ℓ/ℓ_F.

- By comparison with experiments the exponent *n* is often found to be in the vicinity of 0.7 (cf. Williams ,1985)

- Attempts to justify a single exponent on the basis of dimensional analysis, however, fall short even of Damköhler's pioneering work who had recognized the existence of two different regimes in premixed turbulent combustion

- There is a large amount of data on turbulent burning velocities in the literature

- Correlations of this material, mostly presented in terms of the burning velocity ratio s_T/s_L plotted as function of v'/s_L, called the burning velocity diagram

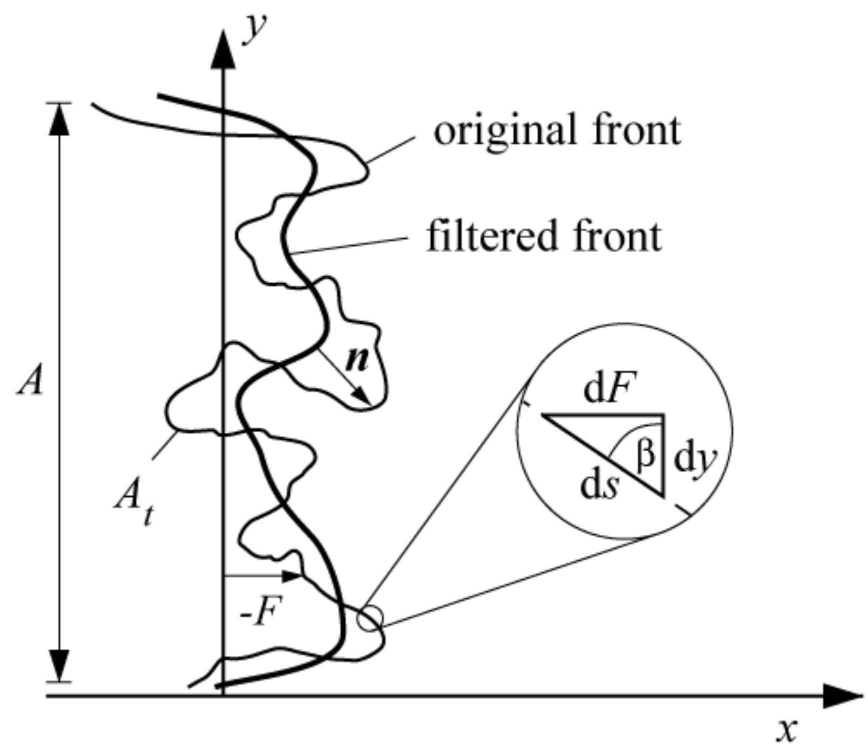


- When experimental data from different authors are collected in such a diagram, they usually differ considerably

- In the review articles by Bray (1990) and Bradley (1992), the many physical parameters that affect the turbulent burning velocity are discussed

A Model Equation for the Flame Surface Area Ratio

- It was stated previously that the mean gradient $\bar{\sigma} = \overline{|\nabla G|}$ represents the flame surface area ratio



- In the two-dimensional illustration, the instantaneous flame surface area A_T is identified with the length of the line $G=G_0$, where here the G-field is defined as $G = x + F(y)$

- The blow-up shows that a differential section dS of that line and the corresponding differential section dy of the cross sectional area A are related to each other by $$\frac{\mathrm{d}S}{\mathrm{d}y} = \frac{1}{|\cos\beta|}$$

- On the other hand, in two dimensions the gradient magnitude σ is given by

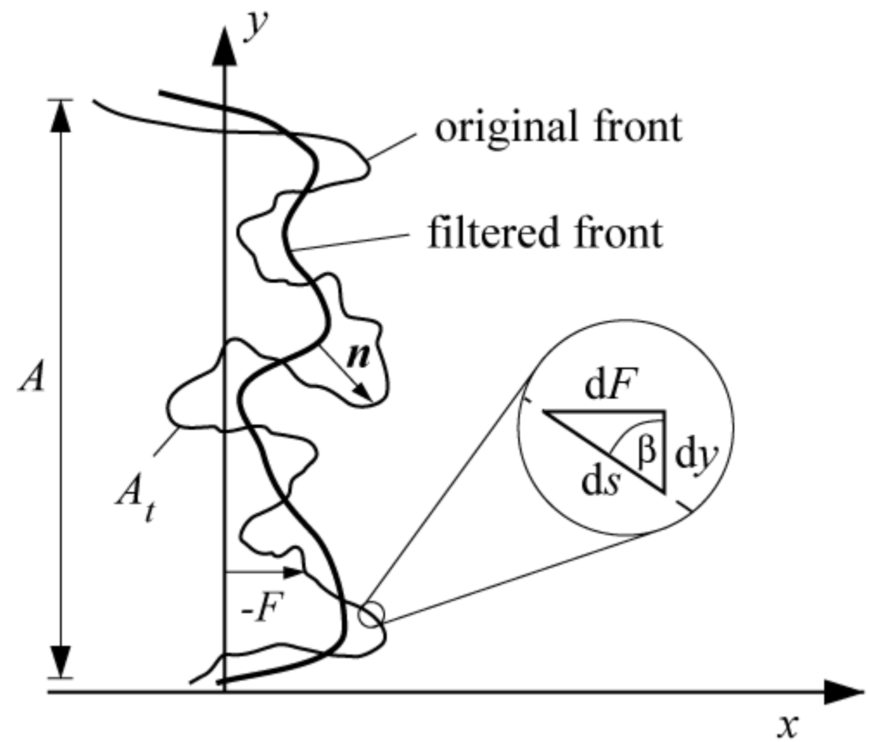
$$\sigma = \left(1 + \left(\frac{\partial F}{\partial y}\right)^2\right)^{1/2}$$

- It can be seen that $\partial F / \partial y = \tan\beta$ which using $1 + \tan^2\beta = \frac{1}{\cos\beta}$ relates σ to the angle β as

$$\sigma = \frac{1}{|\cos\beta|}$$

and therefore the differential flame surface area ratio is equal to the gradient σ:

$$\frac{\mathrm{d}S}{\mathrm{d}y} = \sigma$$



- We now want to derive a modeled equation for the flame surface area ratio in order to determine the turbulent burning velocity

- An equation for σ can be derived from

$$\rho\frac{\partial G}{\partial t} + \rho \boldsymbol{v} \cdot \nabla G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

- For illustration purpose we assume constant density and constant values of s_L^0 and D

- Applying the Nabla-operator to both sides of

$$\rho\frac{\partial G}{\partial t} + \rho\boldsymbol{v}\cdot\boldsymbol{\nabla}G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

and multiplying this with $-\boldsymbol{n} = \boldsymbol{\nabla}G/\sigma$

one obtains

$$\frac{\partial\sigma}{\partial t}+\boldsymbol{v}\cdot\boldsymbol{\nabla}\sigma = -\boldsymbol{n}\cdot\boldsymbol{\nabla}\boldsymbol{v}\cdot\boldsymbol{n}\,\sigma+s_L^0(\kappa\sigma+\boldsymbol{\nabla}^2G)+D\,\boldsymbol{n}\cdot\boldsymbol{\nabla}(\kappa\sigma)$$

- The terms on the RHS are
    1. straining by the flow field, which amounts to a production of flame surface area
    2. Term containing the laminar burning velocity has a similar effect as kinematic restoration has in the variance equation
    3. Last term is proportional to *D* and its effect is similar to that of scalar dissipation in the variance equation

- Introducing appropriate models for the unclosed terms (Peters, 1999), the resulting model equation for the unconditional quantity $\bar{\sigma}_t$ covering both regimes, is written as

$$
\begin{aligned}
\bar{\rho}\frac{\partial\bar{\sigma}_t}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\bar{\sigma}_t \;&=\; \boldsymbol{\nabla}_{||}\cdot\left(\bar{\rho}D_t\boldsymbol{\nabla}_{||}\,\bar{\sigma}_t\right) \\
&\quad + c_0\bar{\rho}\frac{(-\widetilde{\boldsymbol{v}''\boldsymbol{v}''}):\boldsymbol{\nabla}\tilde{\boldsymbol{v}}}{\tilde{k}}\,\bar{\sigma}_t + c_1\bar{\rho}\frac{D_t(\boldsymbol{\nabla}\tilde{G})^2}{\widetilde{G''^2}}\,\bar{\sigma}_t \\
&\quad - c_2\bar{\rho}\frac{s_L^0\bar{\sigma}_t^2}{(\widetilde{G''^2})^{1/2}} - c_3\bar{\rho}\frac{D\bar{\sigma}_t^3}{\widetilde{G''^2}}.
\end{aligned}
$$

- The terms on the RHS are
  1. First term is turbulent transport
  2. The second term is production of flame surface area ratio due to mean velocity gradients
  3. The last three terms represent turbulent production, kinematic restoration, and scalar dissipation of the flame surface area ratio, respectively

- The production term due to velocity gradients is in general much smaller than production by turbulence and will be neglected
- Then, using using the assumption that production equals dissipation in the equation for the flame surface area ratio leads to an algebraic expression allowing the determination of the turbulent burning velocity

- Using

$$\ell_{F,t} = \left. \frac{(\widetilde{G''^2}(\boldsymbol{x},t))^{1/2}}{|\nabla \tilde{G}|} \right|_{\tilde{G}=G_0}$$

this relation becomes a quadratic equation for $\bar{\sigma}_t$

$$c_1 \frac{D_t}{\ell_{F,t}^2} - c_2 \frac{s_L^0}{\ell_{F,t}} \frac{\bar{\sigma}_t}{|\nabla \tilde{G}|} - c_3 \frac{D}{\ell_{F,t}^2} \frac{\bar{\sigma}_t^2}{|\nabla \tilde{G}|^2} = 0.$$

- In Peters (2000) it is shown that the turbulent burning velocity s_T is related to the mean flame surface area ratio as

$$(\rho s_T^0) \mid \nabla \tilde{G} \mid = (\rho s_L^0) \, \bar{\sigma}_t.$$

providing a quadratic equation for s_T

- The difference Δs between the turbulent and the laminar burning velocity is then

$$\Delta s = s_T^0 - s_L^0 = s_L^0 \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|}$$

- Taking only the positive root in the solution of

$$\frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} + \frac{a_4 b_3^2}{b_1} \frac{\ell}{\ell_F} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - a_4 b_3^2 \frac{v' \ell}{s_L^0 \ell_F} = 0$$

this leads to the algebraic expression for Δs

$$\frac{\Delta s}{s_L^0} = -\frac{a_4 b_3^2}{2\, b_1} \frac{\ell}{\ell_F} + \left[\left(\frac{a_4 b_3^2}{2\, b_1} \frac{\ell}{\ell_F} \right)^2 + a_4 b_3^2 \frac{v' \ell}{s_L^0 \ell_F} \right]^{1/2}$$

- The modeling constants used in the final equations are summarized in a table

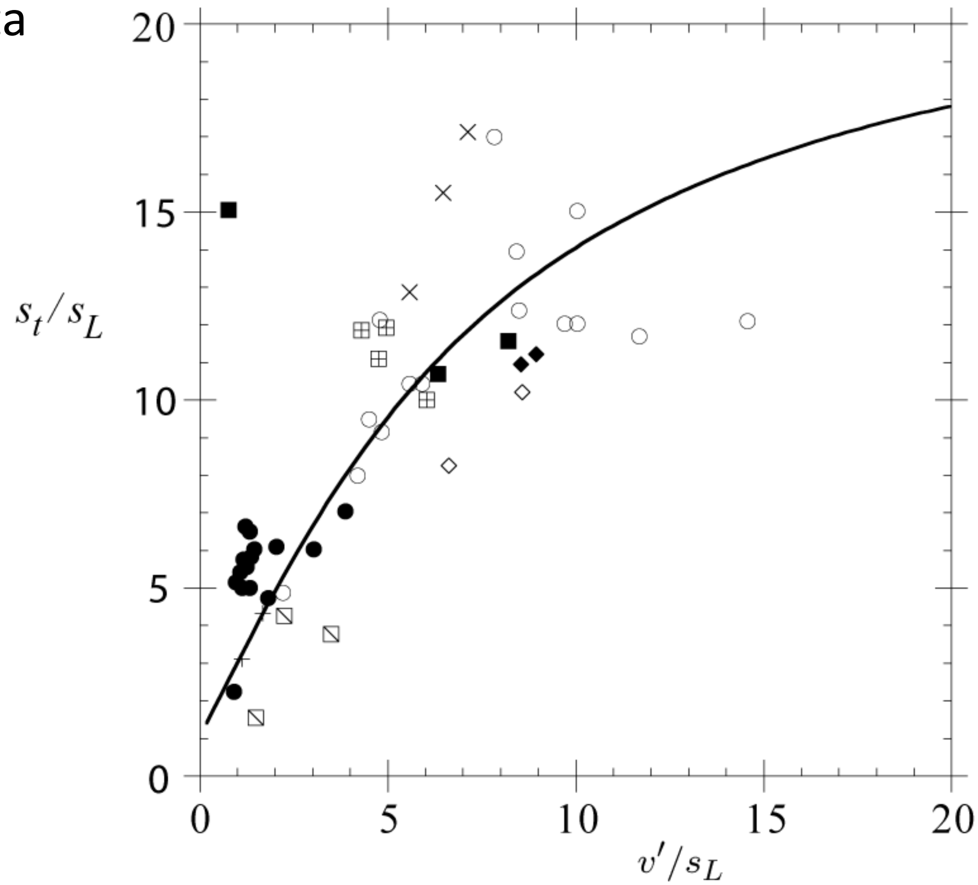
constant	equation	suggested value	origin
a_1	$\tilde{\varepsilon} = a_1 v'^3/\ell$	0.37	Bray (1990)
a_2	$\tilde{k} = a_2 v'^2$	1.5	definition
a_3	$\tau = a_3 \ell/v'$	4.05	$\tau = \tilde{k}/\tilde{\varepsilon}$
a_4	$D_t = a_4 v' \ell$	0.78	$D_t = \nu_t/0.7$
b_1	$s_T = b_1 v'$	2.0	experimental data
b_2	$\ell_{F,t} = b_2 \ell$	1.78	$(2a_3a_4/c_s)^{1/2}$
b_3	$s_T^0/s_L^0 = b_3(D_t/D)^{1/2}$	1.0	Damköhler (1940)
c_0	$c_0 = c_{\varepsilon 1} - 1$	0.44	standard value
c_1	Eq. (13.18)	4.63	DNS
c_2	Eq. (13.18)	1.01	$a_4c_1/(b_1b_2)$
c_3	Eq. (13.18)	4.63	$c_1 = c_3$
c_s	Eq. (12.47)	2.0	spectral closure

- Note that b_1 is the only constant that has been adjusted using experimental data from turbulent burning velocity while the constant b_3 was suggested by Damköhler (1940). The constant c_1 was obtained from DNS and all other constants are related to constants in standard turbulence models

- If

$$\frac{\Delta s}{s_L^0} = -\frac{a_4 b_3^2}{2\,b_1}\frac{\ell}{\ell_F} + \left[\left(\frac{a_4 b_3^2}{2\,b_1}\frac{\ell}{\ell_F}\right)^2 + a_4 b_3^2 \frac{v'\ell}{s_L^0 \ell_F}\right]^{1/2}, \quad \mathrm{Re}_t = \frac{v'\ell}{s_L^0 \ell_F}$$

  is compared with experimental data as in the burning velocity diagram, the turbulent Reynolds number appears as a parameter



- From the viewpoint of turbulence modeling this seems disturbing, since in free shear flows any turbulent quantity should be independent of the Reynolds number in the large Reynolds number limit

- The apparent Reynolds number dependence turns out to be an artifact, resulting from the normalization of Δs by $s_L{}^0$, which is a molecular quantity whose influence should disappear in the limit of large Reynolds numbers and large values of v'/s_L

- If the burning velocity difference Δs is normalized by v' rather than by $s_L{}^0$, it may be expressed as a function of the turbulent Damköhler number

$$\mathrm{Da}_t = s_L^0 \ell / v' \ell_F$$

- One obtains the form

$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2}$$

- This is Reynolds number independent and only a function of a single parameter, the turbulent Damköhler number

- In the limit of large scale turbulence

$$\ell/\ell_F \to \infty \qquad \text{or} \qquad \mathrm{Da}_t \to \infty$$

it becomes Damköhler number independent

- In the small scale turbulence limit

$$\ell/\ell_F \to 0 \qquad \text{or} \qquad \mathrm{Da}_t \to 0$$

it is proportional to the square root of the Damköhler number

- A Damköhler number scaling has also been suggested by Gülder (1990) who has proposed

$$\frac{\Delta s}{v'} = 0.62\,\mathrm{Da}_t^{1/4}$$

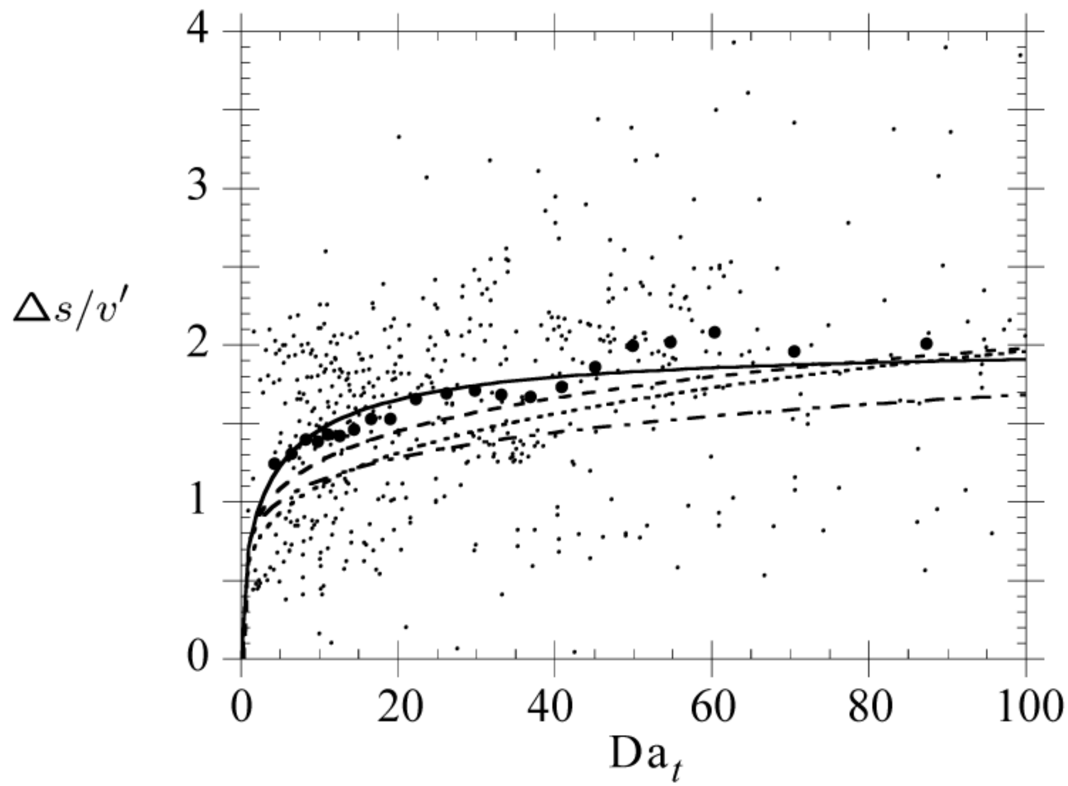
as an empirical fit to a large set of burning velocity data

- The correlations

$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\,b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\,b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2} \qquad \frac{\Delta s}{v'} = 0.62\,\mathrm{Da}_t^{1/4}$$

$$\frac{\Delta s}{v'} = 1.53\left(\frac{s_L^0}{v'}\right)^{0.3}\mathrm{Da}_t^{0.15}\mathrm{Le}^{-0.3} - \frac{s_L^0}{v'}$$

from various sources are compared amongst each other and with data from the experimental data collection



- The data points show a large scatter, which is due to the fact that the experimental conditions were not always well defined

- Since unsteady effects have been neglected in deriving

$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2}$$

  only data based on steady state experiments were retained from this collection

- These 598 data points and their averages within fixed ranges of the turbulent Damköhler number are shown as small and large dots