Turbulent Premixed Combustion

CEFRC Combustion Summer School
2014

Prof. Dr.-Ing. Heinz Pitsch
Example: LES of a stationary gas turbine
Course Overview

Part II: Turbulent Combustion

- Turbulence
- Turbulent Premixed Combustion
- Turbulent Non-Premixed Combustion
- Modelling Turbulent Combustion
- Applications

- Scales of Turbulent Premixed Combustion
- Regime-Diagram
- Turbulent Burning Velocity
Scales of Turbulent Premixed Combustion

- Integral turbulent scales
  \[ l_t = c_1 \frac{k^{3/2}}{\bar{\varepsilon}}, \quad u' = \sqrt{\frac{2\bar{\nu}}{3}k}, \quad \tau = \frac{l_t}{u'} \sim \frac{\bar{k}}{\bar{\varepsilon}} \]

- Smallest turbulent scales/Kolmogorov scales
  \[ \eta = \left( \frac{\nu^3}{\bar{\varepsilon}} \right)^{1/4}, \quad u_\eta = (\nu \bar{\varepsilon})^{1/4}, \quad t_\eta = \left( \frac{\nu}{\bar{\varepsilon}} \right)^{1/2} \]

- Flame thickness and time, reaction zone thickness
  \[ l_F = \frac{D}{s_L} = \frac{\lambda_b}{\rho_u c_p s_L}, \quad t_F = \frac{l_F}{s_L} = \frac{D}{s_L^2}, \quad l_\delta \ll l_F \]
Dimensionless Quantities in Premixed Turbulent Combustion

- **Turbulent Reynolds number**

\[ Sc = \frac{\nu}{D} = 1 \quad \Rightarrow \quad Re_t = \frac{l_t}{l_F} \frac{u'}{s_L} \]

- **Turbulent Damköhler number**

\[ Da_t = \frac{\tau}{t_F} = \frac{l_t}{l_F} \frac{s_L}{u'} \]

- **Karlovitz number** (interaction of small-scale turbulence with the flame)

\[ Ka = \frac{t_F}{\eta} = \frac{l_F^2}{\eta^2} = \sqrt{\frac{l_F}{l_t} \left( \frac{u'}{s_L} \right)^3} \quad \text{und} \quad Ka_\delta = \frac{l_F^2}{\eta^2} = \delta^2 Ka \]

\[ \delta \approx 0.1 \]
Course Overview

Part II: Turbulent Combustion

• Turbulence
• Turbulent Premixed Combustion
• Turbulent Non-Premixed Combustion
• Modelling Turbulent Combustion
• Applications

• Scales of Turbulent Premixed Combustion
  • Regime-Diagram
• Turbulent Burning Velocity
Regime Diagram

Corrugated Flamelet Regime

\[ Re_t = \frac{l_t}{l_F} \frac{u'}{s_L} \]
\[ Ka^2 = \left( \frac{l_t}{l_F} \right)^{-1} \left( \frac{u'}{s_L} \right)^3 \]
\[ Da_t = \frac{l_t}{l_F} \left( \frac{u'}{s_L} \right)^{-1} \]
Regime Diagram: Corrugated Flamelets

- $Ka < 1 \rightarrow \eta > l_F$
  - Interaction of a very thin flame with a turbulent flow
  - Assumption: infinitely thin flame (compared to turbulent scales)

Buschmann (1996)
Regime Diagramm: Broken Reaction Zones Regime

\[ Re_t = \frac{l_t}{l_F} \frac{u'}{s_L} \]

\[ Ka^2 = \left( \frac{l_t}{l_F} \right)^{-1} \left( \frac{u'}{s_L} \right)^3 \]

\[ Da_t = \frac{l_t}{l_F} \left( \frac{u'}{s_L} \right)^{-1} \]
Regime Diagramm: Broken Reaction Zones Regime

- $K\alpha_\delta > 1 \rightarrow \eta < l_\delta$
  - Smallest turbulent eddies enter the reaction zones
  - Turbulent transport $\rightarrow$ radicals are removed from reaction zone
  - Local extinguishing in the inner reaction zone
    - Overall extinguishing of the flame front is possible

Two-dimensional slices from three-dimensional simulations of low- and high-Karlovitz-supernovae flames, respectively. The left-hand panel in each case is burning rate, and the right-hand panel is temperature.

Source: A. J. Aspden et al. (JFM 2011)
Regime Diagramm: Thin Reaction Zones Regime

\[ Re_t = \frac{l_t u'}{l_F s_L} \]

\[ Ka^2 = \left( \frac{l_t}{l_F} \right)^{-1} \left( \frac{u'}{s_L} \right)^3 \]

\[ Da_t = \frac{l_t}{l_F} \left( \frac{u'}{s_L} \right)^{-1} \]
Regime Diagramm: Thin Reaction Zones Regime

- $Ka > 1$ und $K_\delta < 1 \rightarrow l_\delta < \eta < l_F$
  - With $l_\delta \approx 0,1l_F \rightarrow Ka \approx 100K_\delta$
  - Turbulent mixing inside preheat zone
  - Assumption: infinitely thin reaction zone (compared to turbulent scales)

thin reaction zone

thickened preheat zone

temperature distribution from DNS of a premixed turbulent flame
Regime Diagram: Résumé

<table>
<thead>
<tr>
<th>Case</th>
<th>A40</th>
<th>B40</th>
<th>C40</th>
<th>D40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalence ratio ($\phi$)</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Flame speed ($u_\infty$) (m s$^{-1}$)</td>
<td>$2.24 \times 10^{-1}$</td>
<td>$2.24 \times 10^{-1}$</td>
<td>$2.24 \times 10^{-1}$</td>
<td>$2.24 \times 10^{-1}$</td>
</tr>
<tr>
<td>Flame width ($\ell_{f}$) (m)</td>
<td>$6.29 \times 10^{-4}$</td>
<td>$6.29 \times 10^{-4}$</td>
<td>$6.29 \times 10^{-4}$</td>
<td>$6.29 \times 10^{-4}$</td>
</tr>
<tr>
<td>Domain width ($L$) (m)</td>
<td>$3.14 \times 10^{-3}$</td>
<td>$3.14 \times 10^{-3}$</td>
<td>$3.14 \times 10^{-3}$</td>
<td>$3.14 \times 10^{-3}$</td>
</tr>
<tr>
<td>Domain height ($H$) (m)</td>
<td>$2.51 \times 10^{-2}$</td>
<td>$2.51 \times 10^{-2}$</td>
<td>$2.51 \times 10^{-2}$</td>
<td>$2.51 \times 10^{-2}$</td>
</tr>
<tr>
<td>Integral length scale ($l$) (m)</td>
<td>$3.14 \times 10^{-4}$</td>
<td>$3.14 \times 10^{-4}$</td>
<td>$3.14 \times 10^{-4}$</td>
<td>$3.14 \times 10^{-4}$</td>
</tr>
<tr>
<td>Length ratio ($l/l_{f}$)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>RMS velocity ($\bar{u}$) (m s$^{-1}$)</td>
<td>0.825</td>
<td>3.83</td>
<td>7.34</td>
<td>23.9</td>
</tr>
<tr>
<td>Velocity ratio ($\bar{u}/s_{L}$)</td>
<td>3.69</td>
<td>17.1</td>
<td>32.9</td>
<td>106.8</td>
</tr>
<tr>
<td>Karlovitz number ($K_{\alpha \delta}$)</td>
<td>10</td>
<td>100</td>
<td>266</td>
<td>1562</td>
</tr>
<tr>
<td>Damköhler number ($Da_{L}$)</td>
<td>$1.36 \times 10^{-1}$</td>
<td>$2.92 \times 10^{-2}$</td>
<td>$1.52 \times 10^{-2}$</td>
<td>$4.68 \times 10^{-3}$</td>
</tr>
<tr>
<td>Levels of refinement</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Effective resolution ($N$)</td>
<td>$128^2 \times 1024$</td>
<td>$128^2 \times 1024$</td>
<td>$128^2 \times 1024$</td>
<td>$256^2 \times 2048$</td>
</tr>
<tr>
<td>Cell width ($\Delta x$) (m)</td>
<td>$2.45 \times 10^{-5}$</td>
<td>$2.45 \times 10^{-5}$</td>
<td>$2.45 \times 10^{-5}$</td>
<td>$2.45 \times 10^{-5}$</td>
</tr>
<tr>
<td>Kolmogorov length ($\eta$) (m)</td>
<td>$4.33 \times 10^{-5}$</td>
<td>$1.37 \times 10^{-5}$</td>
<td>$8.41 \times 10^{-6}$</td>
<td>$3.47 \times 10^{-6}$</td>
</tr>
<tr>
<td>Cell Kolmogorov length ($\eta_{\Delta x}$) (m)</td>
<td>$7.36 \times 10^{-6}$</td>
<td>$7.36 \times 10^{-6}$</td>
<td>$7.36 \times 10^{-6}$</td>
<td>$7.36 \times 10^{-6}$</td>
</tr>
<tr>
<td>Effective Kolmogorov length ($\eta_{L}$) (m)</td>
<td>$4.33 \times 10^{-5}$</td>
<td>$1.51 \times 10^{-5}$</td>
<td>$11.2 \times 10^{-6}$</td>
<td>$5.12 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

**TABLE 2.** Turbulent flame properties for the four simulations at equivalence ratio $\phi = 0.40$.

Source: A. J. Aspden et al. (JFM 2011)
Figure 8. Two-dimensional vertical slices through three-dimensional simulations showing density, burning rate and temperature at $\varphi = 0.40$, respectively. The density, burning rate and temperature ranges are [0.2, 1.02] kg m$^{-3}$, [0.64] kg m$^{-3}$ s$^{-1}$ and [298, 1600] K, respectively.

Source: A. J. Aspden et al. (JFM 2011)
Part II: Turbulent Combustion

- Turbulence
- Turbulent Premixed Combustion
- Turbulent Non-Premixed Combustion
- Modelling Turbulent Combustion
- Applications
- Scales of Turbulent Premixed Combustion
  - Regime-Diagram
  - Turbulent Burning Velocity
Turbulent Burning Velocity

Comparison: Laminar/Measured Burning Velocity

Laminar burning velocity of iso-octane

\[ s_L \approx 15 \text{ m/s} \]

Exemplary measurements in gasoline engine with tumble generator of flame velocity at spark plug position during full load (Source: Merker, „Grundlagen Verbrennungsmotoren“)
Comparison: Laminar/Measured Burning Velocity

Experimental data of $s_T$ vs. wrinkled laminar-flame theories of turbulent flame propagation (data from Turns 2000)

$\approx$ factor 30

$v_{ Flam } = 0.40 \text{ m/s}$

$v_{ Flam } = 0.20 \text{ m/s}$
Turbulent Burning Velocity

- Main problem for turbulent premixed combustion: 
  Quantification of turbulent burning velocity \( s_T \)
- \( s_T \): Velocity which quantifies the propagation of the turbulent flame front into unburnt mixture
- Distinction of two limiting cases by Damköhler (1940)
  1. Large scale turbulence ↔ corrugated flamelets
  2. Small scale turbulence ↔ thin reaction zones
Turbulent Burning Velocity: Corrugated Flamelets

- Instantaneous flame front
  - Flame surface area $A_T$
  - Propagates locally with laminar burning velocity $s_L$ into unburnt mixture

- Mean flame front
  - Mean flame surface area $A$
  - Propagates with turbulent burning velocity $s_T$
Turbulent Burning Velocity: Corrugated Flamelets

- With the mass flux through $A$ and $A_T$

$$\dot{m} = \rho_u s_L A_T = \bar{\rho}_u s_T A$$

- Assume constant density in the unburnt mixture (assumption) ($\rho_u = \bar{\rho}_u$)

$$\frac{s_T}{s_L} = \frac{A_T}{A}$$

- Wrinkling of the laminar flame ($A_T \uparrow$) $\Rightarrow$ increase of $s_T$
Turbulent Burning Velocity: Corrugated Flamelets

• Turbulence $\rightarrow$ flame surface area $\uparrow$

• Using an analogy with a Bunsen flame

\[ s_L = u_u \sin \alpha \quad \text{hier} \quad \sin \alpha = \frac{s_L}{u'} \quad \Rightarrow \quad \frac{A_T}{A} \sim \frac{d}{s_L} = \frac{u'}{A} \]

• Limit for $u' \rightarrow 0$

\[ s_T = \frac{A_T}{A} = 1 + \frac{u'}{s_L} \]

• Internal combustion engine:
  - Engine speed $n \uparrow \rightarrow$ burning velocity $s_T \uparrow$ due to

\[ u' \sim u_{\text{piston}} \sim n \]

$\rightarrow$ High engine speed achievable
Turbulent Burning Velocity: large-scale turbulence

- In experiments often used empirical relation

\[
\frac{s_T}{s_L} = 1 + C \left( \frac{u'}{s_L} \right)^n
\]

- Constant C experimentally determined
- Typical values: 0.5 < n < 1.0

- From experimental data →
  - For small \( u' \), \( s_T \sim u' \) applies
    - Consistent with Damköhler theory
  - Increase of turbulent intensity
    - \( s_T \) grows linearly
    - With further increase less than linear
Turbulent Burning Velocity: Thin Reaction Zones

- Reduced increase of turbulent burning velocity
  → second limiting case of Damköhler
- Thin reaction zones/small-scaled turbulence
- In analogy to

  \[ s_L = \sqrt{\frac{D}{t_c}} \]

  \[ s_T = \sqrt{\frac{D_t}{t_c}} \]

  Damköhler uses

  \[ \frac{s_T}{s_L} = \sqrt{\frac{D_t}{D}} = \sqrt{\frac{0.78 u' l_t}{s_L l_F}} \]

  - \( t_c \): chemical time scale
  - Dimensional analysis \( D_t \sim u' l_t \)
  - Constant of proportionality 0.78

  consistent with experimental data
Turbulent Burning Velocity

- Damköhler-limits can be combined to a single formula (Peters, 1999):

\[
\frac{s_T}{s_L} = 1 - \alpha \frac{l_t}{l_F} + \sqrt{\left(\frac{\alpha l_t}{l_F}\right)^2 + 4\alpha \frac{u' l_t}{s_L l_F}}
\]

- constant \( \alpha = 0,195 \)

- Low turbulence intensity →

\[
\frac{s_T}{s_L} = 1 + 2 \frac{u'}{s_L}
\]

- High turbulence intensity →

\[
\frac{s_T}{s_L} = 1 + \sqrt{0,78 \frac{u' l_t}{s_L l_F}}
\]
Turbulent Burning Velocity

• By rearranging this formula with \( Da_t = (l_t s_L)/(l_F u') \) →

\[
\frac{s_T - s_L}{u'} = -\alpha Da_t + \sqrt{\alpha^2 Da_t^2 + 4\alpha Da_t}
\]

• Limit for high Damköhler number →

\[
\lim_{Da_t \to \infty} \frac{s_T - s_L}{u'} = 2
\]

comparison with experimental data
Summary

Part II: Turbulent Combustion

- Turbulence
- Turbulent Premixed Combustion
- Turbulent Non-Premixed Combustion
- Modelling Turbulent Combustion
- Applications

- Scales of Turbulent Premixed Combustion
- Regime-Diagram
- Turbulent Burning Velocity