Turbulent Non-Premixed Combustion

CEFRC Combustion Summer School
2014

Prof. Dr.-Ing. Heinz Pitsch
Course Overview

Part II: Turbulent Combustion

- Turbulence
- Turbulent Premixed Combustion
- Turbulent Non-Premixed Combustion
- Modelling Turbulent Combustion
- Applications
- Laminar Jet Diffusion Flames
- Turbulent Jet Diffusion Flames
Laminar Jet Diffusion Flames
Round Laminar Diffusion Flame

- **Fuel** enters into the combustion chamber as a *round jet*
- Forming mixture is ignited
- Example: Flame of a gas lighter
  - Only *stable* if dimensions are small
  - Dimensions too large: *flickering due to influence of gravity*
  - Increasing the jet momentum → Reduction of the relative importance of gravity (buoyancy) in favor of momentum forces
  - At high velocities, hydrodynamic instabilities gain increasing importance: *laminar-turbulent transition*
Laminar Diffusion Flame: Influence of Gravity

1g

0g
Round Laminar Diffusion Flame

- **Starting point**: Conservation equations for stationary axisymmetric boundary layer flow without buoyancy
  
  - **Continuity**:
    
    \[
    \frac{\partial (\rho u_z r)}{\partial z} + \frac{\partial (\rho u_r)}{\partial r} = 0
    \]

  - **Momentum equation in z-direction**
    
    \[
    \rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r \frac{\partial u_z}{\partial r} = -r \frac{\partial p}{\partial z} + \frac{\partial}{\partial r} \left( \mu r \frac{\partial u_z}{\partial r} \right)
    \]

  - **Mixture fraction**
    
    \[
    \rho u_z r \frac{\partial Z}{\partial z} + \rho u_r \frac{\partial Z}{\partial r} = \frac{\partial}{\partial r} \left( \frac{\mu}{Sc} r \frac{\partial Z}{\partial r} \right)
    \]
Round Laminar Diffusion Flame

- Schmidt number $Sc = \frac{\mu}{\rho D}$
- Farfield area
  - $r \to \infty$: $u_z = u_r = 0$
  - From $z$-momentum equation $\Rightarrow \frac{dp}{dz} = 0$
- Boundary layer flow:
  \[
  \rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r \frac{\partial u_z}{\partial r} = -r \frac{\partial p}{\partial z} + \frac{\partial}{\partial r} \left( \mu r \frac{\partial u_z}{\partial r} \right)
  \]
- Incompressible round jet
  - Quiescent ambient
  - Constant density
  - No buoyancy
  $\Rightarrow$ Similarity solution
- Similarity coordinate $\eta = \frac{r}{z}$
  (Schlichting, „Boundary Layer Theory“)
Round Laminar Diffusion Flame

- If density not constant → Transformation

\[ \zeta = z + a, \quad \eta = \frac{\sqrt{2 \int_0^r \frac{\rho}{\rho_\infty} r \, dr}}{\zeta} \]

- \( a \): Distance of the virtual origin of the jet from the nozzle exit

- For \( \rho = \text{const. und } a \to 0 \)

\[ \zeta = z, \quad \eta = \frac{r}{z} \]

- Implies linear spreading of the round jet
Round Laminar Diffusion Flame

- Introduction of a stream function $\Psi$

$$\rho u_z r = \frac{\partial \Psi}{\partial r}, \quad \rho u_r r = -\frac{\partial \Psi}{\partial z}$$

→ Continuity equation identically satisfied

- Applying the transformation rules

$$\zeta = z + a, \quad \eta = \frac{\sqrt{2 \int_0^r \frac{\rho}{\rho_\infty} rdr}}{\zeta} \quad \rightarrow \frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta} + \frac{\partial \eta}{\partial z} \frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r} \frac{\partial}{\partial \eta}$$

to the convective terms in the momentum and mixture fraction equations yields

$$\rho u_z r \frac{\partial}{\partial z} + \rho u_r r \frac{\partial}{\partial r} = \frac{\partial \eta}{\partial r} \left( \frac{\partial \Psi}{\partial \eta} \frac{\partial}{\partial \zeta} - \frac{\partial \Psi}{\partial \zeta} \frac{\partial}{\partial \eta} \right)$$
Round Laminar Diffusion Flame

- Such manipulations are always possible for two-dimensional stationary boundary layer flows, if a stream function and a similarity coordinate $\zeta \neq f(r)$ can be introduced.

- The diffusive terms become

$$\frac{\partial}{\partial r} \left( \mu r \frac{\partial}{\partial r} \right) = \mu_\infty \frac{\partial}{\partial r} \frac{\partial}{\partial \eta} \left( C \eta \frac{\partial}{\partial \eta} \right)$$

- $C$: Chapman-Rubesin-Parameter

$$C = \frac{\rho \mu r^2}{2 \mu_\infty \int_0^r \rho r dr}$$

- For constant density (with $\eta = r/\zeta$ and $\mu = \mu_\infty$): $C = 1$
Formal transformation of the momentum and concentration equations and assumption that \( C = f(\zeta, \eta) \)

Ansatz for stream function

\[
\Psi = \mu_\infty \zeta F(\zeta, \eta)
\]

and for the velocities

\[
u_z = \frac{\partial F/\partial \eta}{\eta} \frac{\mu_\infty}{\rho_\infty \zeta}, \quad \rho u_r r = -\mu_\infty \left( \zeta \frac{\partial F}{\partial \zeta} + F - \eta \frac{\partial F}{\partial \eta} \right)
\]

\( u_z \) und \( u_r \) can be expressed as a function of the nondimensional stream function \( F \) and its derivatives
Round Laminar Diffusion Flame

- From the momentum equation

\[
\rho u_z r \frac{\partial u_z}{\partial z} + \rho u_r r \frac{\partial u_z}{\partial r} = \frac{\partial}{\partial r} \left( \mu r \frac{\partial u_z}{\partial r} \right)
\]

\rightarrow

\[
\zeta \left( \frac{\partial F}{\eta} \frac{\partial}{\partial \zeta} \frac{\partial F}{\eta} - \frac{\partial F}{\partial \zeta} \frac{\partial}{\partial \eta} \frac{\partial F}{\eta} \right) - \frac{\partial}{\partial \eta} \left( F \frac{\partial F}{\eta} \right) = \frac{\partial}{\partial \eta} \left( C_\eta \frac{\partial}{\partial \eta} \frac{\partial F}{\eta} \right)
\]

- Similarity solution only exists, if \( F \neq f(\zeta) \)
- Then, \( u_z \) is proportional to \( 1/\zeta \) (see previous slide)
  \rightarrow velocity decreases linearly with \( 1/(z + a) \)
- Prerequisites: Boundary conditions and \( C \) are independent of \( z \)
  (e.g. \( u_z = 0 \) and \( u_r = 0 \) for \( \eta \rightarrow 0 \))
Round Laminar Diffusion Flame

- Equation for the nondimensional stream function

\[- \frac{\partial}{\partial \eta} \left( F \frac{\partial F}{\partial \eta} \right) = \frac{\partial}{\partial \eta} \left( C \eta \frac{\partial}{\partial \eta} \left( \frac{\partial F}{\partial \eta} \right) \right)\]

- Let \( \omega = Z(z,r)/Z_a(z) \), ratio of the mixture fraction \( Z_a(z) \) to its value at \( r = 0 \)

- Applying the same transformations to the \( \omega \)-equation yields

\[\zeta \left( \frac{\partial F}{\partial \eta} \frac{\partial \omega}{\partial \zeta} - \frac{\partial F}{\partial \zeta} \frac{\partial \omega}{\partial \eta} \right) + \zeta \frac{\partial F}{\partial \eta} \omega \frac{\partial \ln(Z_a)}{\partial \zeta} - F \frac{\partial \omega}{\partial \eta} = \frac{\partial}{\partial \eta} \left( \frac{C}{Sc} \eta \frac{\partial \omega}{\partial \eta} \right)\]

- In case of a similarity solution

\[-F \frac{\partial \omega}{\partial \eta} = \frac{\partial}{\partial \eta} \left( \frac{C}{Sc} \eta \frac{\partial \omega}{\partial \eta} \right)\]
Round Laminar Diffusion Flame

• If $C = \text{const.}$:

$$F = \frac{C (\gamma \eta)^2}{1 + (\gamma \eta)^2/4}, \quad \omega = \left( \frac{1}{1 + (\gamma \eta)^2/4} \right)^{2Sc}$$

• The assumption $C = \text{const.}$ holds if

$$C = \frac{\rho \mu r^2}{2 \mu_\infty \int_0^r \rho r dr} \quad \rightarrow \quad C = \frac{\rho \mu}{\rho_m \mu_\infty}$$

and $\rho \mu/\rho_m \mu_\infty = \text{const.}$.

• $C = \text{const.}$ Often not a good assumption, since \(\mu \sim T^{0.7}\) und \(\rho \sim T^{-1}\)
Round Laminar Diffusion Flame

- Constant of integration $\gamma$ can be determined from the condition that the jet momentum is independent of $\zeta$
- Substitution of the solution into the momentum balance

\[
\int_0^\infty \rho u_z^2 r dr = \rho_0 u_{z,0}^2 \frac{d^2}{8}
\]

yields

\[
\gamma^2 = \frac{3}{64} \rho_0 \frac{Re^2}{C^2}
\]

- $\rho_0$: density of the fuel stream
- Reynolds number $Re = u_{z,0}d/\nu_\infty$
Round Laminar Diffusion Flame

• Analogously for the mixture fraction (with \( Z_0 = 1 \))

\[
\int_{0}^{\infty} \rho u_z Z r dr = \rho_0 u_{z,0} \frac{d^2}{8}
\]

→ Mixture fraction on the centerline \( Z_a(z) = Z(z,r=0) \):

\[
Z_a(z) = \frac{1 + 2Sc}{32} \rho_0 \frac{Re}{\rho_\infty} \frac{d}{C} \zeta
\]

→ \( Z_a \) decreases with \( 1/\zeta \) (as the velocity)
Round Laminar Diffusion Flame

- Determination of the flame contour $r$ as function of $z$ from the condition

$$Z(z, r) = Z_a \omega(\eta) = Z_{st}$$

- Flame contour intersects centerline, $r = 0$, if $Z_a = Z_{st}$

- Corresponding value of $z$ defines the flame length

$$Z_a(z) = \frac{1 + 2Sc}{32} \frac{\rho_0}{\rho_{\infty}} \frac{Re}{C} \frac{d}{\zeta} \rightarrow L = \frac{1 + 2Sc}{32Z_{st}} \frac{\rho_0}{\rho_{\infty} C} \frac{u_0 d^2}{\nu} - a$$

- Valid for laminar jet flames without buoyancy
Round Laminar Diffusion Flame

• For a given nozzle diameter, \( L \) increases linearly with the Reynolds number \( Re \)

\[ Sc_L = 0.72 \]
Course Overview

Part II: Turbulent Combustion

• Turbulence
• Turbulent Premixed Combustion
• **Turbulent Non-Premixed Combustion**
• Modelling Turbulent Combustion
• Applications

• Laminar Jet Diffusion Flames
• Turbulent Jet Diffusion Flames
Turbulent Jet Diffusion Flame

- Shear flow at nozzle exit
- Flow instabilities (Kelvin-Helmholtz-instabilities) → laminar-turbulent transition
- Ring shaped turbulent shear layer propagates in radial direction
- Merging after 10 to 15 nozzle diameters downstream
- Streamlines are parallel in the potential core
- Velocity profile reaches self similar state after 20-30 nozzle diameters
Round Turbulent Diffusion Flame

- Linear reduction of velocity along central axis
- Linear increase of jet width
- Assumption: fast chemical reaction

$\rightarrow$ Scalar quantities such as temperature, concentration and density as function of mixture fraction $Z$

- Turbulent flow with variable density $\rightarrow$ Favre-averaged boundary layer equations
Linear Propagation of (turbulent) Jet
Round Turbulent Diffusion Flame

- **Assumptions:**
  - Axisymmetric jet flame
  - Neglecting buoyancy
  - Neglecting molecular transport as compared to turbulent transport
  - Turbulent transport modeled by Gradient Transport model
  - $Sc_t = v_t/D_t$
- **Using Favre averaging and the boundary layer assumption we obtain a system of two-dimensional axisymmetric equations**
Round Turbulent Diffusion Flame

- **Continuity equation**

\[
\frac{\partial (\bar{\rho} \bar{u}_z r)}{\partial z} + \frac{\partial (\bar{\rho} \bar{u}_r r)}{\partial r} = 0
\]

- **Momentum equation in z-direction**

\[
\bar{\rho} \bar{u}_z r \frac{\partial \bar{u}_z}{\partial z} + \bar{\rho} \bar{u}_r r \frac{\partial \bar{u}_z}{\partial r} = \frac{\partial}{\partial r} \left( \bar{\rho} \bar{\nu}_t r \frac{\partial \bar{u}_z}{\partial r} \right)
\]

- **Mean mixture fraction**

\[
\bar{\rho} \bar{u}_z r \frac{\partial \bar{Z}}{\partial z} + \bar{\rho} \bar{u}_r r \frac{\partial \bar{Z}}{\partial r} = \frac{\partial}{\partial r} \left( \bar{\rho} \bar{\nu}_t \frac{\partial \bar{Z}}{Sc_t \partial r} \right)
\]
Round Turbulent Diffusion Flame

- Requires solving of equations for $k$ and $\varepsilon$ to determine $\nu_t$
- Round turbulent jet: $\nu_t$ approximately constant
- Analogous for round laminar jet:

\[
\frac{\partial(ru_z r)}{\partial z} + \frac{\partial(ru_r r)}{\partial r} = 0
\]
\[
u_z r \frac{\partial u_z}{\partial z} + \nu_r r \frac{\partial u_z}{\partial r} = \frac{\partial}{\partial r} \left( \mu r \frac{\partial u_z}{\partial r} \right)
\]
\[
u_z r \frac{\partial Z}{\partial z} + \nu_r r \frac{\partial Z}{\partial r} = \frac{\partial}{\partial r} \left( \mu r \frac{\partial Z}{\partial r} \right)
\]

\[
\frac{\partial(r\tilde{u}_z r)}{\partial z} + \frac{\partial(r\tilde{u}_r r)}{\partial r} = 0
\]
\[
\nu_t r \frac{\partial \tilde{u}_z}{\partial z} + \nu_t r \frac{\partial \tilde{u}_z}{\partial r} = \frac{\partial}{\partial r} \left( \nu_t r \frac{\partial \tilde{u}_z}{\partial r} \right)
\]
\[
\nu_t r \frac{\partial \tilde{Z}}{\partial z} + \nu_t r \frac{\partial \tilde{Z}}{\partial r} = \frac{\partial}{\partial r} \left( \nu_t r \frac{\partial \tilde{Z}}{\partial r} \right)
\]
Round Turbulent Diffusion Flame

• Special case: Jet in quiescent ambient
  – Treatment of turbulent equations like those in a laminar round jet case
  – Using the laminar theory

• Similarity coordinate

  \[ \eta = \frac{\sqrt{2 \int_0^r \frac{\rho}{\rho_\infty} r dr}}{z + a} \]

• Chapman-Rubesin-Parameter

  \[ C = \frac{\rho \nu r^2}{2 \mu_\infty \int_0^r \rho r dr} \]

  \[ \rightarrow \]

  \[ C = \frac{\bar{\rho}^2 \nu_t r^2}{2 \rho_\infty \nu_{t,ref} \int_0^r \bar{\rho} r dr} \]
Round Turbulent Diffusion Flame

• Turbulent Chapman-Rubesin-Parameter approximately constant→

\[ \tilde{u}_z = \frac{2C\gamma^2\nu_{t,ref}}{\zeta \left(1 + (\gamma\eta)^2 / 4\right)^2} \]

• Integration constant \( \gamma \), containing fuel density and reference viscosity

\[ \gamma^2 = \frac{3}{64} \frac{\rho_0}{\rho_\infty C^2} \left(\frac{u_{z,0} d}{\nu_{t,ref}}\right)^2 \quad \left( \text{laminar: } \gamma^2 = \frac{3}{64} \frac{\rho_0}{\rho_\infty} \frac{Re^2}{C^2} \right) \]

• The Favre-averaged velocity decreases proportional to \( 1/\zeta = 1/(z + a) \), just like in the laminar case
Round Turbulent Diffusion Flame

- Mean mixture fraction

\[
\tilde{Z} = \frac{\tilde{Z}_a}{(1 + (\gamma \eta)^2 / 4)^{2Sc_t}}
\]

with

\[
\tilde{Z}_a = \frac{1 + 2Sc_t}{32} \frac{\rho_0}{\rho_\infty C} \left( \frac{u_{z,0}}{\nu_{t,\text{ref}}} \right) \frac{d}{\zeta} \quad \left( \text{laminar: } Z_a = \frac{1 + 2Sc}{32} \frac{\rho_0}{\rho_\infty \frac{Re}{C} \frac{d}{\zeta}} \right)
\]

\[\rightarrow \text{ Mixture fraction decreases proportional to } 1/(z + a) \text{ on the jet axis}\]

\[\rightarrow \text{ Progression of profiles along jet axis resembles those of the laminar case}\]

- Also applies to the contour of the stoichiometric mixture
Round Turbulent Diffusion Flame

- Flame length $L$ of round turbulent diffusion flame: Distance $z$ from the nozzle, where the mean mixture fraction on the axis equals $Z_{st}$.

\[
\frac{L + a}{d} = \frac{1 + 2Sc_t}{32Z_{st}} \left( \frac{u_{z,0}d}{\nu_{t,ref}} \right) \frac{\rho_0}{\rho_\infty C}
\]

- Comparison with experimental correlations (Hawthorne, Weddel and Hottel (1949))

\[
\frac{L + a}{d} = \frac{5.3}{Z_{st}} \sqrt{\frac{\rho_0}{\rho_\infty}}
\]

- With $u_{z,0}d/\nu_{t,ref} = 70$ and $Sc_t = 0.72$
- Complete agreement for $C = (\rho_0 \rho_{st})^{1/2}/\rho_\infty$
Round Turbulent Diffusion Flame

\[
\frac{L + a}{d} = \frac{1 + 2Sc}{32Z_{st}} \frac{\rho_0 u_0 d}{\rho_\infty C \nu}
\]

\[
\text{const.} \approx 70
\]

\[
\frac{L + a}{d} = \frac{1 + 2Sc_t}{32Z_{st}} \frac{\rho_0 u_0 d}{\rho_\infty C \nu_{t,\text{ref}}}
\]
Experimental Data: Round Turbulent Diffusion Flame

- Comparison of experimental results and simulations with chemical equilibrium

- Concentration of radicals and emissions cannot be described by infinitely fast chemistry
Summary

Part II: Turbulent Combustion

- Turbulence
- Turbulent Premixed Combustion
- Turbulent Non-Premixed Combustion
- Modelling Turbulent Combustion
- Applications

- Laminar Jet Diffusion Flames
- Turbulent Jet Diffusion Flames