Combustion dynamics
Lecture 1a

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Princeton summer school, June 2016

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Many thanks to Ed Law for inviting me to give this course in this stimulating setting. It is a pleasure to be here and share ideas and some of our research with you.

Thanks also to Nicolas Noiray, Paul Palies, Frédéric Boudy, Jean-François Bourgouin, Jonas Moeck, Kevin Prieur and Davide Laera for their many contributions.

The focus is on combustion dynamics with special attention to swirling flames and annular systems dynamics which are important for gas turbines. To cover this topic one needs more than the fifteen hours allocated to this set of lectures but it is already a reasonable amount of time. I’ll try to cover fundamentals and some practical applications.
General introduction and background

- Introductory comments
- A few examples
- Why is combustion so susceptible to instabilities
- Classification
- Combustion dynamics timeline
- Objectives
The Eiffel tower was built by Gustave Eiffel in 1889.

On February 24th 1954, at the controls of a Mystere IV, Constantin Rozanoff is the first French pilot to break the sound barrier with a French built airplane.
December 17, 1903 First motored flight of the Wright brothers Orville and Wilbur Wright at Kitty Hawk (North Carolina)
Louis Blériot crosses the English channel on July 25, 1909 in his Blériot XI aircraft.

The Blériot XI flies at Le Bourget in 2009.
Thermo-acoustic instabilities

New technologies promote acoustic coupling problems

<table>
<thead>
<tr>
<th>High pressure</th>
<th>Increased efficiency</th>
<th>High energy densities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compact design</td>
<td>Gain in weight</td>
<td>Less well damped</td>
</tr>
<tr>
<td>Lean combustion</td>
<td>Low NOx</td>
<td>Stabilization issues</td>
</tr>
</tbody>
</table>

Domestic boiler

~ 10 kW$_{th}$

Process heater

~ 1 MW$_{th}$

Gas turbine

~ 100 MW$_{th}$

Powerplant

~ 1 GW$_{th}$

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Case 1

Domestic boiler

During unstable operation, lateral walls of this boiler are “breathing” highlighting large pressure oscillations within the system.

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Combustion dynamics issues in gas turbines

Combustion dynamics degrades operation of many practical systems and in extreme cases leads to failure.

**Challenges:**

1. Gain an understanding of the processes driving and coupling combustion instabilities
2. Develop models for the nonlinear dynamics observed in practice: limit cycles, triggering, mode switching, hysteresis...
3. Derive predictive analytical and numerical tools for combustion dynamics
4. Design novel instability control systems (passive, dynamical, active)

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Difficulties are related to the reduced stability of lean premixed combustors widely used in gas turbines.

Analysis must take into account physical and geometrical complexities including:

1. A turbulent swirling flame,
2. Resonant acoustic characteristics of the system,
3. Reduced damping rates,
4. Complicated boundary conditions.
Numerical simulation of instabilities has to deal with complex swirling injection configurations, turbulent flows, acoustic flame coupling, dynamics of upstream and downstream components.

Lean premixed combustion generates low levels of NOx but is susceptible to pressure coupling and instability.

Predictive tools are needed to design stable combustors. Such tools are not available but their development is an objective for the future.
Thermoacoustic interactions in gas turbine combustors

- Air and fuel supply
- Flow turbulence
- Flow instabilities
- Equivalence ratio
- Heat release

Adapted from Paschereit et al (1998)
**Stable regime**: the combustion zone (luminous region) features small stochastic fluctuations around its mean location due to turbulence. Radiated noise remains weak and broadband: “combustion roar”.

**Unstable regime**: Large synchronized motions with a strong harmonic content. Intensification of luminosity near the wall: enhanced heat fluxes to the boundaries. Oscillations induce flame flashback.
Flow perturbation is produced

This induces a combustion perturbation

Acoustic feedback links the unsteady combustion process to flow perturbation

The system is unstable if the gain exceeds the damping
Basic interactions leading to combustion instabilities

- Upstream dynamics
- Feed line dynamics
- Impedance conditions
- Injection
- Mixing
- Atomization/vaporization/mixing
- Stabilization Flame holding
- Organized vortex structures
- Flame wall interactions
- Flame/vortex interactions
- Heat release
- Flame/vortex interactions
- Acoustics
- Entropy waves
- Exhaust impedance conditions

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Organ pipe instabilities

Stable regime

Unstable regime

Turbulent fluctuations
Small amplitudes
Broadband noise
Combustion noise

High amplitude self-sustained cyclic oscillation. The frequency depends on:
- the flame position within the tube
- the tube length
- the boundary conditions

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Vortex driven instabilities in a premixed combustor ($f=530$ Hz)

Stable operation

Unstable operation

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Schlieren images of the unsteady flow (flow from right to left)

Heat release images (C$_2$ radical emission)

Numerical simulation of vortex driven instability. Flow from left to right (flame is thickened, flow is forced at the oscillation frequency $f=530$ Hz)

Angelberger et al (1999, 2001)
Vortex driven combustion instability

Experiment: schlieren image of the flow (Poinsot et al. 1987)

Simulation: temperature distribution
Case 5  Turbulent flame response to acoustic waves

Driver units

Plane acoustic waves

Flame stabilizer

Combustor

Turbulent premixed propane/air flame

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Fuel mass fraction (external acoustic modulation)

Schlieren images of premixed turbulent ducted flame

Air/propane


Computation (C. Nottin, 1999)
Acoustically coupled combustion instabilities (thermo-acoustic instabilities)

Flow \rightarrow \text{Combustion} \rightarrow \text{Acoustics} \rightarrow \text{Burner acoustics}
Why is combustion so susceptible to instabilities? Some standard reasons:

(1) The power density associated with combustion is sizable. A small fraction of this power is sufficient to drive the oscillations.

- **Power level**: 2.5 GW
- **Power Density**: $E_c \approx 50 \text{ GW m}^{-3}$

A fluctuation of 20% in pressure (about 2 MPa) corresponds to a power density $E_a \approx 0.4 \text{ MW m}^{-3}$.

The acoustic power is a small fraction of the thermal power:

$$E_a/E_c \approx 10^{-5}$$
(2) Combustion involves time lags. Reactants introduced in the chamber at one instant are converted into burnt gases at a later time.

Consider the following model including a restoring force with delay
\[
\frac{d^2 x}{dt^2} + 2\zeta \omega_0 \frac{dx}{dt} + \omega_0^2 x(t - \tau) = 0
\]

Assume that the time lag is small and expand the last term in a Taylor series up to first order
\[
\frac{d^2 x}{dt^2} + (2\zeta - \omega_0 \tau) \omega_0 \frac{dx}{dt} + \omega_0^2 x = 0
\]

The model features a negative damping coefficient if
\[
2\zeta < \omega_0 \tau
\]

The system is unstable when the time lag is sufficiently large.

When the distance between the hot water tap and the shower cap is too long it introduces an excessive delay. This may give rise to growing oscillations in temperature resulting from the user lack of understanding of the situation.
Luigi Crocco (1909-1986) one of the founders of combustion instability theory, professor at Princeton for many years. He spent the later part of his life in Paris and was a Professor at Ecole Centrale Paris for a few years.

H.S. Tsien (Tsien Hsue-Shen or Qian Xuesen) (1911-2009), went to study at Caltech under the supervision of von Karman, one of the founders of the Jet Propulsion Laboratory, and later « Father of China’s Space Program ».

Frank Marble (1918-2014), professor at Caltech, jet propulsion pionneer.

\[
\frac{d\tau}{dt} = n \frac{p(t - \tau) - p(t)}{\bar{p}}
\]
Resonant interactions may readily occur in the weakly damped geometries used in modern combustors.

Among the possible coupling modes, acoustics is dominant.

If the frequency is low,
\[ \lambda > d \]
wave propagation is longitudinal and this gives rise to system instabilities.

If the frequency is high,
\[ \lambda < d \]
the coupling may involve transverse modes giving rise to chamber instabilities.
System Instabilities
- Resonant modes of the chamber
- \( \lambda \approx \) typical transverse dimension
- High frequency range

Intrinsic Instabilities
- Depend on the combustion process itself
- High frequency range

System Instabilities
- Involve the entire system
- Longitudinal wave propagation
- Low frequency range

Classified by (Barrère and Williams, PCI 1969)

Inlet

Combustor

Chamber

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Georges Darrieus

Lev Landau
Theoretical and experimental investigations for basic understanding of combustion instability in high performance devices (rocket engines, jet engines…)

1945

1970

Synthesis, theoretical modeling, further experiments

Marcel Barrère

1980

Detailed experiments, active control demonstrations

Forman Williams

Ben Zinn

1990

Detailed experiments, scale-up

Luigi Crocco

H.S. Tsien

Frank Marble

Fred Culick

Vigor Yang

Ann Dowling

Thierry Schuuller

Thierry Poinsot

Tim Lieuwen

CCD for gas turbines, numerical modeling, active control scale-up

LES

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Objectives

1. Identify the main mechanisms governing combustion instabilities

2. Illustrate these mechanisms by experiments, that can also serve to validate predictions

3. Derive theoretical and modelling tools to analyze combustion dynamics.

4. Provide fundamental elements for the prediction of linear and nonlinear stability of combustors
Elements of acoustics

Basic equations of linear acoustics
Plane waves in one dimension
Harmonic waves
Plane modes in a duct
Harmonic spherical waves
Acoustic energy balance
Acoustics of reactive flows

Accounting for heat release fluctuations
Compact flames
Acoustic energy balance
Equations of reactive flows
The unified framework
Theoretical methods in flame dynamics
Combustion dynamics
Lecture 1b

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The thermodynamic state is determined by two thermodynamic variables.

The fluid is ideal so that its viscosity and heat conductivity may be taken equal to zero.

Chemical reactions are absent and there is no volumetric addition of mass or heat.

There are no volume forces (volume forces are negligible).
Begin with the Euler set of equations

**Mass**

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0
\]

**Momentum**

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p
\]

**Energy in entropy form**

\[
\rho T \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla s \right) = 0
\]

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Because the fluid is bivariant the entropy $S$ may be expressed in terms of two other thermodynamic variables. For example $s = s(p, \rho)$ or equivalently one may write $p = p(\rho, s)$.

For example, in the case of a perfect gas the state equation takes the forms

\[ s = c_v \ln\left(\frac{p}{p^\gamma}\right) \quad \text{or} \quad p = \rho^\gamma e^{s/c_v} \]
\[ \gamma = c_p/c_v \]

This equation indicates that $\frac{ds}{dt} = 0$ which is consistent with the fact that there is no entropy production associated with volumetric heat release, viscous dissipation and heat conduction.
If the medium is homogeneous its entropy is constant everywhere at the initial instant and because its material derivative is identically zero, the entropy will remain constant and equal to its initial value at all times. Hence, the acoustic disturbances will propagate in the medium at a constant entropy $s = s_0$

and the state equation will take the form $p = p(\rho, s_0)$

Now, consider a disturbance of the ambient state. The field variables may be cast in the form of a sum of the ambient value and a perturbation

$$p = p_0 + p_1, \quad v = v_0 + v_1, \quad s = s_0 + s_1$$
Here \( \mathbf{v}_0 = 0 \) and \( p_0, \rho_0, s_0 \)
are constants linked by the state equation \( p_0 = p(\rho_0, s_0) \)

From the previous analysis of the balance equation for entropy
one immediately deduces that the
entropy perturbation vanishes identically \( s_1 = 0 \)

One may now substitute the perturbed expressions into the
balance equations of mass and momentum and in the equation
of state

\[
\frac{\partial}{\partial t} (\rho_0 + \rho_1) + \nabla \cdot (\rho_0 + \rho_1) \mathbf{v}_1 = 0
\]

\[
(\rho_0 + \rho_1) \left( \frac{\partial}{\partial t} + \mathbf{v}_1 \cdot \nabla \right) \mathbf{v}_1 = -\nabla (p_0 + p_1)
\]

\[
p_0 + p_1 = p(\rho_0 + \rho_1, s_0)
\]
For small perturbation in pressure, density and velocity, it is easy to distinguish terms of order zero, one and two. The terms of order zero vanish identically. The first order approximation obtained by neglecting higher order terms leads to the following equations

\[
\frac{\partial}{\partial t} \rho_1 + \nabla \cdot \rho_0 \mathbf{v}_1 = 0
\]

\[
\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla p_1 = 0
\]

A Taylor-series expansion of the state equation yields

\[
p_0 + p_1 = p(\rho_0, s_0) + \left( \frac{\partial p}{\partial \rho} \right)_0 \rho_1 + \frac{1}{2} \left( \frac{\partial^2 p}{\partial \rho^2} \right)_0 \rho_1^2 + \ldots
\]

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Retaining only first order terms in this expansion one obtains

\[ p_1 = c^2 \rho_1 \quad \text{where} \quad c^2 = (\partial p / \partial \rho)_0 \]

The derivative of pressure with respect to density at constant entropy has the dimensions of velocity square. From thermodynamics it can be shown that this quantity is positive. It will be shown later on that this derivative is actually the square of the speed of sound.

The linear acoustic equations take the form

\[
\begin{align*}
\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 &= 0 \\
\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla p_1 &= 0 \\
p_1 &= c^2 \rho_1
\end{align*}
\]
Determine the speed of sound in air at a temperature $T=298.15$ K

For a perfect gas $p = \rho \gamma e^{s/c_v}$

$$c^2 = (\gamma \rho \gamma^{-1}) \exp s/c_v \quad c^2 = \gamma p_0/\rho_0$$

Now the state equation for a perfect gas writes

$p = \rho r T \quad r = \mathcal{R}/W$

The speed of sound for a perfect gas is then given by

$c = (\gamma r T_0)^{1/2}$

$\mathcal{R} = 8314 \text{ J kmol}^{-1} \text{ K}^{-1} \quad W = 29 \text{ kg kmol}^{-1}$

$r = 8314/29 = 287 \text{ J kg}^{-1} \text{ K}^{-1}$

$$c = (1.4)(287)(298.15)^{1/2} = 346.1 \text{ m s}^{-1}$$
There are other useful forms of the basic system of equations. It is first convenient to eliminate the density perturbation from the first equation by making use of the third relation. This yields

\[ \frac{1}{c^2} \frac{\partial p_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \]

\[ \rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla p_1 = 0 \]

\[ p_1 = c^2 \rho_1 \]

The acoustic problem is now specified by the first two equations. The third relation gives the density perturbation in terms of the pressure perturbation.
Another system may be obtained by eliminating the velocity perturbation from the first two equations. This is achieved by taking the time derivative of the linearized mass balance and subtracting the divergence of the linearized momentum balance

\[ \nabla^2 p_1 - \frac{1}{c^2} \frac{\partial^2 p_1}{\partial t^2} = 0 \]
\[ \rho_0 \frac{\partial v_1}{\partial t} + \nabla p_1 = 0 \]
\[ p_1 = c^2 \rho_1 \]

It is worth noting that the wave equation by itself does not allow the solution of most acoustic problems. It is in general necessary to use the linearized momentum equation to define the boundary conditions at the limits of the domain.
It is worth reviewing at this point the fundamental solution of the wave equation in one dimension. In this particular case the velocity perturbation has a single component and the set of linearized equations reduces to

\[
\frac{\partial p}{\partial t} + \rho_0 c^2 \frac{\partial v}{\partial x} = 0
\]

\[
\rho_0 \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} = 0
\]

\[
p = c^2 \rho
\]

Index 1 designating perturbed quantities has been eliminated from the previous equations. This simplified notation is not ambiguous and may be adopted from here-on.
The wave equation becomes in this case

\[
\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0
\]

\[
(\frac{\partial}{\partial x} - \frac{1}{c} \frac{\partial}{\partial t})(\frac{\partial}{\partial x} + \frac{1}{c} \frac{\partial}{\partial t})p = 0
\]

The factored form of the wave equation suggests the following change of variable

\[
\xi = t - x/c, \quad \eta = t + x/c
\]

Introducing these relations in the wave equation yields

\[
-\frac{4}{c^2} \frac{\partial^2 p}{\partial \xi \partial \eta} = 0
\]
The general solution of this partial differential equation takes the form of a sum

\[ p = f(\xi) + g(\eta) \]

\[ p = f(t - x/c) + g(t + x/c) \]

It is a simple matter to show that the acoustic velocity corresponding to this pressure field takes the form

\[ v(x, t) = \frac{1}{\rho_0 c} [f(t - x/c) - g(t + x/c)] \]

\( f(t - x/c) \) represents a wave traveling to the right at the speed of sound

\( g(t + x/c) \) represents a wave traveling to the left at the speed of sound
The perturbation has the form of a pulse

\[ p(x, t_1) \]

\[ p(x, t_2) \]

\[ f(t - x/c) \] represents a wave traveling to the right at the speed \( c \).
\( \rho_0 c \) is the specific acoustic impedance of the medium

\[
\rho_0 c = (1.18)(346.1) = 408 \text{ kg m}^{-2} \text{ s}^{-1} = 408 \text{ Rayl}
\]

The unit of acoustic impedance is the Rayl in honor of J.W. Strutt, Lord Rayleigh, one of the founders of modern acoustics
Harmonic waves

Harmonic waves are such that their variation in time is of the form

\[ p(x, t) = p_\omega(x)e^{-i\omega t} \]
\[ \rho(x, t) = \rho_\omega(x)e^{-i\omega t} \]
\[ \mathbf{v}(x, t) = \mathbf{v}_\omega(x)e^{-i\omega t} \]

The complex number representation adopted for these waves follows the standard conventions. If one wishes to determine the actual pressure field at point and time it is sufficient to take the real part of the complex number which specifies the perturbation. For example:

\[ p(x, t) = \text{Re}\{p_\omega(x)e^{-i\omega t}\} \]
One may now derive the field equations governing harmonic disturbances. This is easily achieved by substituting the representation in the linearized acoustic equations obtained previously.

One finds that the time derivative \( \partial / \partial t \) must be replaced by a factor \( -i \omega \).

The common factor \( e^{-i \omega t} \) may be dropped from all equations. This process yields:

\[
-i \omega p_\omega + \rho_0 c^2 \nabla \cdot \mathbf{v}_\omega = 0
\]
\[
-\rho_0 i \omega \mathbf{v}_\omega + \nabla p_\omega = 0
\]
\[
p_\omega = c^2 \rho_\omega
\]
The wave equation is replaced by

$$\nabla^2 p_\omega + (\omega^2/c^2)p_\omega = 0$$

which is often written in the form

$$\nabla^2 p_\omega + k^2 p_\omega = 0 \quad \text{where} \quad k = \omega/c$$

This Helmholtz equation specifies the spatial dependence of the complex field amplitude.
Harmonic waves in one dimension

The set of equations for one dimensional wave propagation is

\[
\frac{d^2 p}{dx^2} + k^2 p = 0
\]

\[
-i\omega v + \frac{dp}{dx} = 0
\]

The pressure field is then a combination of two waves

\[
p = A \exp(+ikx) + B \exp(-ikx)
\]

and the velocity field is given by

\[
v = \frac{1}{\rho_0 c} \left[ A \exp(+ikx) - B \exp(-ikx) \right]
\]
The first wave in the previous expressions propagates towards positive $x$ while the second wave travels in the negative $x$ direction.

Problem 1 : Radiation in an infinite channel

A piston is placed at one end of an infinite duct and imposes an acoustic velocity of the form

$$v(0, t) = v_0 \cos \omega t$$

Find the acoustic field generated by the piston in this device.

To deal with this problem it is convenient to work with complex representations.
\[ v(0, t) = \text{Re}[v_0 \exp(-i\omega t)] \]

Since the duct is infinite there is only traveling wave propagating away from the piston in the positive x direction.

\[ p(x, t) = \text{Re}[A \exp(ikx - i\omega t)] \]
\[ v(x, t) = \text{Re}[\frac{A}{\rho_0 c} \exp(ikx - i\omega t)] \]

To satisfy the condition at the piston

\[ v(0, t) = \text{Re}[\frac{A}{\rho_0 c} \exp(-i\omega t)] = \text{Re}[v_0 \exp(-i\omega t)] \]
As a consequence

\[ A = \rho_0 c v_0 \]

and the pressure field is given by

\[ p(x, t) = \text{Re}[\rho_0 c v_0 \exp(ikx - i\omega t)] \]

or

\[ p(x, t) = \rho_0 c v_0 \cos(kx - \omega t) \]
Problem 2: Plane modes in a duct

\[ x = 0 \quad \quad \quad \quad \quad \quad x = l \]

It is now interesting to examine plane modes in a duct. We consider a closed duct with an open end at \( x = l \).

The pressure field satisfies the Helmholtz equation

\[ \frac{d^2 p}{dx^2} + k^2 p = 0 \]

subject to the following boundary conditions

\[ \left( \frac{\partial p}{\partial x} \right)_0 = 0 \quad \quad \quad \quad \quad p(l) = 0 \]

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The pressure field is of the form

\[ p = A \exp(+ikx) + B \exp(-ikx) \]

The first condition is satisfied if

\[ A = B \]

To fulfil the second condition

\[ \exp(ikl) + \exp(-ikl) = 0 \]

or equivalently

\[ \cos(kl) = 0 \]

This takes the form of a dispersion relation \[ \mathcal{D}(\omega) = 0 \]

It provides the eigenfrequencies of this system

\[ k_n = (2n + 1) \frac{\pi}{2l} \]
This yields

\[ f_n = (2n + 1) \frac{c}{4l} \]

and the corresponding eigenmodes

\[ \psi_n(x) = \cos(k_n x) \]

The wavelength is given in this case by

\[ \lambda_n = \frac{4l}{2n + 1} \]

\[ \lambda_0 = 4l \quad \lambda_1 = \frac{4l}{3} \quad \lambda_2 = \frac{4l}{5} \]
Problem 3: Propagation in ducts with variable cross section

\[ l_j = x_{j+1} - x_j \]

\[ p_j = a_j \exp[i k_j (x - x_j)] + b_j \exp[-i k_j (x - x_j)] \]

\[ v_j = \frac{a_j}{\rho_j c_j} \exp[i k_j (x - x_j)] - \frac{b_j}{\rho_j c_j} \exp[-i k_j (x - x_j)] \]
\[ p_{j+1} = a_{j+1} \exp[ik_j(x - x_{j+1})] + b_{j+1} \exp[-ik_j(x - x_{j+1})] \]
\[ v_{j+1} = \frac{a_{j+1}}{\rho_{j+1}c_{j+1}} \exp[ik_j(x - x_{j+1})] - \frac{b_{j+1}}{\rho_{j+1}c_{j+1}} \exp[-ik_j(x - x_{j+1})] \]

At the area change the pressure and volume flow rates are continuous:

\[ p_j(x_{j+1}) = p_{j+1}(x_{j+1}) \]
\[ S_j v_j(x_{j+1}) = S_{j+1} v_{j+1}(x_{j+1}) \]

This yields

\[ a_j e^{ik_j l_j} + b_j e^{-ik_j l_j} = (a_{j+1} + b_{j+1}) \]
\[ \frac{S_j}{\rho_j c_j} (a_j e^{ik_j l_j} - b_j e^{-ik_j l_j}) = \frac{S_{j+1}}{\rho_{j+1} c_{j+1}} (a_{j+1} - b_{j+1}) \]
Defining

\[ \beta_j = \frac{S_j}{S_{j+1}} \frac{\rho_{j+1} c_{j+1}}{\rho_j c_j} \]

the previous expressions become

\[ a_{j+1} + b_{j+1} = a_j e^{ik_j l_j} + b_j e^{-ik_j l_j} \]
\[ a_{j+1} - b_{j+1} = \beta_j (a_j e^{ik_j l_j} - b_j e^{-ik_j l_j}) \]

and one obtains

\[ a_{j+1} = \frac{1}{2} [(1 + \beta_j) a_j e^{ik_j l_j} + (1 - \beta_j) b_j e^{-ik_j l_j}] \]
\[ b_{j+1} = \frac{1}{2} [(1 - \beta_j) a_j e^{ik_j l_j} + (1 + \beta_j) b_j e^{-ik_j l_j}] \]
Harmonic spherical waves

We look for solutions of the Helmholtz equation in three dimensions which only depend on the radius $r$

The pressure field satisfies

$$p = p(r)$$

or equivalently

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{dp}{dr}) + k^2 p = 0$$

or equivalently

$$\frac{d^2 (rp)}{d r^2} + k^2 (pr) = 0$$
There are two solutions to this wave equation

\[ p(r) = \frac{A}{r} \exp(ikr) \]  \text{travels outwards}

\[ p(r) = \frac{B}{r} \exp(-ikr) \]  \text{travels inwards}

The acoustic velocity has only a radial component which is given by

\[ \rho_0 \frac{\partial v}{\partial t} + \frac{\partial p}{\partial r} = 0 \]
For the wave traveling outwards, the velocity is given by

\[
v = \frac{1}{\rho_0 i \omega} \frac{\partial p}{\partial r}
\]

or equivalently

\[
v = \frac{1}{\rho_0 i \omega} A (ik - \frac{1}{r}) \frac{\exp(ikr)}{r}
\]

In the farfield

\[
v \approx \frac{1}{\rho_0 c} A \frac{\exp(ikr)}{r}
\]
Problem 4: Acoustic radiation by a pulsating sphere

A sphere of radius $a$ pulsates harmonically. The acceleration of the surface of the sphere is specified

$$\ddot{w}(a, t) = \tilde{W} \exp(-i\omega t)$$

Determine the pressure radiated by this sphere

The pressure field is an outgoing spherical wave

$$p(r) = A \frac{1}{r} \exp(i kr)$$

On the sphere, the radial acceleration is specified and the linearized momentum equation must be satisfied
Now

\[ \rho_0 \frac{\partial v}{\partial t} + \frac{\partial p}{\partial r} = 0 \]

so that

\[ \ddot{w} = \frac{\partial v}{\partial t} \]

By imposing this condition at \( r=a \) one finds :

\[ \rho_0 \ddot{W} = -A \left( ik - \frac{1}{a} \right) \frac{\exp(ika)}{a} \]
It is convenient to introduce the volume acceleration

\[ A = \frac{\rho_0 \dddot{W} a^2}{(1 - ika)} \exp(-ika) \]

\[ p(r, t) = \frac{\rho_0 \dddot{W} a^2}{(1 - ika)} \frac{1}{r} \exp[ik(r - a) - i\omega t] \]

\[ \ddot{Q} = 4\pi \dddot{W} a^2 \]

\[ p(r, t) = \frac{\rho_0 \ddot{Q}}{4\pi(1 - ika)} \frac{1}{r} \exp[ik(r - a) - i\omega t] \]
If the radius of the sphere is small compared to the wavelength

\[ ka << 1 \]

this expression reduces to

\[
p(r, t) = \frac{\rho_0 \ddot{Q}}{4\pi r} \exp[ikr - i\omega t]
\]

This is the sound field radiated by a point source featuring a specified volume acceleration \( \ddot{Q} \)

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A conservation equation for the acoustic energy may be obtained from the linearized equations describing the acoustic field. One may start from

\[
\frac{\rho_1}{\rho_0} \left( \frac{1}{c^2} \frac{\partial p_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} \right) + \mathbf{v}_1 \cdot (\rho_0 \frac{\partial \mathbf{v}}{\partial t} + \nabla p_1) = 0
\]
This equation becomes

$$\frac{\partial}{\partial t} \left( \frac{1}{2} \rho_0 v_1^2 + \frac{1}{2} \frac{p_1^2}{\rho_0 c^2} \right) + \nabla \cdot p_1 v_1 = 0$$

$$\mathcal{E} = \frac{1}{2} \rho_0 v_1^2 + \frac{1}{2} \frac{p_1^2}{\rho_0 c^2} \quad \mathcal{F} = p_1 v_1$$

With these definitions the balance equation may be cast in the form

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \mathcal{F} = 0$$

This expression closely resembles to the balance equations of fluid dynamics. It is also in the same form as Pointing's theorem of electromagnetic theory.
Sound pressure level (dB)

\[ SPL = 20 \log_{10} \frac{p_{rms}}{p_{ref}} \]

\[ p_{ref} = 2 \times 10^{-5} \ Pa \]

Intensity level (dB)

\[ IL = 10 \log_{10} \frac{I}{I_{ref}} \]

\[ I_{ref} = 10^{-12} \ W m^{-2} \]
The sound intensity in the far field is given by

\[ I = \frac{p^2}{\rho_0 c} \]

In air

\[ \rho_0 c \simeq 400 \text{ Rayl} \]

and the sound intensity corresponding to the reference pressure used to define the sound pressure level is given by

\[ I_{ref} = \frac{(2 \times 10^{-5})^2}{400} = 10^{-12} \text{ W m}^{-2} \]

Thus the sound pressure level and the intensity level are nearly equal

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Combustion dynamics
Lecture 2a

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Princeton summer school, June 2016
Consider the set of acoustic equations but this time including a nonsteady heat release source term in the energy balance.

\[ \frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \]

\[ \rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla p_1 = 0 \]

\[ \rho_0 T_0 \left( \frac{\partial}{\partial t} s_1 \right) = \dot{q}_1 \]

The state equation \( p = p(\rho, s) \) may be differentiated

\[ dp = \left( \frac{\partial p}{\partial \rho} \right)_s d\rho + \left( \frac{\partial p}{\partial s} \right)_{\rho} ds \]

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\[ p_1 = \left( \frac{\partial p}{\partial \rho} \right)_s \rho_1 + \left( \frac{\partial p}{\partial s} \right)_\rho s_1 \]

\[ c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s \quad \alpha = \left( \frac{\partial p}{\partial s} \right)_\rho \]

\[ p_1 = c^2 \rho_1 + \alpha s_1 \]

For a perfect gas  \[ p = \rho^\gamma \exp(s/c_v) \]

\[ c^2 = \frac{\gamma p}{\rho} = \gamma r T \quad \alpha = \frac{p}{c_v} = (\gamma - 1) \rho T \]
The density perturbation may be expressed in terms of pressure and entropy perturbations

\[ \rho_1 = \frac{1}{c^2} p_1 - \frac{\alpha}{c^2} s_1 \]

This relation may be introduced in the balance of mass

\[ \frac{1}{c^2} \frac{\partial p_1}{\partial t} - \frac{\alpha}{c^2} \frac{\partial s_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \]

Using the perturbed energy equation

\[ \rho_0 T_0 \left( \frac{\partial}{\partial t} s_1 \right) = \dot{q}_1 \]

One obtains

\[ \frac{1}{c^2} \frac{\partial p_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = \frac{\alpha}{\rho_0 T_0} \frac{1}{c^2} \dot{q}_1 \]

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The previous equation may be combined with the perturbed momentum balance

\[
\frac{\partial}{\partial t} \left( \frac{1}{c^2} \frac{\partial p_1}{\partial t} \right) + \rho_0 \nabla \cdot \mathbf{v}_1 = \frac{\alpha}{\rho_0 T_0} \frac{1}{c^2} \dot{q}_1
\]

\[- \nabla \cdot \rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla p_1 = 0\]

\[
\frac{1}{c^2} \frac{\partial^2 p_1}{\partial t^2} - \nabla^2 p_1 = \frac{\alpha}{\rho_0 T_0} \frac{1}{c^2} \frac{\partial \dot{q}_1}{\partial t}
\]

One obtains a wave equation with a source term

\[
\frac{1}{c^2} \frac{\partial^2 p_1}{\partial t^2} - \nabla^2 p_1 = \gamma - 1 \frac{\partial \dot{q}_1}{\partial t}
\]
The compact flame case

\[ \frac{1}{c^2} \frac{\partial p_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = \frac{\alpha}{\rho_0 T_0} \frac{1}{c^2} \dot{q}_1 \]

Now

\[ \frac{\alpha}{\rho_0 T_0} = \frac{p_0}{\rho_0 c_v T_0} = \gamma - 1 \]

Thus

\[ \frac{1}{c^2} \frac{\partial p_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = \frac{\gamma - 1}{c^2} \dot{q}_1 \]

Or equivalently

\[ \frac{1}{\rho_0 c^2} \frac{\partial p_1}{\partial t} + \nabla \cdot \mathbf{v}_1 = \frac{\gamma - 1}{\rho_0 c^2} \dot{q}_1 \]

Now \( \rho c^2 = \gamma p \) is constant across the flame
Integrating the last expression on a volume including the flame

\[
\int_V \frac{1}{\rho_0 c^2} \frac{\partial p_1}{\partial t} dV + \int_V \nabla \cdot \mathbf{v}_1 dV = \int_V \frac{\gamma - 1}{\rho_0 c^2} \dot{q}_1 dV
\]

Or equivalently

\[
\frac{1}{\rho_0 c^2} \int_V \frac{\partial p_1}{\partial t} dV + \int_V \nabla \cdot \mathbf{v}_1 dV = \frac{\gamma - 1}{\rho_0 c^2} \int_V \dot{q}_1 dV
\]

If the flame is compact, the first term vanishes. The second term may be transformed using Green’s theorem yielding

\[
S_2 v_2' - S_1 v_1' = \frac{\gamma - 1}{\rho_0 c^2} \dot{Q}' \quad \text{where} \quad \dot{Q}' = \int_V \dot{q}' dV
\]

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Assume that the surfaces on the upstream and downstream sides are equal

\[ v'_2 - v'_1 = \frac{\gamma - 1}{\rho_0 c^2} \frac{1}{S} \frac{\dot{Q}'}{\dot{\bar{Q}}} \]

From the definition of the flame transfer function

\[ \mathcal{F}(\omega) = \frac{\dot{Q}'/\bar{Q}}{v'/\bar{v}} \]

\[ \dot{Q}' = \bar{Q} \mathcal{F}(\omega) v'/\bar{v} \]

\[ v'_2 - v'_1 = \frac{\gamma - 1}{\rho_0 c^2} \frac{\bar{Q}}{S\bar{v}} \mathcal{F}(\omega) v'_1 \]

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Now
\[ \overline{\dot{Q}} = \dot{m} c_p (T_b - T_u) \]

and
\[ \rho_0 c^2 S \overline{v} = \dot{m} \gamma r T_u \]

\[ \frac{\gamma - 1}{\rho_0 c^2} \frac{\overline{\dot{Q}}}{S \overline{v}} = \frac{\gamma - 1}{\gamma r} c_p \frac{T_b - T_u}{T_u} = \frac{T_b}{T_u} - 1 \]

Hence
\[ v'_2 - v'_1 = \left( \frac{T_b}{T_u} - 1 \right) \mathcal{F}(\omega) v'_1 \]
Acoustic energy balance

\[ p_1 \frac{1}{\rho_0 c^2} \frac{\partial p_1}{\partial t} + \nabla \cdot \mathbf{v}_1 = \frac{\gamma - 1}{\rho_0 c^2} \dot{q}_1 \]

\[ \mathbf{v}_1 \cdot \rho_0 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla p_1 = 0 \]

\[ \frac{\partial}{\partial t} \left( \frac{1}{2} \frac{p_1^2}{\rho_0 c^2} + \frac{1}{2} \rho_0 v_1^2 \right) + \nabla \cdot p_1 \mathbf{v}_1 = \frac{\gamma - 1}{\rho_0 c^2} p_1 \dot{q}_1 \]

\[ \mathcal{E} = \frac{1}{2} \frac{p_1^2}{\rho_0 c^2} + \frac{1}{2} \rho_0 v_1^2 \quad \mathcal{F} = p_1 \mathbf{v}_1 \quad \mathcal{S} = \frac{\gamma - 1}{\rho_0 c^2} p_1 \dot{q}_1 \]

Acoustic energy density \quad Acoustic energy flux \quad Source term

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The energy balance should include a term associated with damping processes and takes the final form

\[
\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \mathbf{F} = S - D
\]
Taking the average of the energy balance over a period of oscillation one obtains

$$\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{F} = S - D$$

$$S = \frac{\gamma - 1}{\rho_0 c^2} \frac{1}{T} \int_T \rho_1 \dot{q}_1 dt$$

If the source term $S$ is positive it tends to increase the acoustic energy density. However this energy density will grow locally if the source term is greater than the damping term and the acoustic energy flux leaving the local volume


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The energy balance may be integrated over a volume $V$ containing the reactive region:

$$
\int_V \frac{\partial E}{\partial t} dV + \int_V \nabla \cdot \mathbf{F} dV = \int_V S dV - \int_V D dV
$$

Now

$$\int_V \nabla \cdot \mathbf{F} dV = \int_A \mathbf{F} \cdot \mathbf{n} dA$$

So that

$$\int_V \frac{\partial E}{\partial t} dV = \int_V S dV - \int_V D dV - \int_A \mathbf{F} \cdot \mathbf{n} dA$$

The acoustic energy in the control volume increases if

$$\int_V S dV > \int_V D dV - \int_A \mathbf{F} \cdot \mathbf{n} dA$$
A gain is obtained if pressure and heat-release fluctuations are in phase (Rayleigh, 1878)

\[ \frac{1}{T} \int_{0}^{T} p'q' dt > 0 \]

Local Rayleigh index in a lean premixed combustor (Lee et al 2000)

driving \((R>0)\) damping \((R<0)\)
Equations of combustion acoustics

\[
\begin{align*}
\frac{d\rho}{dt} &= -\rho \nabla \cdot \mathbf{v} \\
\rho \frac{d\mathbf{v}}{dt} &= -\nabla p + \nabla \cdot \mathbf{\tau} \\
\rho c_p \frac{dT}{dt} &= \dot{Q} + \frac{dp}{dt} + \mathbf{\tau} \cdot \nabla \mathbf{v} - \nabla \cdot \mathbf{J}^H \\
\rho \frac{dY_k}{dt} &= \dot{\omega}_k - \nabla \cdot \mathbf{J}_k^D
\end{align*}
\]

\[
\mathbf{J}_k^D = \rho Y_k \mathbf{V}_k^D, \quad \mathbf{J}^H = -\lambda \nabla T + \sum_{k=1}^{N} \rho Y_k \mathbf{V}_k^D h_k
\]

\[
\dot{Q} = -\sum_{k=1}^{N} h_k \dot{\omega}_k
\]

Mass, Momentum, Energy, Species, Heat release rate
Starting from the state equation for the mixture

\[ p = \rho r_g T, \text{ where } r_g = \frac{R}{W} \]

\[ \frac{1}{W} = \sum_{k=1}^{N} \frac{Y_k}{W_k} \]

one obtains

\[ dT = \left( \frac{\partial T}{\partial p} \right)_{\rho,Y_k} dp + \left( \frac{\partial T}{\partial \rho} \right)_{p,Y_k} d\rho + \left( \frac{\partial T}{\partial Y_k} \right)_{\rho,p} dY_k \]

\[ dT = \frac{T}{p} dp - \frac{T}{\rho} d\rho - T \sum_{k=1}^{N} \frac{W}{W_k} dY_k \]
Combining the previous expression with the balance equations for energy and species one obtains

\[
\frac{1}{\gamma p} \frac{dp}{dt} - \frac{1}{\rho} \frac{d\rho}{dt} = \frac{\dot{Q}}{\rho c_p T} + W \frac{d}{dt} \left( \frac{1}{W} \right) \\
+ \frac{1}{\rho c_p T} \left[ \nabla \cdot \lambda \nabla T + \tau : \nabla \mathbf{v} - \sum_{k=1}^{N} \rho Y_k c_p k \mathbf{V}_k^D \cdot \nabla T \right]
\]

Together with the balance of mass and momentum, this expression yields a wave equation for the logarithm of the pressure

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Wave equation in a reactive flow

\[
\nabla \cdot \left( \frac{c^2}{\gamma} \nabla \ln p \right) - \frac{d}{dt} \left( \frac{1}{\gamma} \frac{d}{dt} \ln p \right) = \nabla \cdot \left( \frac{1}{\rho} \nabla \cdot \tau \right)
\]

\[- \frac{d}{dt} \left( \frac{1}{\rho c_p T} \left[ \nabla \cdot \lambda \nabla T + \tau : \nabla \mathbf{v} - \sum_{k=1}^{N} \rho Y_k c_p \mathbf{V}_k^D \cdot \nabla T \right] \right)\]

\[- \frac{d}{dt} \left( \frac{\dot{Q}}{\rho c_p T} \right) - \frac{d}{dt} \left[ W \frac{d}{dt} \left( \frac{1}{W} \right) \right] - \nabla \mathbf{v} : \nabla \mathbf{v}\]

Combustion noise source associated with nonsteady heat release

Combustion source associated with changes in molar composition

Aerodynamic noise source

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Simplified wave equation

\[ \nabla \cdot \left( \frac{c^2}{\gamma} \nabla \ln p \right) - \frac{d}{dt} \left( \frac{1}{\gamma} \frac{d}{dt} \ln p \right) = \]

\[ - \frac{d}{dt} \left( \frac{\dot{Q}}{\rho c_p T} \right) - \frac{d}{dt} \left[ W \frac{d}{dt} \left( \frac{1}{W} \right) \right] - \nabla v : \nabla v \]

\[ \dot{Q} = - \sum_{k=1}^{N} \dot{\omega}_k h_k = \left( - \Delta h_f^0 \right) \dot{\omega} \]

Heat release rate

\[ [ML^{-1}T^{-3}] \]

\[ \dot{\omega} \]

Reaction rate

\[ [ML^{-3}T^{-1}] \]
Combustion noise source associated with nonsteady heat release

Truffaut and Searby (1998) propose an alternative expression for the second source term

\[ \frac{d}{dt} \left( \frac{\dot{Q}}{\rho c_p T} \right) \]

where \( n \) is the molar concentration

\[ n = \sum_{k=1}^{N} n_k = \rho \sum_{k=1}^{N} \frac{Y_k}{W_k} \]

Using this expression one finds that

\[ \frac{\dot{n}}{n} = \frac{1}{n} \frac{dn}{dt} = W \frac{d}{dt} \left( \frac{1}{W} \right) + \frac{1}{\rho} \frac{d\rho}{dt} \]

which contains an extra term

\[ \frac{d}{dt} \left[ W \frac{d}{dt} \left( \frac{1}{W} \right) \right] \]

Combustion noise source associated with changes in molar composition

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Linearized wave equation

The previous wave equation may be linearized by writing

\[ \ln p \simeq \frac{p'}{\bar{p}} \]

and assuming that the mean pressure is essentially constant (combustion is nearly isobaric). One obtains

\[
\nabla \cdot \left( \bar{c}^2 \nabla p' \right) - \frac{\partial^2 p'}{\partial t^2} = -\frac{\partial}{\partial t} \left[ (\gamma - 1) \dot{Q}' \right] - \gamma \bar{p} \nabla \mathbf{v} : \nabla \mathbf{v} + \frac{\gamma \bar{p}}{\bar{W}} \frac{\partial^2 W'}{\partial t^2}
\]

- Nonsteady heat release source term
- Aerodynamic sound
- Changes in molar composition

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An alternative formulation of the wave equation

Low Mach number limit \( \frac{d}{dt} \sim \frac{\partial}{\partial t} \)

By developing the logarithm of the pressure and using \( \gamma p = \rho c^2 \) one obtains

\[
\nabla \cdot \left( \frac{1}{\rho} \nabla p \right) - \frac{\partial}{\partial t} \left( \frac{1}{\rho c^2} \frac{\partial}{\partial t} p \right) = \\
- \frac{\partial}{\partial t} \left( \frac{\dot{Q}}{\rho c_p T} \right) - \frac{\partial}{\partial t} \left[ W \frac{\partial}{\partial t} \left( \frac{1}{W} \right) \right] - \nabla v : \nabla v
\]

This expression can be rearranged by adding on both sides the left hand side terms where the density and the sound speed are replaced by their uniform ambient values

\[
\nabla \cdot \left( \frac{1}{\rho_0} \nabla p \right) - \frac{\partial}{\partial t} \left( \frac{1}{\rho_0 c_0^2} \frac{\partial}{\partial t} p \right)
\]
One obtains

\[
\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = \nabla \cdot \left[ \left( \frac{1}{\rho c^2} - \frac{1}{\rho_0 c_0^2} \right) \frac{\partial p}{\partial t} \right] - \rho_0 \nabla \cdot \left[ \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right) \nabla p \right] - \frac{\partial}{\partial t} \left[ W \frac{\partial}{\partial t} \left( \frac{1}{W} \right) \right] - \rho_0 \frac{\partial}{\partial t} \left( \frac{\dot{Q}}{\rho c_p T} \right)
\]

The reasoning parallels that used by Lighthill in his theory of aerodynamic sound. This was used by Howe and by Dowling.

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Noting that
\[
\frac{1}{\rho c_p T} = \frac{\gamma - 1}{\rho_0 c_0^2}
\]
And only keeping the heat release source term and one finally obtains the following equation
\[
\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = -\frac{\gamma - 1}{c_0^2} \frac{\partial \dot{Q}'}{\partial t}
\]
Assuming that radiation takes place in an unconfined domain and only keeping the source term associated with perturbations in heat release one obtains

\[ p'(\mathbf{r}, t) = \frac{\gamma - 1}{4\pi c_0^2} \int_V \frac{1}{|\mathbf{r} - \mathbf{r}_0|} \frac{\partial}{\partial t} \dot{Q}'(\mathbf{r}_0, t - |\mathbf{r} - \mathbf{r}_0|/c_0) dV(\mathbf{r}_0) \]

When the observation point is in the farfield of a compact flame the previous expression becomes (Strahle (1985))

\[ p'(\mathbf{r}, t) = \frac{\gamma - 1}{4\pi c_0^2 r} \frac{\partial}{\partial t} \int_V \dot{Q}'(\mathbf{r}_0, t - r/c_0) dV(\mathbf{r}_0) \]
Theoretical description of instabilities

Considering acoustics as the central mechanism one derives a wave equation which yields a unified framework.
The unified analytical framework

\[ \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = h(\bar{p}, \bar{v}, p, v, ...) \]

Wave equation

Source term

\[ n \cdot \nabla p = -f \]

Boundary condition

Using simplifying assumptions one may show that

\[ h = - \frac{(\gamma - 1)}{c^2} \frac{\partial \dot{q}}{\partial t} \]

Source term

where \( \dot{q} \) is the nonsteady heat release rate

The acoustic field is expanded in a series of normal modes where the eigenfunctions $\psi_n$ satisfy a Helmholtz problem

$$p(x, t) = \bar{p} \sum_{n=1}^{N} \eta_n(t) \psi_n(x)$$

where the eigenfunctions $\psi_n$ satisfy a Helmholtz problem

$$\nabla^2 \psi_n + \left(\frac{\omega_n^2}{c^2}\right) \psi_n = 0$$

$$n \cdot \nabla \psi_n = 0$$

$\omega_n$ are the modal eigenfrequencies
\[ \psi_m \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = h \]

\[ -p \nabla^2 \psi_n + \frac{\omega^2_n}{c^2} \psi_n = 0 \]

\[ \psi_m \nabla^2 p - p \nabla^2 \psi_n \left. \right| - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \psi_m - \frac{\omega^2_n}{c^2} p \psi_m = h \psi_m \]

Integrating over the volume of the system

\[ \int_V \left( \psi_m \nabla^2 p - p \nabla^2 \psi_n \right) dV \left. \right| - \frac{\bar{p}}{c^2} \sum_{n=1}^{N} \int_V \frac{d^2 \eta_n}{dt^2} \psi_n \psi_m dV \]

\[ - \frac{\bar{p}}{c^2} \omega^2_n \sum_{n=1}^{N} \eta_n \int_V \psi_n \psi_m dV = \int_V h \psi_m dV \]
Using Green’s identity one may write
\[
\int_{V} (\psi_m \nabla^2 p - p \nabla^2 \psi_n) dV = \int_{A} (\psi_m \nabla p - p \nabla \psi_n) \cdot \mathbf{n} dA
\]
\[
= - \int_{A} \psi_m f dA
\]

The normal modes are orthogonal so that
\[
\int_{V} \psi_n \psi_m dV = 0 \quad \text{when} \ m \neq n
\]
\[
= \Lambda_n \quad \text{when} \ m = n
\]

These expressions may be used to get a set of coupled equations for the modal amplitudes.
The modal amplitudes satisfy a set of second order equations

\[
\frac{d^2 \eta_n}{dt^2} + \omega_n^2 \eta_n = F_n
\]

where

\[
F_n = -\frac{c^2}{\bar{\rho} \Lambda_n} \left[ \int_V h \psi_n dV + \int_A f \psi_n dA \right]
\]

and

\[
\Lambda_n = \int_V \psi_n^2 dV
\]

This yields a framework for the determination of the \( \eta_n \) amplitudes. It is however difficult to specify the source terms arising from the coupling between acoustics and combustion (\( h \) and \( f \) functions)

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It is also difficult to derive source terms associated with control

\[ h \rightarrow h + h_c \]

\[ f \rightarrow f + f_c \]

control term

The unified framework is useful for theoretical investigations and it has been extensively employed to formulate low order combustion control models

\[ \frac{d\eta}{dt} = A\eta + Bu \]

The normal modes are orthogonal

\[ \psi_m \nabla^2 \psi_n + \frac{\omega_n^2}{c^2} \psi_n = 0 \]

\[ -\psi_n \nabla^2 \psi_m + \frac{\omega_n^2}{c^2} \psi_m = 0 \]

\[ \psi_m \nabla^2 \psi_n - \psi_m \nabla^2 \psi_m + (k_n^2 - k_m^2) \psi_n \psi_m = 0 \]

This expression is now integrated over the volume \( V \) of the domain of interest

\[ \int_V (\psi_m \nabla^2 \psi_n - \psi_m \nabla^2 \psi_m) dV + \]

\[ (k_n^2 - k_m^2) \int_V \psi_n \psi_m dV = 0 \]
Using Green’s identity and the boundary condition imposed to the normal modes one obtains

\[
\int_V (\psi_m \nabla^2 \psi_n - \psi_n \nabla^2 \psi_m) dV = \int_A (\psi_m \nabla \psi_n - \psi_n \nabla \psi_m) \cdot \mathbf{n} dA = 0
\]

and one finds that

\[
\int_V \psi_m \psi_n dV = 0 \text{ if } k_n \neq k_m
\]
Considering combustion as the central process, the modeling focuses on the flame response to acoustic waves. The analysis emphasizes fluid flow and flame dynamics.
Adopting the second viewpoint, there is no unique framework. The flame dynamics is the central issue.

The flame may be treated as a thin front separating cold and hot gases and its position is a variable.

Problem is formulated in terms of \([u_1, u_2, p, \eta]\).
The flame may be treated as a thin front and its position is described with a $G$-equation

$$\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d |\nabla G|$$


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Flame motion calculated with the G-equation

Schlieren imaging of modulated flame

Conical flame dynamics (1)

\[
v = \bar{v} + a \cos \omega t \quad a/\bar{v} = 0.2
\]

\[
u = 0
\]
Important axial gradient, important radial velocity
The uniform velocity fluctuation assumption is inadequate
When this is used, it leads to a bad representation of the transfer function
Conical flame dynamics (2)

\[ u = \frac{1}{2} ka(x - x_0) \sin(ky - \omega t) \]
\[ v = \bar{v} + a \cos(ky - \omega t) \]

\[ a / \bar{v} = 0.2 \]

\[ k = \omega / \bar{v} \]
\[ \phi = 1.05 \quad S_L = 0.39 \text{ ms}^{-1} \]
\[ \bar{v} = 0.97 \text{ ms}^{-1} \quad v' = 0.19 \text{ ms}^{-1} \]

Flame dynamics and combustion noise: progress and challenges


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Contents

- Introduction
- Combustion noise fundamentals and scaling
- Mechanisms of sound radiation from perturbed flames
- Confinement effects, noise and instabilities
- Computational combustion acoustics (CCA)
Noise radiation interacts with the flow leading to unstable oscillations

- Flame dynamics
- Noise sources
- Radiated field
- Scaling rules
- Spectral content
- Noise control

- Flame dynamics
- Coupling mechanisms
- Conditions leading to instability
- Level of oscillation
- Prediction of instability
- Passive and active control
The flow is not modified by the radiated sound

The flow is modified by the radiated sound
Indirect noise associated with entropy waves

\[ \sigma = s'/c_p \]

\[ P_2^+ = \left( \frac{M_2 - M_1}{2} \right) \left[ \frac{(1/2)\sigma}{1 + (1/2)(\gamma - 1)M_1} \right] \]

Hot spots
Entropy waves
Pressure perturbations

<table>
<thead>
<tr>
<th>Combustion zone</th>
<th>Propagation zone</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Diagram" /></td>
<td><img src="image.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

- **Direct noise:**
  - unsteady heat production
  - unsteady momentum production

- **Indirect noise:**
  - accelerated entropy-wave
  - accelerated vorticity-wave

Additionally:
- unsteady entropy production (hot spots)
- generation of vorticity

(from Schemel, Thiele, Bake, Lehmann and Michel (2004))
Interactions in combustion instabilities


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Early work on combustion noise

The soap bubble is filled with a mixture of fuel and air.

The farfield pressure field is due to the volume acceleration induced by non-steady combustion (Thomas and Williams 1966)

\[ p'(r, t) = \frac{\rho_\infty}{4\pi r} \frac{d^2 \Delta V}{dt^2} \]
The farfield can also be expressed in terms of the volumetric rate of consumption of reactants (Hurle et al. (1968), Price et al. (1968))

\[ p'(r, t) = \frac{\rho_\infty}{4\pi r} \left( \frac{\rho_u}{\rho_b} - 1 \right) \left[ \frac{dq}{dt} \right]_{t-\tau} \]

\( \rho_u / \rho_b \) is the volumetric expansion ratio

\( \tau = r / c_0 \) is the time required for acoustic propagation

\( q \) is the volumetric rate of consumption of reactants
In the premixed case and for lean conditions the volumetric rate of reactants consumption can be determined from the emission intensity of free radicals

\[ q = kI \]

Photomultiplier

The sound pressure field can be deduced from the light intensity radiated by the flame

\[ p'(r, t) = \frac{\rho_\infty}{4\pi r} \left( \frac{\rho_u}{\rho_b} - 1 \right) k \left[ \frac{dI}{dt} \right]_{t-\tau} \]
For wrinkled flames, it is also possible to introduce the flame area (Abugov and Obrezkov (1978), Clavin and Siggia (1991))

\[ p'(r, t) = \frac{\rho_\infty}{4\pi r} \left( \frac{\rho_u}{\rho_b} - 1 \right) S_L \left[ \frac{dA}{dt} \right]_{t-\tau} \]

This is useful for theoretical analysis and has been employed to examine the spectral content of the radiated sound.

The radiated power may be obtained from an estimate of the variance of the rate of change of flame surface area

\[ W_a = \frac{\rho_\infty}{4\pi c_\infty} \left( \frac{1}{\rho_b} - \frac{1}{\rho_u} \right)^2 (\rho_u S_L)^2 \left( \frac{dA}{dt} \right)^2 \]
Strahle (1971, …1985) provides alternative expressions of the sound radiated by flames

\[
p'(r, t) = c_0^2 \rho'(r, t) = -\frac{1}{4\pi r} \frac{\partial^2}{\partial t^2} \int_V \rho'_T \left( r_0, t - \frac{r}{c_0} \right) dV(r_0)
\]

\[
\rho'(r, t) = \frac{\overline{\rho}_1}{4\pi r} \int_{S_1} \frac{\partial \mathbf{v}_t}{\partial t} \left( r_0, t - \frac{r}{c_0} \right) \cdot \mathbf{n}_0 dS(r_0)
\]

\[
p'(r, t) = \frac{\overline{\rho}_1}{4\pi r} \frac{\gamma - 1}{\gamma p} (-\Delta h_f^0) \int_{V_c} \frac{\partial \dot{\omega}}{\partial t} \left( r_0, t - \frac{r}{c_0} \right) dV(r_0)
\]

\[
p'(r, t) = \frac{\gamma - 1}{4\pi c_0^2} \int_V \frac{1}{|r - r_0|} \frac{\partial}{\partial t} \dot{Q}'(r_0, t - |r - r_0|/c_0) dV(r_0)
\]
Only keeping the heat release source term, the following equation governs the noise emission from the flame:

\[ \nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = -\frac{\gamma - 1}{c_0^2} \frac{\partial \dot{Q}'}{\partial t} \]
Assuming that radiation takes place in an unconfined domain and only keeping the source term associated with perturbations in heat release one obtains

\[ p'(\mathbf{r}, t) = \frac{\gamma - 1}{4\pi c_0^2} \int_V \frac{1}{|\mathbf{r} - \mathbf{r}_0|} \frac{\partial}{\partial t} Q'(\mathbf{r}_0, t - |\mathbf{r} - \mathbf{r}_0|/c_0) dV(\mathbf{r}_0) \]

When the observation point is in the farfield of a compact flame the previous expression becomes (Strahle (1985))

\[ p'(\mathbf{r}, t) = \frac{\gamma - 1}{4\pi c_0^2 r} \frac{\partial}{\partial t} \int_V Q'(\mathbf{r}_0, t - r/c_0) dV(\mathbf{r}_0) \]
The classical expression of Hurle and Price

\[ p'(r, t) = \frac{\rho_{\infty}}{4\pi r} \left( \frac{\rho_u}{\rho_b} - 1 \right) \left[ \frac{dq}{dt} \right]_{t-\tau} \]

is equivalent to that obtained previously

\[ p'(r, t) = \frac{\gamma - 1}{4\pi c_0^2 r} \frac{\partial}{\partial t} \int_V \dot{Q}'(r_0, t - r/c_0) dV(r_0) \]

when the flame is compact and is formed by premixed reactants. This is shown by noting that :

\[ \rho_0 \left( \frac{\rho_u}{\rho_b} - 1 \right) q = \rho_0 \left( \frac{T_b}{T_u} - 1 \right) q = \frac{\gamma - 1}{c_0^2} \int \dot{Q}' dV \]
Combustion noise scaling laws

The power radiated by the flame may be expressed as

$$W_a = \frac{\overline{p' r^2}}{\rho_0 c_0} 4\pi r^2 = \frac{(\gamma - 1)^2}{4\pi \rho_0 c_0^5} \int_V \frac{\partial \dot{Q}'}{\partial t} (r'_0, t) \frac{\partial \dot{Q}'}{\partial t} (r''_0, t) d\mathbf{r}_0' d\mathbf{r}_0''$$

Changing variables

$$r'_0 = r'_0$$
$$\xi_0 = r''_0 - r'_0$$

one obtains

$$W_a = \frac{\overline{p' r^2}}{\rho_0 c_0} 4\pi r^2 = \frac{(\gamma - 1)^2}{4\pi \rho_0 c_0^5} \int_V \frac{\partial \dot{Q}'}{\partial t} (r'_0, t) \frac{\partial \dot{Q}'}{\partial t} (r'_0 + \xi_0, t) d\mathbf{r}_0' d\xi_0$$
One obtains the following estimate

\[ W_a = \frac{p'^2}{\rho_0 c_0} 4\pi r^2 = \frac{(\gamma - 1)^2}{4\pi \rho_0 c_0^5} f_c^2 \dot{Q}'_\text{max} V V_{\text{cor}} \]

where \( f_c \) is a characteristic frequency

\[ \dot{Q}'_\text{max} = \eta \dot{m}_F h / V \]
Premixed combustion noise correlation established by Rajaram and Lieuwen (2006). (See also the work of Belliard and Truffaut)
Premixed combustion noise correlation established by Rajaram and Lieuwen (2006)
Perturbed flame noise radiation

Compact premixed flame characterized by a thin reaction surface

Heat release rate is proportional to the flame surface area

Perturbed flame area is proportional to emission intensity from free radicals (CH*, OH*)

\[ p_\infty (r, t) = k_1 (r) \left[ \frac{dI}{dt} \right]_{t-\tau} = k_2 (r) \left[ \frac{dA}{dt} \right]_{t-\tau} \]

k1 and k2 are known coefficients

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Experimental setup

**Flames stabilization**
- Burner rim  conical and M
- Central rod  V and M flame
- Perforated plate CCF

**Burner**
- Grid+ nozzle  laminar flow
- Loudspeaker  ac. Perturbation
- Axisymmetric

**Diagnostics**
- Microphone  phased locked
- PM+ filter  noise emission
- LDV  light emission
- PIV  axial velocity
- ICCD camera  velocity field
- Flame surface

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Perturbed conical flames

Low frequency modulation

\[ \omega_\ast = \frac{\omega R}{S_L \cos \alpha_0} \]

methane-air flame, \( \Phi = 0.95 \), \( v_0 = 0.96 \) m.s\(^{-1}\), \( \omega_\ast = 5 \)
Perturbed conical flames

Intermediate frequency modulation

methane-air flame, $\Phi = 0.95$, $v_0 = 0.96$ m.s$^{-1}$, $\omega_* = 15$

$\omega_* = \frac{\omega R}{S_L \cos \alpha_0}$
Noise emission from conical flame

Conical flames

\[ \frac{v_{rms}}{\overline{v}} = 0.47 \]
\[ \frac{A_{rms}}{\overline{A}} = 0.11 \]

- Strong harmonic forcing
- Moderate noise production

Maximum noise emission results from a \textit{fast rate of production} of the flame surface area (a-b)

\( \text{(a) CF: } f_e = 50 \ \text{Hz, } \overline{v} = 1.7 \ \text{m s}^{-1}, \)
\( v_{rms} = 0.8 \ \text{m s}^{-1}, \Phi = 1.11 \)

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Mutual flame interaction

Self-induced instability of a flame spreading from an annular burner

The M flame is stabilized both on the central rod and the burner rim


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Noise emission from M flame

Maximum noise emission corresponds to a fast rate of destruction of the flame surface area due to mutual annihilation of neighboring flame elements.

\[ v_{rms}/\bar{v} = 0.17, \quad A_{rms}/\bar{A} = 0.19 \]

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Noise emission from M-flame

Power spectral density of acoustic pressure during flame/flame interaction

\[
PSD(dB) = 10 \log_{10} \left[ PSD(Pa^2/Hz) \Delta f / P_{ref}^2 \right]
\]

\[\Phi = 1.13, \quad v = 1.71 \text{ m s}^{-1}, \quad = 150 \text{ Hz}, \quad v' = 0.50 \text{ m s}^{-1}\]
Flame vortex interactions

Visualization of the flame-vortex interaction (inverted conical flame) obtained by applying an Abel transform to direct images of light emission by the flame.

Vorticity fields provided by phase averaged PIV.

\[ \Phi = 0.8, \quad v = 1.87 \text{ m s}^{-1}, \quad f = 150 \text{ Hz}, \quad v' = 0.15 \text{ m s}^{-1} \]

Flame vortex interactions

\[ \Phi = 0.8, \quad v = 1.87 \text{ m s}^{-1}, \quad f = 150 \text{ Hz}, \quad v' = 0.15 \text{ m s}^{-1} \]

Flame vortex interactions

\[ \Phi = 0.8, \quad v = 1.87 \text{ m s}^{-1}, \quad f = 150 \text{ Hz}, \quad v' = 0.15 \text{ m s}^{-1} \]

Flame vortex interactions

\[ \Phi = 0.8, \, v = 1.87 \, \text{m s}^{-1}, \, f = 150 \, \text{Hz}, \, v' = 0.15 \, \text{m s}^{-1} \]

Noise emission from V-flame

Maximum noise production corresponds to a fast rate of destruction of the flame surface area due to flame vortex interaction.

\( \frac{v_{\text{rms}}}{\bar{v}} = 0.26 \)
\( \frac{A_{\text{rms}}}{\bar{A}} = 0.43 \)

(c)VF : \( f_e = 100 \text{ Hz, } \bar{v} = 2.3 \text{ m s}^{-1}, \)
\( v_{\text{rms}} = 0.6 \text{ m s}^{-1}, \Phi = 1.11 \)

Noise emission from V flame

Power spectral density of acoustic pressure during flame vortex interaction

\[ PSD(dB) = 10 \log_{10} \left[ \frac{PSD(Pa^2/Hz) \Delta f}{p_{ref}^2} \right] \]

\( \Phi = 0.92, \ v = 2.56 \text{ m s}^{-1}, \ f = 170 \text{ Hz}, \ v' = 0.30 \text{ m s}^{-1} \)

Noise emission from a collection of small conical flames

Maximum and minimum pressure peaks result from both fast rates of production (a-b) and destruction (c-d) of the flame surface area.

(d) CSCF: $f_e = 400 \text{ Hz}$, $\bar{v} = 5.0\text{ m s}^{-1}$, $v_{rms} = 0.9\text{ m s}^{-1}$, $\Phi = 0.95$

Flame interaction with solid boundaries

Water cooled plate

Burner nozzle

Overall sound level as a function of modulation frequency

With plate-with flame

No plate-with flame

Lab noise level

Self-induced oscillation occur as a function of plate to nozzle distance

Confinement effects, noise and instabilities

Most of the work on combustion noise concerns radiation from unconfined flames.

In many applications combustion takes place in a confined environment and sound is radiated from the system inlet and exhaust.

Confinement induces interactions between the radiated Field and the sources of sound.

1. Weak coupling manifested by enhancement of sound by system resonances
2. Strong coupling in which energy fed in the burner eigenmodes destabilizes the reactive flow
The analysis may be carried out by expanding the perturbed field on a basis of eigenmodes (Zinn (1972), Culick (1980...))

\[ p'(\mathbf{r}, t) = \sum a_n(t)\psi_n(\mathbf{r}) \]

The eigenmodes satisfy the homogeneous equation

\[ \nabla \cdot \bar{c}^2 \nabla \psi_n + \omega_n^2 \psi_n = 0 \]

The modal amplitudes are given

\[ \frac{d^2 a_n}{dt^2} + \omega_n^2 a_n = F_n \]

where

\[ F_n = \frac{\bar{c}^2}{\Lambda_n} \left[ \int_V \frac{\gamma - 1}{\bar{c}^2} \frac{\partial \dot{Q}'}{\partial t} \psi_n(\mathbf{r}_0) dV(\mathbf{r}_0) + \int_S f \psi_n(\mathbf{r}_0) dS(\mathbf{r}_0) \right] \]

Heat release rate \hspace{1cm} Boundary condition
Boundaries have two main effects:

1. They modify the reflection response of the system.
2. They alter the flame dynamics.

The first item is treated in S. Candel et al. (2007) « Computational Flame Dynamics ». Invited lecture, ECCOMAS Computational Combustion. Delft and in various papers in combustion acoustics workshop.

The second item is illustrated by systematic variations of lateral confinement [Birbaud et al. (2007)]

\[ \xi = \frac{d}{D} = \frac{\text{Injector diam}}{\text{Flame tube diam}} \]
\[ \xi = 0. \quad \xi = 0.32 \quad \xi = 0.64 \]

\[ \frac{d(\text{LDV})}{dt}, \text{mic} (\text{arb. units}) \quad \frac{d(I_{\text{OH}^+})}{dt}, \text{mic} (\text{arb. units}) \]

\[ \xi = 0. \quad \xi = 0.64 \]
Computational combustion acoustics

Much of the current work in combustion noise is concerned with the development of computational tools for the estimation of sound radiated by flames.

Computational Combustion Acoustics (CCA) is in a relatively naissant state and borrows many tools and concepts from two more mature fields:

1. Computational aeroacoustics (CAA)
2. Computational flame dynamics (CFD)

Prediction of sound radiated by flames also raises new issues:

1. It is important to identify and determine the noise sources in the flame region.
2. The second problem is to propagate sound from the flame zone to the farfield.

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Computational Combustion Acoustics (CCA) is in a relatively nascent state.

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Computational strategies

Source modeling

- Statistical methods
- Stochastic methods
- LES and DNS

Propagation

- Green’s function
- Wave extrapolation
- Propagation using LEE

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Case 1: V-flame submitted to equivalence ratio modulations
Case 1: V-flame submitted to equivalence ratio modulations
Case 2 : Turbulent conical flame

Close-up view of the region of interest

Global view of the computational domain

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Reaction rate distributions in the flame
Conclusions and perspectives

- Progress in combustion noise has been quite substantial.

- Sources of indirect combustion noise identified in the 70’s are now the subject of further evaluation based on modern computational tools.

- Direct combustion noise sources also identified during the same period have been studied in a series of detailed investigations of perturbed flames.

- Basic expressions providing the sound field in terms of heat release rate fluctuations are now well validated in the premixed case.

- Early correlations for the radiated sound power and spectral densities have been improved.
Conclusions and perspectives

- Much of the recent effort has concerned the development of computational tools for combustion noise estimation.
- Initial efforts in this direction are promising but this field is less advanced than computational aeroacoustics (because of the added difficulties of combustion).
- One of the future challenges will be to devise reliable tools which will be able to retrieve data from well controlled experiments.
- The general objective will be to devise coupled methodologies providing accurate radiated sound fields, power and spectral densities.
- One may hope that this timely workshop will serve these goals well.
Combustion dynamics
Lecture 3

S. Candel, D. Durox, T. Schuller

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Université Paris-Saclay

Princeton summer school, June 2016

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Flow perturbation is produced

This induces a combustion perturbation

Acoustic feedback links the unsteady combustion process to flow perturbation

The system is unstable if the gain exceeds the damping
A simplified instability model
In region 1 (upstream of the flame)

\[ p_1 = A_1 \exp(ikx) + B_1 \exp(-ikx) \]
\[ v_1 = \frac{1}{\rho_0 c} [A_1 \exp(ikx) - B_1 \exp(-ikx)] \]

In region 2 (downstream of the flame)

\[ p_2 = A_2 \exp(ikx) + B_2 \exp(-ikx) \]
\[ v_2 = \frac{1}{\rho_0 c} [A_2 \exp(ikx) - B_2 \exp(-ikx)] \]
Across the flame, the pressure is continuous, the jump in velocity fluctuations is governed by fluctuations in heat release rate which caused by velocity fluctuations.

\[ p_1 = p_2 \]
\[ v_2 - v_1 = \mathcal{F}(\omega)v_1 \]

In this expression \( \mathcal{F}(\omega) \) designates the flame transfer function multiplied by \( (T_b/T_u) - 1 \)

The left and right conditions correspond to rigid walls

\[ v_1(0) = 0 \quad v_2(l) = 0 \]
Dispersion relation

\[ \sin(kl) + \mathcal{F}(\omega) \cos(kb) \sin(ka) = 0 \]

It is convenient to define

\[ \mathcal{H}(\omega) = \sin(kl) \]
\[ \mathcal{L}(\omega) = \sin(ka) \cos(kb) \]

In the absence of a flame, the resonant modes are given by

\[ \mathcal{H}(\omega_0) = 0 \]

The first root corresponds to

\[ \omega_0 = \pi c/l \quad f_0 = c/(2l) \quad \lambda = 2l \]

Half wave mode
Assuming that the flame response is weak and expanding to first order one obtains

\[
\mathcal{H}(\omega_0) + \left[ \frac{d\mathcal{H}}{d\omega} \right]_{\omega_0} \omega_1 + \mathcal{F}(\omega_0) \mathcal{L}(\omega_0) = 0
\]

Since \( \mathcal{H}(\omega_0) = 0 \)

One obtains the first order estimate

\[
\omega_1 = - \frac{\mathcal{F}(\omega_0) \mathcal{L}(\omega_0)}{\left[ d\mathcal{H}/d\omega \right]_{\omega_0}}
\]

\( \mathcal{L}(\omega_0) = \sin(\pi a/l) \cos(\pi b/l) \)

\[ \left[ d\mathcal{H}/d\omega \right]_{\omega_0} = (l/c) \cos(\omega_0 l/c) = -(l/c) \]
This can be written in the form
\[
\frac{\omega_1 l}{c} = \mathcal{F}(\omega_0) \sin(\pi a/l) \cos(\pi b/l)
\]

The imaginary part of this expression defines the growth rate
\[
\frac{\omega_{1i} l}{c} = \text{Im}[\mathcal{F}(\omega_0)] \sin(\pi a/l) \cos(\pi b/l)
\]

Writing the transfer function in terms of a gain and phase
\[
\mathcal{F}(\omega) = G(\omega) e^{i\phi(\omega)}
\]

one obtains
\[
\frac{\omega_{1i} l}{c} = G(\omega) \sin[\phi(\omega_0)] \sin(\pi a/l) \cos(\pi b/l)
\]
The sign of the imaginary part of the angular frequency defines the stability of this system. If the sign is positive, the system is unstable.

\[
\frac{\omega_{1i} l}{c} = G(\omega) \sin[\phi(\omega_0)] \sin(\pi a/l) \cos(\pi b/l)
\]

In general \( b/l > 1/2 \) hence \( \cos(\pi b/l) < 0 \)

and the first mode will be linearly unstable if

\[
\pi < \phi(\omega_0) < 2\pi \quad \text{modulo} \quad 2\pi
\]
One may now consider a situation where the duct is closed on the upstream side and open downstream. This case may be worked out as before. One has to change the boundary condition at $x = l$.

The dispersion relation becomes

$$\mathcal{H}(\omega) = \cos kl - \mathcal{F}(\omega) \sin ka \sin kb = 0$$
In the absence of a flame the dispersion relation becomes

\[ \cos kl = 0 \]

The first root of this expression is given by

\[ \omega_0 = \frac{\pi c}{2l} \]

corresponding to

\[ f_0 = \frac{c}{4l} \]

\[ \lambda = 4l \]

Quarter wave mode

Assuming that the flame response is weak and expanding to first order one obtains

\[ \mathcal{H}(\omega_0) + \left[ \frac{d\mathcal{H}}{d\omega} \right]_{\omega_0} \omega_1 + \mathcal{F}(\omega_0) \mathcal{L}(\omega_0) = 0 \]

where

\[ \mathcal{L}(\omega) = -\sin ka \sin kb \]
Since \( \mathcal{H}(\omega_0) = 0 \)

One obtains the first order estimate

\[
\omega_1 = -\frac{\mathcal{F}(\omega_0) \mathcal{L}(\omega_0)}{[d\mathcal{H}/d\omega]_{\omega_0}}
\]

This yields after some calculations

\[
\omega_1 = -\frac{c}{l} \mathcal{F}(\omega_0) \sin k_0 a \sin k_0 b
\]

The imaginary part of the angular frequency perturbation is given by

\[
\omega_{1i} = -\frac{c}{l} G(\omega_0) \sin \phi \sin \frac{\pi a}{2l} \sin \frac{\pi b}{2l}
\]

The system is unstable if \( \pi < \phi < 2\pi \) modulo \( 2\pi \)
Sensitive time lag concepts

Time lag analysis
An equation for the sensitive time lag
Heat release rate fluctuations
Intrinsic low frequency instability analysis of rocket engines
Luigi Crocco (1909-1986) one of the founders of combustion instability theory, was a professor at Princeton for many years. He spent the later part of his life in Paris and was a Professor at Ecole Centrale Paris for a few years.

H.S. Tsien (Tsien Hsue-Shen or Qian Xuesen) one of rocket propulsion pionneers (1911-2009), went to study at Caltech under the supervision of Theodore von Karman, he was one of the founders of the Jet Propulsion Laboratory, and later "Father of China’s Space Program".

Frank Marble (1918-2014) jet propulsion pionneer and eminent adviser.
Consider a mass of propellant burning between times $t$ and $t+dt$

$$\dot{m}_b(t) dt$$

This mass must be equal to the mass injected from $t - \tau$ to $t - \tau + d(t - \tau)$
\[ \dot{m}_b(t) dt = \dot{m}_i(t - \tau) d(t - \tau) \]

- This yields a relation between the mass rate of burning and the mass rate of propellants injected in the chamber

\[ \dot{m}_b(t) = \dot{m}_i(t - \tau)(1 - \frac{d\tau}{dt}) \]

- If the time lag is constant the last term vanishes and the mass rate of burning reflects the mass rate of injected propellants with a delay

\[ \dot{m}_b(t) = \dot{m}_i(t - \tau) \]

- In general the delay is not constant but is sensitive to the values of the main parameters governing the conversion rate of propellants in the chamber
An equation for the sensitive time lag

An equation for the time lag may be derived by considering the process which is involved in the conversion of propellants into combustion products. Consider a function $f(p, T_g)$ which globally describes this process.

This function will depend on pressure and gas temperature and may also depend on other parameters. To fix the ideas this function may be considered to represent the rate of heat transfer to the propellant. Vaporization of the liquid propellant will be achieved when a certain amount of heat designated by $C$ will have been transferred to the liquid. This may be described by stating that the sum of this function over the time lag will have to be equal to $C$

$$\int_{t-\tau}^{t} f(p, T_g) dt' = C = \text{const.}$$

The dependence of the time lag with respect to the state parameters is made more explicit by differentiating the previous equation with respect to time
\[
f[p(t), T_g(t)] - f[p(t - \tau), T_g(t - \tau)](1 - \frac{d\tau}{dt}) = 0
\]

Assuming that pressure and temperature remain close to their mean values, the function \(f\) may be expanded in a Taylor series

\[
f[p(t), T_g(t)] = f(\bar{p}, \bar{T}_g) + \frac{\partial f}{\partial p}(p - \bar{p}) + \frac{\partial f}{\partial T_g}(T_g - \bar{T}_g)
\]

\[
f[p(t - \tau), T_g(t - \tau)] = f(\bar{p}, \bar{T}_g) + \frac{\partial f}{\partial p}(p(t - \tau) - \bar{p}) + \frac{\partial f}{\partial T_g}(T_g(t - \tau) - \bar{T}_g)
\]

Inserting these expressions in the previous relation one finds that

\[
\frac{d\tau}{dt} = \frac{\partial \ln f}{\partial \ln p} \frac{p(t - \tau) - p(t)}{\bar{p}} + \frac{\partial \ln f}{\partial \ln T_g} \frac{T_g(t - \tau) - T_g(t)}{\bar{T}_g}
\]

The dependence of the time lag on the gas pressure and temperature in the chamber now appears explicitly.
It is convenient to define two interaction indices

\[ n = \frac{\partial \ln f}{\partial \ln p}, \quad q = \frac{\partial \ln f}{\partial \ln T_g} \]

One obtains the following expression for the rate of change of the time lag

\[ \frac{d\tau}{dt} = n \frac{p(t - \tau) - p(t)}{\bar{p}} + q \frac{T_g(t - \tau) - T_g(t)}{\bar{T}_g} \]

It is often considered that the burnt gas temperature remains essentially constant so that the second term vanishes. The rate of change of the time lag then becomes

\[ \frac{d\tau}{dt} = n \frac{p(t - \tau) - p(t)}{\bar{p}} \]
It is now possible to combine the two main expressions obtained previously

\[ \dot{m}_b(t) = \dot{m}_i(t - \tau)(1 - \frac{d\tau}{dt}) \]

\[ \frac{d\tau}{dt} = \frac{\dot{n} p(t - \tau) - p(t)}{\bar{p}} \]

One considers fluctuations around the mean value and one finds after a few calculations

\[ \frac{\dot{m}_b'(t)}{\dot{m}} = \frac{\dot{m}_i'(t - \tau)}{\dot{m}} - \frac{d\tau}{dt} \]

If the injected mass flow rate is constant, the fluctuation in burnt gas flow rate is given by

\[ \frac{\dot{m}_b'(t)}{\dot{m}} = \frac{\dot{n} p(t) - p(t - \tau)}{\bar{p}} \]
One may interpret this result in terms of relative heat release fluctuations by noting that

\[
\frac{\dot{m}_b'(t)}{\bar{m}} = \frac{\dot{q}'(t)}{\bar{q}}
\]

One obtains

\[
\frac{\dot{q}'(t)}{\bar{q}} = n \frac{p(t) - p(t - \tau)}{\bar{p}}
\]

An expression which is often used in analytical studies of instabilities coupled by longitudinal modes involves a delayed velocity perturbation impinging on the flame

\[
\frac{\dot{q}'(t)}{\bar{q}} = n \frac{u'(t - \tau)}{\bar{u}}
\]
This corresponds to a transfer function

\[ F(\omega) = \frac{\dot{q}'/\bar{q}}{u'/\bar{u}} = ne^{i\omega \tau} \]

The gain is constant and the phase depends linearly on frequency. This is a simplified description of what is found experimentally and theoretically but may be used to simplify the analysis.
As a first application of the time lag concept let us consider the low frequency instabilities (chugging instabilities) of rocket motors

(1) The gas temperature in the chamber is constant and uniform even in the presence of pressure oscillations. (2) The gas pressure is uniform in the combustion chamber and that it oscillates with a small amplitude around its mean value. (3) The time lag between injection and combustion exhibits the dependence described previously.

A balance of mass written for the thrust chamber indicates that

$$\frac{dM_g}{dt} = \dot{m}_b - \dot{m}_e$$

Now, in the steady state the gas mass in the chamber is constant and the mass rates of burning and ejection are equal.
\[
\bar{m}_b = \bar{m}_e = \bar{m}
\]

Introducing the fractional burning and discharge rates

\[
\mu_b = \frac{\dot{m}_b - \bar{m}}{\bar{m}}
\]

\[
\mu_e = \frac{\dot{m}_e - \bar{m}}{\bar{m}}
\]

The mass balance equation becomes

\[
\theta_g \frac{d}{dt} \left( \frac{M_g}{M_g} \right) = \mu_b - \mu_e
\]

\(\theta_g\) represents the average residence time that the burned gas spends in the chamber

It is convenient to introduce a dimensionless time \(z = t/\theta_g\)
\begin{equation}
\frac{d}{dz}\left(\frac{M_g}{M_g}\right) = \mu_b - \mu_e
\end{equation}

And to define a dimensionless time lag as well
\begin{equation}
\tilde{\tau} = \frac{\tau}{\theta_g}
\end{equation}

To simplify the notation the tilde will be deleted in what follows

The relative rate of burning obtained previously is first substituted in the mass balance
\begin{equation}
\frac{d}{dz}\left(\frac{M_g}{M_g}\right) = n[\varphi(z) - \varphi(z - \tau)] + \mu_i(z - \tau) - \mu_e(z)
\end{equation}

Now consider the mass of gas stored in the chamber
\begin{equation}
M_g = \int_V \rho_g dV = \int_V \frac{p}{RT_g} dV
\end{equation}
Because the pressure and temperature are both uniform in the chamber

\[ M_g = \frac{p}{RT_g} \int_V dV = \frac{pV}{RT_g} \]

\[ M_g/M_g = p/p = 1 + \varphi \]

\[ \frac{d\varphi}{dz} = n[\varphi(z) - \varphi(z - \tau)] + \mu_i(z - \tau) - \mu_e(z) \]

Consider the fractional variation of the mass flow ejected through the nozzle

\[ \mu_e(z) = (\dot{m}_e - \bar{m})/\bar{m} \]
In the low frequency range, the nozzle behaves as a compact element and it may be described in terms of a succession of equilibrium flows

\[ \dot{m}_e = K \frac{p}{(T_g)^{1/2}} \]

Since the temperature is constant in the chamber one finds that

\[ \frac{\dot{m}_e}{m_e} = \frac{p}{\bar{p}} \quad \text{and} \quad \mu_e(z) = \varphi(z) \]

The mass balance equation finally becomes

\[ \frac{d\varphi}{dz} = n[\varphi(z) - \varphi(z - \tau)] - \varphi(z) + \mu_i(z - \tau) \]

This equation governs the low frequency instabilities of a monopropellant engine

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Intrinsic rocket instabilities

If the injection rate is constant in time, and in particular if it is not influenced by the processes taking place in the chamber the dynamic behavior of the system is governed by

\[
\frac{d\varphi}{dz} + (1 - n)\varphi(z) + n\varphi(z - \tau) = 0
\]

\[
z = t/\theta_g, \quad \theta_g = M_g/\dot{m}, \quad n = \left(\frac{\partial \ln f}{\partial \ln p}\right)_{\bar{p}, \bar{T}_g}
\]

To examine the stability of this system one may take the Laplace transform of this equation or equivalently set the relative pressure fluctuation in the form

\[
\varphi(z) = Ae^{sz}
\]

This yields the characteristic equation
\[ s + (1 - n) + ne^{-s\tau} = 0 \]

One may solve the characteristic equation and discuss the sign of the real part of the roots obtained to determine the regions of stability. For this one may write

\[ s = \Lambda + i\Omega \]

This yields the following set of equations

\[ \Lambda + (1 - n) + ne^{-\Lambda\tau} \cos \Omega\tau = 0 \]
\[ \Omega - ne^{-\Lambda\tau} \sin \Omega\tau = 0 \]

Neutral stability is achieved when \( \Lambda = 0 \)

\[ 1 - n + n \cos \Omega_* \tau_* = 0 \]
\[ \Omega_* - n \sin \Omega_* \tau_* = 0 \]
\( \tau_* \) designates the time delay for neutral oscillation

The angular frequency of neutral oscillations is easily determined from these expressions

\[
(1 - n)^2 + \Omega_*^2 = n^2 \quad \text{and hence} \quad \Omega_*^2 = 2n - 1
\]

Because this angular frequency must be real it turns out that no neutral oscillations may exist if \( 0 < n < 1/2 \). When \( n > 1/2 \), the stability boundary may be obtained from the previous equation

\[
\Omega_* \tau_* = \cos^{-1} \left[ \frac{(n - 1)}{n} \right]
\]

\[
n = \frac{1}{1 - \cos \Omega_* \tau_*}
\]
Stability diagram

Unstable

\[
\begin{align*}
\gamma &\sim \frac{1}{2} \\
0 &\leq \Omega \leq 2\pi
\end{align*}
\]
Perturbed flames

Experiments on conical flames
Mutual interactions of flame sheets
Representing the flame dynamics using the G-equation
Flame transfer function concepts
Effects of equivalence ratio perturbations

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Acoustically coupled combustion instabilities \textit{(thermo-acoustic instabilities)}

Flame dynamics

(A)

Flow \rightarrow Combustion

(B)

Acoustics \rightarrow Combustion

(C)

Burner acoustics

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The flame can be stabilized in three different configurations. (film by Daniel Durox, EM2C)

Sensitivity of flames to acoustic waves

Experiments indicate that flames are sensitive to perturbations and that their response depends on the flame geometry, modulation frequency, type of perturbation and amplitude

Interactions leading to heat release rate disturbances

LES of the unsteady flow in a combustion chamber featuring a self-sustained oscillation

This film shows the reaction rate field

1. Flame interactions with the flow
   A. Velocity and mixture composition disturbances
   B. Mutual flame annihilation leading to destruction of flame surface area
   C. Anchoring devices used to stabilize the flame
   D. Combustion chamber walls confining the flame

2. Flame interactions with solid boundaries
It is instructive to examine simple situations before dealing with the more complex practical configurations. This is exemplified by experiments on the flame response to acoustic modulations generated in the fresh mixture.
Methane-air flame perturbed by upstream acoustic modulations

$\phi = 0.95 \quad f=75 \text{ Hz}$

Theoretical and experimental determinations of the transfer function of a laminar premixed flame.
\[ \omega_\ast = \frac{\omega R}{S_L \cos \alpha_0} \]

\[ \Phi = 0.95, \; v_0 = 0.96 \text{ m.s}^{-1}, \; \omega_\ast = 15 \]

methane-air flame

Schlieren visualisations

\[ \alpha_0 \]

\[ R \]

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$\Phi = 1.13$, $v = 1.71 \text{ m s}^{-1}$, $f = 150 \text{ Hz}$, $v' = 0.5 \text{ m s}^{-1}$
Many types of interactions have been identified by considering different flame geometries:

- Conical flame
- Cylindrical flame
- V-flame
- Swirling flame
- M-flame
- Flame on a wall

Lateral confinement effects on dynamics of inverted flames

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The G-equation and its perturbation

The flame is described by a level set. One level corresponds to the flame position

\[ G(x, t) = G_0 \]

This expression may be differentiated with respect to time

\[ \frac{dG(x, t)}{dt} = \frac{\partial G}{\partial t} + \mathbf{w} \cdot \nabla G = 0 \]

The unit normal vector is directed towards the fresh stream

\[ \mathbf{n} = -\nabla G / |\nabla G| \]

\[ \mathbf{w} = \mathbf{v} + S_d \mathbf{n} \]

\[ G(x, t) = G_0 \]

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Using the previous expressions for the normal and the absolute flame velocity one obtains

\[
\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d |\nabla G| \]

This expression can now linearized by introducing small perturbations around the mean value

\[
G = G_0 + G_1 \\
\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1 \\
S_d = S_{d0} + S_{d1}
\]
Retaining only terms up to first order one finds that
\[
|\nabla G_0 + \nabla G_1| = |\nabla G_0| + n_0 \cdot \nabla G_1
\]
\[
\frac{\partial G_1}{\partial t} + v_0 \cdot \nabla G_0 + v_1 \cdot \nabla G_0 + v_0 \cdot \nabla G_1 = (S_{d0} + S_{d1})(|\nabla G_0| + n_0 \cdot \nabla G_1)
\]
(1) - Transport equation for the mean $G_0$ field
\[
n_0 = -\frac{\nabla G}{|\nabla G|} \quad v_0 \cdot \nabla G_0 = S_{d0}|\nabla G_0|
\]
\[
S_{d0} - v_0 \cdot n_0 = 0
\]
(2) - Transport equation for the perturbed $G_1$ field

$$\frac{\partial G_1}{\partial t} + \mathbf{v}_1 \cdot \nabla G_0 + \mathbf{v}_0 \cdot \nabla G_1 = S_{d0} \mathbf{n}_0 \cdot \nabla G_1 + S_{d1} |\nabla G_0|$$

which may be rearranged in the form

$$\frac{\partial G_1}{\partial t} + (\mathbf{v}_0 - S_{d0} \mathbf{n}_0) \cdot \nabla G_1 = -\mathbf{v}_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0|$$

Making use of the result obtained at zero-th order one may write

$$\mathbf{v}_0 - S_{d0} \mathbf{n}_0 = \mathbf{v}_0 - (\mathbf{v}_0 \cdot \mathbf{n}_0) \mathbf{n}_0 = \mathbf{v}_{0t}$$

This is the velocity vector projected on the plane tangent to the flame

$$\frac{\partial G_1}{\partial t} + \mathbf{v}_{0t} \cdot \nabla G_1 = -\mathbf{v}_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0|$$

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Recalling that \( S_{d0} = v_0 \cdot n_0 \)

The right hand side of the previous equation may be written in the form

\[
-v_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0| = (v_1 - \frac{S_{d1}}{S_{d0}} v_0) \cdot n_0 |\nabla G_0|
\]

One obtains in this way the following equation

\[
\frac{\partial G_1}{\partial t} + v_{0t} \cdot \nabla G_1 = (v_1 - \frac{S_{d1}}{S_{d0}} v_0) \cdot n_0 |\nabla G_0|
\]

Burning velocity and velocity perturbations generate disturbances of the flame position in the normal direction, which are then convected along the flame front by the component of the mean local flow velocity \( v_{0t} \) parallel to the mean flame front.
Flame transfer function concepts
The flame response can be characterized in terms of a transfer function

\[ \mathcal{F}(\omega) = \frac{\dot{Q}' / \dot{Q}}{u' / U} \]

But the flame transfer function (FTF) only provides the linear growth rate in the analysis of instabilities,
Flowrate disturbances

Experiments carried out at a fixed mixture composition when the flame is submitted to harmonic flowrate modulations.

\[
\frac{\dot{Q}_1}{\dot{Q}_0} = \int_A \frac{dA_1(\Phi, \nu)}{A_0}
\]

Heat release rate fluctuations
flame surface area fluctuations

Flame surface wrinkles produce heat release rate disturbances

Methane/air
Phi=0.95, u’/u~0.3, u~1 m/s

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Harmonic mixture composition oscillations are convected and wrinkle the flame (no acoustic forcing):
(1) Fluctuations in the burning rate
(2) Flame surface area disturbances
(3) Feedback on the flow field
Giving rise to large heat release rate oscillations (nonlinear)

Heat release rate fluctuations

Volumetric heat release rate controlled by the fuel supply (only lean flames are considered)

\[ \dot{Q} = \int_A Y_F \rho S_d dA(\phi, v)(-\Delta h_f^0) \]

Fuel heating value (J/kg) is constant

\[ \dot{m}_f = Y_F \rho S_d \]

Fuel mass burning rate
- equivalence ratio
- stretch effects

\[ dA(\Phi, v) \]

Flame surface area
- equivalence ratio
- velocity

\[
\frac{\dot{Q}_1}{\dot{Q}_0} = \frac{\int \dot{m}_f (\phi, \epsilon) dA_0}{\dot{m}_f_0 \int dA_0} + \frac{\int dA_1(\Phi, v)}{\int dA_0}
\]

Heat release rate fluctuations

mass burning rate fluctuations averaged over the flame surface area

Flame surface area fluctuations

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Growth rates are relatively weak : in many applications (but not all!) other mechanisms dominate

Exceptions : Oxy-fuel welding torch  D-L instability

Darrieus-Landau  Thermo-diffusive

Markstein (1964)
Clavin et al. (1990)
Buckmaster and Ludford (1982)
Sivashinski (1976…)
Law (2008)
Clanet & Searby (1998)
Searby et al. (2001)
First example

Domestic boiler

During unstable regime, walls are “breathing” highlighting large pressure oscillations within the combustion
Stable regime: the combustion zone (luminous zone) features small stochastic fluctuations around its mean location (effects of turbulence). Radiated noise remains weak and broad band: “combustion roar”.

Unstable regime: Large synchronized motions with a large peak noise emission. Intensification of luminosity near the wall: higher heat fluxes transferred. Induce flame flashback.
Third example

Azimuthally coupled instabilities in an annular combustor equipped with 16 matrix burners

Standing mode @ f=380 Hz

Spinning mode @ f=498 Hz
Acoustically induced combustion Instabilities (thermo-acoustic instabilities)
Flowrate disturbances

Experiences made with a fixed mixture composition when the flame is submitted to harmonic flowrate modulations.

Heat release rate fluctuations

\[ \frac{\dot{Q}_1}{\dot{Q}_0} = \int_A \frac{dA_1(\Phi, \nu)}{A_0} \]

Flame surface area fluctuations

\( f = 75 \text{ Hz} \)

Flame surface wrinkles produce heat release rate disturbances

\( f = 150 \text{ Hz} \)

Methane/air

\( \phi = 0.95, \ u'/u \sim 0.3, \ u \sim 1 \text{ m/s} \)

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Equivalence ratio inhomogeneities

\[ \phi = \phi_0 + \phi' \]

Equivalence ratio perturbations

Lieuwen and Zinn (1998)

\[ \tau_i + \tau_{conv} + \tau_c = (2n - 1) \frac{T}{2} \]
Combustion dynamics of flames interacting with equivalence ratio perturbations

Conical flame perturbed by equivalence ratio modulations

Velocity field induced on the upstream side of the flame by interactions with equivalence ratio modulations

Combustion dynamics of flames interacting with equivalence ratio perturbations

Conical flame perturbed by equivalence ratio modulations

Velocity field induced on the upstream side of the flame by interactions with equivalence ratio modulations

Inverted flame perturbed by equivalence ratio oscillations

Premixed flow with equivalence ratio perturbations

\[ \phi(t) = \phi_0 + \phi_1 \sin \omega t \]

\[ \phi_0 = 0.8 \text{ and } \phi_1 = 0.1 \]
Harmonic mixture composition oscillations are convected and wrinkle the flame (no acoustic forcing):

1. Fluctuations in the burning rate
2. Flame surface area disturbances
3. Feedback on the flow field

Result in large heat release rate oscillations (nonlinear)

Volumetric heat release rate controlled by the fuel supply
Only lean flames are considered

\[ \dot{Q} = \int_A Y_F \rho S_d dA(\phi, v)(-\Delta h_f^0) \]

Fuel heating value (J/kg) is constant

\[ \dot{m}_f = Y_F \rho S_d \]

Fuel mass burning rate
- equivalence ratio
- stretch effects

\[ \frac{\dot{Q}_1}{\dot{Q}_0} = \frac{\int \dot{m}_{f1}(\phi, \epsilon) dA_0}{\dot{m}_{f0} \int dA_0} + \frac{\int dA_1(\Phi, v)}{\int dA_0} \]

Heat release rate fluctuations
mass burning rate fluctuations averaged over the flame surface area
flame surface area fluctuations

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Mixture composition disturbances lead to fuel mass burning rate perturbations

\[
\frac{\int \dot{m}_{f1}(\phi, \epsilon) \, dA_0}{\dot{m}_{f0} \int dA_0} = \frac{m(\phi_0) \int \phi_1 \, dA_0}{\phi_0 \int \phi \, dA_0}
\]

\[
m(\phi_0) = \left[ \frac{\partial (\rho/\rho_0)}{\partial (\phi/\phi_0)} + \frac{\partial (Y_f/Y_{f0})}{\partial (\phi/\phi_0)} + \frac{\partial (S_L/S_{L0})}{\partial (\phi/\phi_0)} \right]_{\phi=\phi_0}
\]

\[
<< 1 \quad \sim 1 \quad a
\]

\[
m(\phi_0) \simeq 1 + a
\]

Example: \(\text{CH}_4/\text{air}\)

\[
\phi_0 = 0.8 \quad a = \frac{\partial (S_L/S_{L0})}{\partial (\phi/\phi_0)} = 2.30
\]
Fluctuations of the flame displacement speed

Mixture composition oscillations

\[ \phi = \phi_0 + \phi_1(t) \]

\[ \phi_1(t) = \Phi \exp(-i\omega t) \]

The flame speed describes (twisted) cycles around steady conditions for increasing modulation frequencies

Lauvergne & Egolfopoulos (2000)
Response of lean premixed flames to mixture composition disturbances

\[ \phi_0 < 1 \quad \dot{m}_f = Y_F \rho S_d \]

\[ \frac{\dot{Q}_1}{\dot{Q}_0} = m(\phi_0) \frac{\int \phi_1 dA_0}{\phi_0 \int dA_0} + \frac{\int dA_1(\Phi, v)}{\int dA_0} \]

Heat release rate fluctuations
mass burning rate fluctuations averaged over the flame surface area
flame surface area fluctuations

\[ m(\phi_0) \approx 1 + a \]

Heat release rate fluctuations result from fuel mass burning rate (function of \( \phi \) and mean flame surface area) AND flame surface area fluctuations (function of \( \phi, v \) and mean flame surface area).
Flame surface area fluctuations

Kinematic description of the flame sheet \((G=0)\) separating the burnt gases \((G>0)\) from the fresh reactants \((G<0)\)

\[
\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d|\nabla G|
\]

\[
S_d(\phi, \epsilon, \kappa, t) \simeq S_L(\phi)
\]

Conical flame

V-flame
- Boyer & Quinard (1990), Dowling (1999), Schuller et al. (2003)

Nonlinear response
- Schuller (2002), Lieuwen (2005), Preetham et al. (2008)

Turbulent flames

Swirling flames
- Palies et al. (2011), Acharya et al. (2012)
The G-equation

The flame is described by a level set. One level corresponds to the flame position

\[ G(x, t) = G_0 \]

This expression may be differentiated with respect to time

\[ \frac{dG(x, t)}{dt} = \frac{\partial G}{\partial t} + \mathbf{w} \cdot \nabla G = 0 \]

\[ \mathbf{n} = -\nabla G / |\nabla G| \]
Using the previous expressions for the normal and the absolute flame velocity one obtains

\[ \frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d |\nabla G| \]

This expression can now linearized by introducing small perturbations around the mean value

\[
G = G_0 + G_1 \\
\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1 \\
S_d = S_{d0} + S_{d1}
\]
Retaining only terms up to first order one finds that

\[ |\nabla G_0 + \nabla G_1| = |\nabla G_0| + n_0 \cdot \nabla G_1 \]

\[ \frac{\partial G_1}{\partial t} + v_0 \cdot \nabla G_0 + v_1 \cdot \nabla G_0 + v_0 \cdot \nabla G_1 = (S_{d0} + S_{d1})(|\nabla G_0| + n_0 \cdot \nabla G_1) \]

(1) - Transport equation for the mean \( G_0 \) field

\[ n_0 = -\frac{\nabla G}{|\nabla G|} \quad v_0 \cdot \nabla G_0 = S_{d0}|\nabla G_0| \]

\[ S_{d0} - v_0 \cdot n_0 = 0 \]
(2) - Transport equation for the perturbed $G_1$ field

$$\frac{\partial G_1}{\partial t} + \mathbf{v}_1 \cdot \nabla G_0 + \mathbf{v}_0 \cdot \nabla G_1 = S_{d0} \mathbf{n}_0 \cdot \nabla G_1 + S_{d1} |\nabla G_0|$$

which may be rearranged in the form

$$\frac{\partial G_1}{\partial t} + (\mathbf{v}_0 - S_{d0} \mathbf{n}_0) \cdot \nabla G_1 = -\mathbf{v}_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0|$$

Making use of the result obtained at zero-th order one may write

$$\mathbf{v}_0 - S_{d0} \mathbf{n}_0 = \mathbf{v}_0 - (\mathbf{v}_0 \cdot \mathbf{n}_0) \mathbf{n}_0 = \mathbf{v}_{0t}$$

This is the velocity vector projected on the plane tangent to the flame

$$\frac{\partial G_1}{\partial t} + \mathbf{v}_{0t} \cdot \nabla G_1 = -\mathbf{v}_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0|$$
Recalling that \( S_{d0} = v_0 \cdot n_0 \)

The right hand side of the previous equation may be written in the form

\[-v_1 \cdot \nabla G_0 + S_{d1} |\nabla G_0| = (v_1 - \frac{S_{d1}}{S_{d0}} v_0) \cdot n_0 |\nabla G_0|\]

One obtains in this way the following equation

\[
\frac{\partial G_1}{\partial t} + v_{0t} \cdot \nabla G_1 = (v_1 - \frac{S_{d1}}{S_{d0}} v_0) \cdot n_0 |\nabla G_0| \]

Burning velocity and velocity perturbations generate disturbances of the flame position in the normal direction, which are then convected along the flame front by the component of the mean local flow velocity \( v_{0t} \) parallel to the mean flame front.

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Equivalence ratio perturbations induce changes in the local displacement velocity

\[ S_d = S_{d0} + S_{d1} \]

such that

\[ S_{d1} = S_{d0}(1 + a \frac{\phi_1}{\phi_0}) \quad \text{where} \quad a = \frac{\phi_0}{S_{d0}} \left( \frac{\partial S_d}{\partial \phi} \right)_{\phi=\phi_0} \]

(1) - Transport equation for the mean $G_0$ field

\[ \mathbf{n}_0 = -\frac{\nabla G_0}{|\nabla G_0|} \quad \mathbf{v}_0 \cdot \nabla G_0 = S_{d0}|\nabla G_0| \]

\[ S_{d0} - \mathbf{v}_0 \cdot \mathbf{n}_0 = 0 \]
Perturbed flame position

\[\frac{\partial G_1}{\partial t} + \left[ v_0 - S_{d0} \frac{\nabla G_0}{|\nabla G_0|} \right] \cdot \nabla G_1 = -v_1 \cdot \nabla G_0 + aS_{d0} |\nabla G_0| \frac{\phi_1}{\phi_0} \]

\[\frac{\partial G_1}{\partial t} + v_0^t \cdot \nabla G_1 = \left( v_1 - a \frac{\phi_1}{\phi_0} v_0 \right) \cdot n_0 |\nabla G_0| \]

\[v_0^t = v_0 - (v_0 \cdot n_0)n_0 \quad \text{mean flow velocity parallel to the mean flame front.} \]

Composition and velocity disturb the flame position in the normal direction. These disturbances are convected along the flame front by the component of the mean local flow velocity \(v_{0t}\) parallel to the mean flame front.
Application to inclined flames

Inclined flames

Many flames are stabilized in a flow featuring a principal direction. The flame sheet then forms an angle with this direction.
Flame reference frame

The flame motion is easier to analyze in a reference frame attached to the flame front

\[ G(X, Y; t) = Y - \xi(X; t) = 0 \]

\[
\frac{\partial \xi}{\partial t} + U_0 \frac{\partial \xi}{\partial X} = V_1(X; t) \\
\xi(0, t) = \xi_0(t)
\]

\( \xi \) normal flame front displacement with respect to the mean position

\( U_0 \) mean flow velocity along the flame front

\( V_1 \) velocity fluctuation normal to the flame front

\( \xi_0 \) normal flame front displacement at the flame base

Normal flame displacement

Solution for normal flame sheet displacement

\[ \xi(X, t) = \frac{1}{U_0} \int_0^X V_1 \left( X', t - \frac{X - X'}{U_0} \right) dX' + \xi_0 \left( t - \frac{X}{U_0} \right) \]

Perturbed velocity field contribution

Anchoring point dynamics

Interference integral for flame front perturbations

Wrinkles (i.e. normal flame displacement \( \xi(X,t) \)) appear as convected by the mean flow with a wavelength \( \lambda = U_0/f \).

This convective wave is modulated by a complex amplitude given by the integral term.

Boyer & Quinard (1990), Schuller et al. (2003), Lee & Lieuwen (2009), Borghesi et al. (2009)
Anchoring point dynamics

Ring modulation and acoustic waves produce the same type of wrinkles along the flame front

Vibrating rod

Acoustic modulation

Ring modulation

Acoustic modulation


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Impact of flow perturbation model

Compact flame submitted to low frequency acoustic forcing

\[ \lambda = \frac{c}{f} \gg L \]

Forcing conditions:
\[ f=62.5 \text{ Hz}, \frac{v'}{v_0}=0.20 \]

Operating conditions:
\[ \Phi=1.05, v_0=0.97 \text{ m/s} \]

---

Predictions with a uniform flow modulation

Predictions with a convective wave perturbation

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Hydrodynamic disturbances

They constitute an important component which determines the time lag of the flame response

$\Phi = 0.85, V_d = 5.2 \text{ m/s}, v' = 1.35 \text{ m/s}, f = 500 \text{ Hz}$

$kL \sim 1 \quad k = \frac{\omega}{u_0}$

$\nu_1 = \tilde{\nu}_1 \cos(\omega t - ky)$

$\nabla \cdot \mathbf{v}_1 = 0$

Flow perturbations are convected by the mean flow and are of hydrodynamic type (incompressible)

De Soete (1964), Baillot et al. (1992), Schuller et al. (2002), Birbaud et al. (2006), Noiray et al. (2006)
The Flame Transfer Function (FTF) describes the flame frequency response in terms of heat release rate disturbances due to the acoustic forcing.

The objective is to decouple the analysis of flow and combustion dynamics (nonlinear problem) from the analysis of the combustor acoustics (linear problem).
The FTF or Flame Impulse Response (FIR) are determined from DNS, LES, low-order models or experiments.
Premixed conical flame FTF submitted to harmonic incoming velocity perturbations in a CH4/air mixture

Experimental FTF determination

Velocity measured by LDV at the burner outlet

$\frac{v_1}{v_0}$

$\frac{\dot{Q}_1}{\dot{Q}_0} \approx \frac{I_1}{I_0}$

Ducruix et al. (2000)
Signal analysis

Mixture kept at constant equivalence ratio
Modulation level kept constant

$\Phi = 0.95$, $v_0 = 1.20 \text{ m/s}$, $v_{1\text{rms}} = 0.19 \text{ m/s}$, $v_{1\text{rms}}/v_0 = 0.16$

The velocity input is harmonic and the flame response (heat release rate fluctuation) remains also harmonic at these two forcing frequencies
FTF reconstruction

The FTF is deduced from cross-spectral power analysis of the input and output signals.

Harmonic velocity disturbances @ \( \omega \)

\[
\frac{v_1}{v_0} \quad F_v(\omega) \quad \frac{\dot{Q}_1}{\dot{Q}_0}
\]

Heat release rate disturbances

Cross-power spectral density of \( x(t) \) and \( y(t) \) examined at the forcing frequency

\[
F_v(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)}
\]

Power spectral density of \( x(t) \) examined at the forcing frequency

A periodogram method helps to improve the signal to noise ratio. Statistical convergence requires a large number of periods (typically more than 100).

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Conical Flame Transfer Function

\[ \frac{\dot{Q}_1}{\dot{Q}_0} \approx \frac{I_1}{I_0} = F_v \frac{v_1}{v_0} \]

\[ F_v = G(\omega) \exp(i\varphi) \]

Gain:
- relative fluctuation amplitude
- G>1 amplification
- G<1 attenuation
- Low pass filter

Phase:
- time lag \( \varphi = \omega T \)
- convective at low frequencies
- saturation at high frequencies

\[ \Phi = 1.05, \]
\[ v_0 = 1.20 \text{ m/s}, \]
\[ v_{1\text{rms}} = 0.19 \text{ m/s} \]

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Acoustic forcing synchronizes large vortices generated in shear layers that are responsible of rapid flame surface destruction when impacting the flame periphery.

Durox et al. (2005)
The flame front motion is controlled by the shear layer dynamics. The time lag corresponds to the travel time taken by a vortex to impinge the flame front (convected at about $V_{\text{max}}/2$). This time lag is barely affected by the input level.
Large scale structures

$f=70 \text{ Hz}$
Large scale structures

\[ f = 150 \text{ Hz} \]
The perturbation level modifies the response of the flame operated at the same conditions

\[ V_{\text{rms}} = 0.14 \, \text{m/s} \quad V_{\text{rms}} = 0.38 \, \text{m/s} \]

When the perturbation level increases saturation occurs: energy is transferred to higher harmonics and the gain examined at the forcing frequency drops.

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V- Flame Transfer Function

\[
\frac{\dot{Q}_1}{\dot{Q}_0} \approx \frac{I_1}{I_0} = F_v \frac{v_1}{v_0}
\]

\[F_v = G(\omega) \exp(i\varphi)\]

Gain:
- relative fluctuation amplitude
- Large overshoot $G > 1$
- Gain reduces with increasing $v_1$
- Low pass filter

Phase:
- time lag $\varphi = \omega \tau$
- convective time lag independent of the input level

Durox et al. (2005)
Self-induced instability of a premixed jet flame impinging on a plate
Most studies concern instabilities of confined flames.

The Rayleigh criterion is often satisfied.

\[ \int_T p'(t)q'(t)dt > 0 \]

Match of frequencies and proper phase difference instabilities.
Instabilities are also observed in the case of unconfined flames

with domestic burners

with radiant burners

with the distance of the piston Schäfer et al. (2000)

The coupling between acoustics and combustion differs from the case of confined flames and it is not well understood.
Experimental set-up

Outlet diameter: 22 mm        L = 100, 164 or 228 mm
CH4 - air. Equivalence ratio: 0.95.
Mean flow velocities: 1.20, 1.44 or 1.68 m/s.
Burner to plate distance (mm)

Sound Pressure level (dB)

$L = 100 \text{ mm}$

$\overline{V}_1 = 1.44 \text{ m/s}$

Steady operation
Sound pressure level (dB) vs. burner to plate distance (mm)

Unsteady operation

$L = 100$ mm

$\overline{V}_1 = 1.44$ m/s
Sound Pressure level (dB)

Unsteady operation

L = 100 mm
\( \bar{V}_1 = 1.44 \) m/s
Unsteady operation

$L = 100 \text{ mm}$
$
\overline{V}_1 = 1.44 \text{ m/s}$
Evolution of the fundamental frequency sawtooth pattern. In this case, the frequency is around 200Hz when the sound pressure level is highest.

\[ L = 100 \text{ mm} \]
\[ v_1 = 1.44 \text{ m/s} \]
Velocity on the axis, at 1.5 mm above the nozzle exit and heat release detected by the PM. At $z = 8.6 \text{ mm}$, the instability is strong.
Acoustic response of the burner

Helmholtz resonator
Bulk oscillation inside the burner

\[ \omega_0^2 = \left( \frac{c^2 S_1}{V L_{eff}} \right) \]

\[ L_{eff} = \int_{in}^{out} \frac{S_1}{S(z)} dz + \delta_e \]

\[ f_o = 202 \text{ Hz} \]
Instability model : Driven Helmholtz resonator

\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + kv_1' = -S_1 \frac{dp_1'}{dt} \]

\[ M = \bar{\rho} S_1 L_{eff} \]  Effective mass of air in the pipe

\[ R = \bar{\rho} S_1 v_1 \]  System damping

\[ k = \bar{\rho} c^2 S_1^2 / V \]  Gas volume stiffness

The resonator is driven by external pressure fluctuations \( p_1' \) at the burner outlet
Instability model: origin of external pressure fluctuations - Model of Price et al. (1969)

\[ p(r, t) = \frac{\rho_0}{4\pi r} \left( \frac{\rho_u}{\rho_b} - 1 \right) \left[ \frac{d\dot{Q}}{dt} \right]_{t-\tau_a} \]

\[ Q \propto I \text{ or } A \]

\( I \) is the light intensity emitted by free radicals

\( A \) is the flame surface area

\( \tau_a \) time delay between the source and the measurement point
Schuller et al. (2001)

Impinging flame with acoustic forcing of the flow

\[ p'(r, t) = K(r) \left[ \frac{dA'}{dt} \right]_{t-\tau_a} \]
The acoustic pressure and the time derivative of the heat release are similar, confirming that the source is suitably identified.
Velocity perturbations are convected by the mean flow along the flame front.

Fluctuations of the flame surface $A(t)$ are induced by these velocity perturbations after a convective delay $\tau_c$. $\tau_c$ is of the order of $\frac{\bar{z}}{v_1}$.

$$A'(t) = n(v'_1)t - \tau_c$$

$n$ characterizes the coupling between the flame surface fluctuation and the velocity perturbation.
\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + kv_1' = -S_1 \frac{dp_1'}{dt} \]

\[
\begin{align*}
p'(r, t) &= K(r) \left[ \frac{dA'}{dt} \right]_{t-\tau_a} \\
A'(t) &= n(v'_1)_{t-\tau_c}
\end{align*}
\]

\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + kv_1' = -S_1 K(r_{21}) n \left[ \frac{d^2 v_1'}{dt^2} \right]_{t-\tau} \]

with \[ \tau = \tau_a + \tau_c \]
\[
\frac{d^2 v'_1}{dt^2} + 2\delta \omega_0 \frac{dv'_1}{dt} + \omega_0^2 v'_1 = -N \left[ \frac{d^2 v'_1}{dt^2} \right]_{t-\tau}
\]

where \( \delta \omega_0 = R/(2M) \quad N = S_1 K(r_{21}) n/M \)

\[ \tau_a \ll \tau_c \implies \tau \approx \tau_c \]

This equation has a solution at the resonant frequency \( f_0 \) if :

\[ \omega_0 \tau = (4m - 1)\pi/2 \quad \text{where } m = 1, 2, \ldots \]

This imposes a condition on the convective delay:

\[ \omega_0 \tau_c \approx \omega_0 \tau = 3\pi/2 \quad (\text{modulo } 2\pi) \]
Thus the time delay between $A'$ (or $I_{CH^*}$) and $v_1$ corresponds to

$$\omega_0 \tau_c = \frac{3\pi}{2} \pmod{2\pi}$$

This is in agreement with the phase measurements when the system oscillates at the resonant frequency.
The system can oscillate in a frequency range around $f_0$

Taking $\delta = 30 \text{ s}^{-1}$ and $N=1$, this equation can be solved as a function of the time delay $\tau$.

Theoretical results are given in terms of a normalized delay $f_0\tau$.

Experimental data obtained for a given distance $Z$ are plotted in the same graph by estimating the delay $\tau_c : \tau_c \propto f(Z)/\overline{v_1}$
\[ \omega_0 \tau_c = \frac{3\pi}{2} \pmod{2\pi} \Rightarrow f_0 \tau = \frac{3\pi}{4}, \frac{7\pi}{4}, \]
Conclusions

- Strong instabilities may be induced when a premixed flame anchored on a burner rim impacts on a plate facing the burner exhaust.

- In this study the burner behaves like a Helmholtz resonator.

- The frequency of oscillation evolves with the burner to plate separation around the fundamental resonance frequency.
Conclusions

- Sudden annihilation of flame surface area produces an intense source of sound

- Flame wall interactions could play a role in the development of combustion instabilities

- Even without a plate, if flame surface variations are important and fast, and if the sound influences the flow velocity, then an instability can be triggered
Determination of the transfer function

thermocouple

cooled plate

LDV

PM

CH* filter

mixture of gases

loudspeaker

mixture of gases
The phase difference between the velocity and the heat release yields a mean convective time. It is of the order of $\frac{\bar{z}}{v_1}$.
\[ \bar{v}_1 = 1.68 \text{ m/s} \]

\[ Z = 8.6 \text{ mm Medium tube} \]
Phase difference between the heat release and the velocity.

When the sound level is the highest, the phase difference is close to $\pi/2$.

Then the heat release lags the velocity with a delay corresponding to $3\pi/2$. 

$$\varphi (v', q')$$

Burner to plate distance (mm)
Phase difference between microphone placed at the burner bottom and the velocity at the burner exhaust.
Zhang & Bray classification (1999)

Cool central core flame

Envelope flame

Envelope flame

Disk flame
Combustion dynamics of inverted conical flames
Combustion Instabilities

Confined flames

Combustion chambers
Gas turbine combustors

Unconfined flames

Domestic burners
Radiant burners

Low emission systems operating in premixed lean modes. Flames are less well stabilized and more susceptible to external perturbations.

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Low emission burner

Fuel+air with swirl

N. Dioc, S. Ducruix, F. Lacas and D. Veynante, EM2C, CNRS
European Program FuelChief

Experimental and numerical investigations on flame dynamics

Lee & Santavicca 2003
Pang et al. 2003

Lieuwen et al. 2003-2004
Shinjo et al. 2003
Huang et al. 2003

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Experimental set-up

Inverted conical flame (ICF)

Steady flame

Diameter 22 mm
CH\textsubscript{4} - air
Eq. Ratio : 0.92
Flow velocity : 2.05 m/s
Self-induced Instability

For certain flow conditions, equivalence ratios and geometry

Self-excited flame at \( f = 172 \) Hz
Eq. Ratio : 0.92
Flow velocity : 2.05 m/s
\( \nu' = 0.14 \) m/s

\( \Delta Q \)
\( \Delta v \)
(431nm)

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Transfer function

\[ \frac{\Delta Q}{Q} = f\left(\frac{\Delta v}{\bar{v}}\right) \quad I \propto Q \]

\[ v' (r = 7 \text{ mm, } z = 0.8 \text{ mm}) \]

Self-excited flame at \( f = 100 \text{ Hz} \)
Eq. ratio : 0.92
Flow velocity : 2.05 m/s
\[ v'_1 = 0.14 \text{ m/s} \]
Transfer function

$$\Phi = 0.92 \quad V_d = 2.05 \text{ m/s}$$

$$I'_{CH*}(t) = G[v'_1]t - \tau_c$$

$\varphi$ is nearly linear
convective lag

$$\tau_c = 8.6 \text{ ms}$$
Flame dynamics

Laser tomography of fresh stream seeded with oil droplets.

\[ f = 70 \text{ Hz} \]

\[ \Phi = 0.8 \]
\[ V_d = 1.87 \text{ m/s} \]
\[ v'_1 = 0.15 \text{ m/s} \]
Flame dynamics

\[ f = 150 \text{ Hz} \]

\[ \Phi = 0.8 \]

\[ V_d = 1.87 \text{ m/s} \]

\[ v'_1 = 0.15 \text{ m/s} \]
Unsteady vorticity field

\[ \Phi = 0.8 \]
\[ V_d = 1.87 \text{ m/s} \quad v'_1 = 0.15 \text{ m/s} \]
\[ f = 150 \text{ Hz} \]

The vortices are convected at a velocity \( V_{\text{max}}/2 \)

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Mechanism of instability

Helmholtz resonator

Bulk oscillation inside the burner

\[ L = 164 \text{ mm} \]

\[ \omega_0^2 = \left( \frac{c^2 S_1}{VL_{eff}} \right) \]

\[ L_{eff} = \int_{in}^{out} \frac{S_1}{S(z)} dz + \delta_e \]

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\[ f_0 = 163 \text{ Hz} \]
Mechanism of instability

Helmholtz resonator with driving

\[ M \frac{d^2 v_1'}{dt^2} + R \frac{dv_1'}{dt} + k v_1' = -S_1 \frac{dp_1'}{dt} \]

\[ M = \bar{\rho} S_1 L_{eff} \quad \text{Effective mass of gases} \]

\[ R = \bar{\rho} S_1 v_1 \quad \text{System damping} \]

\[ k = \bar{\rho} c^2 S_1^2 / V \quad \text{Stiffness of the gas volume} \]

The resonator is driven by external fluctuations \( p_1' \)
Signals measured in self-sustained instability case

$$\Phi = 0.92 \quad V_d = 2.05 \text{ m/s} \quad v'_1 = 0.14 \text{ m/s} \quad f = 172 \text{ Hz}$$

$$p'_1(t) \approx B \left[ I'_{CH^*} \right]_{t-\tau_a}$$

$$p'_1(t) \approx E \left[ dQ'/dt \right]_{t-\tau_a}$$

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Mechanism of instability

\[ I'_{C_{H*}}(t) = G[v'_1] t - \tau_c \]

\[ p'_1(t) = B[I'_{C_{H*}}] t - \tau_a \]

Time lag model
Mechanism of instability

\[ M \frac{d^2 v'_1}{dt^2} + R \frac{dv'_1}{dt} + kv'_1 = -S_1 \frac{dp'_1}{dt} \]

\[ p'_1(t) = B[I'_CH_*]_{t-\tau_a} \]

\[ I'_CH_*(t) = G[v'_1]_{t-\tau_c} \]

\[ \omega_0 = \left[ \frac{S_1 c^2}{V L_e} \right]^{1/2} \]

\[ 2 \delta \omega_0 = \frac{R}{M} \]

\[ \Omega = GB \frac{S_1}{M} \]
Mechanism of instability

\[
\frac{d^2 v'_1}{dt^2} + 2\delta\omega_0 \frac{dv'_1}{dt} + \omega_0^2 v'_1 = -\Omega \left[ \frac{dv'_1}{dt} \right]_{t-\tau_a-\tau_c}
\]

\[\tau_a \ll \tau_c \quad \Rightarrow \quad \tau \approx \tau_c\]

If \(\delta\) and \(\Omega\) are small, a linear analysis indicates that a necessary condition to have an instability is:

\[\omega_0 \tau \text{ belongs to } [\pi/2, 3\pi/2] \text{ modulo } 2\pi\]
Mechanism of instability

Frequency peak at the resonance \( f_o = 163 \, \text{Hz} \)

Instability frequency \( f = 172 \, \text{Hz} \)

\[ \Phi = 0.92 \quad V_d = 2.05 \, \text{m/s} \]

\( \Phi \) = 0.92
Conclusions

- ICF's are sensitive to low frequency acoustic excitations.

- They behave like an amplifier in a broad frequency range.

- The transfer function phase grows linearly: the process involves a convective delay.

- The main wrinkling of the flame front is due to vortex structures created in the shear layer.
Conclusions

- The strong rolling-up of the flame induces a mutual annihilation of neighboring reactive elements.
  - rapid variation of flame surface area
  - important source of pressure wave.

- With ICF's, at low amplitude modulation, entrainement of air modifies the equivalence ratio near the flame tip; the light emission ceases to be proportional to the heat release.
Steady flow streamlines

Average location of the flame front in the absence of perturbation

Averaged image of the flame front positions with a perturbation at 70 Hz

\[ \Phi = 0.8 \]
\[ V_d = 1.87 \text{ m/s} \]

\[ v'_1 = 0.15 \text{ m/s} \]
\[ f = 70 \text{ Hz} \]

The dot corresponds to

\[ h = \tau_c \frac{V_{\text{max}}}{2} \]
Combustion instability

Transfer Function

\[ \Delta v \text{ or } \Delta \Phi \rightarrow \Delta Q \]

Flow → Combustion

Acoustic Feedback

\[ \Delta v \rightarrow \Delta Q \]

\[ \Delta p = f(\Delta Q) \]

In many combustion chambers, pressure fluctuations are too weak to change the flame behavior.

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Axial velocity profiles

\[ \Phi = 0.92 \quad z = 0.7 \text{ mm} \quad V_d = 2.05 \text{ m/s} \quad f = 100 \text{ Hz} \]

The rms reference fluctuations are determined at 7 mm off the axis.
Axial velocity profiles

\[ \Phi = 0.92 \quad z = 0.7 \, \text{mm} \quad V_d = 2.05 \, \text{m/s} \quad f = 100 \, \text{Hz} \]
Gain vs Strouhal number

$\Phi = 0.92 \quad z = 0.7 \text{ mm} \quad V_d = 2.05 \text{ m/s} \quad f = 100 \text{ Hz}$

---

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Gain vs Strouhal number

\[ \Phi = 0.92 \quad z = 0.7 \text{ mm} \quad V_d = 2.05 \text{ m/s} \quad f = 100 \text{ Hz} \]
Gain vs Strouhal number

\( \Phi = 0.92 \quad z = 0.7 \text{ mm} \quad V_d = 2.05 \text{ m/s} \quad f = 100 \text{ Hz} \)
Signals measured under external modulation

\[ \Phi = 0.92 \quad V_d = 2.05 \text{ m/s} \]

\[ v'_1 = 0.30 \text{ m/s} \quad f = 170 \text{ Hz} \]

\[ v'_1 = 0.20 \text{ m/s} \quad f = 100 \text{ Hz} \]
Flamme noise

\[ V_2 = V_1 + \Delta V \]

The thermal expansion creates an increase in volume \( \Delta V \)

Isotropy of the phenomenon
Flamme noise

The flame is like a flow source \( D = d\Delta V/dt \):

\[ \nabla \left( \overline{c^2 p'} \right) - \frac{\partial^2 p'}{\partial t^2} = (\gamma - 1) \frac{\partial Q}{\partial t} \]

\[ p_\infty = \frac{1}{4\pi r} \frac{dQ}{dt} = \frac{1}{4\pi r} \frac{d^2 \Delta V}{dt^2} \]
Flame noise

Flame behaves like a monopolar volume source

\[ Q = \frac{d\Delta V}{dt} : \]

Monopolar radiation

\[ p_\infty = \frac{1}{4\pi r} \frac{dQ}{dt} = \frac{1}{4\pi r} \frac{d^2\Delta V}{dt^2} \]

Confined flames instabilities

If the heat release oscillates in the right frequency range and with an suitable phase

Instabilities
Phase difference between microphone placed at the burner bottom and the velocity at the burner exhaust
The flame response can be characterized in terms of a transfer function

\[ F(\omega) = \frac{\dot{Q}' / \bar{Q}}{u' / \bar{U}} \]

But the flame transfer function (FTF) only provides the linear growth rate in the analysis of instabilities,
It is more informative to use the flame describing function

\[ F(\omega, u') = \frac{\dot{Q}'/\overline{Q}}{u'/\overline{U}} \]

The Flame Describing Function (FDF) extends the transfer function concept to the nonlinear case.

In the FDF the flame response depends on the frequency and amplitude of the incident perturbation.


The flame response may be characterized in terms of a transfer function. The transfer functions depend on the input level.

\[ F(\omega) = \frac{\dot{Q}'/\dot{Q}}{u'/\overline{U}} \]

The flame transfer function (FTF) only provides the linear growth rate.

The flame describing function (FDF) gives access to the nonlinear growth rates. It can be used to determine limit cycle amplitudes and nonlinear features like mode switching, triggering, and hysteresis.

\[ F(\omega, u') = \frac{\dot{Q}/\dot{Q}}{u'/\overline{u}} \]
Nonlinear dynamics and flame describing function concepts
A framework for nonlinear instability analysis

Perforated plate

Flame is anchored on a perforated plate

Multipoint injection combustor

Microphone M1 and photomultiplier

Hot wire probe

Resonant manifold

Equivalence ratio: \( \Phi = 0.86 \)
Volumetric flow rate: \( \dot{m} = 5.4 \times 10^{-3} \text{ kg s}^{-1} \)
Thermal power: 14.4 kW

- Diameter \( D \): 70 mm
- Depth \( L \) easily adjustable: from 90 to 750 mm
- Unconfined reaction layer

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Combustion regimes

Depending on the burner depth $L$ combustion is either stable or unstable.


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Oscillation cycle ($f=530$ Hz) in a typical unstable situation (high amplitude sound radiation > 110 dB at 40 cm from the burner)
Typical self-sustained oscillations

Prediction of limit cycle amplitude constitutes a central challenge

$L = 460\, \text{mm}$
Driving and coupling leading to self-sustained oscillations

Flame describing function

Action

Feedback

Combustion noise and collective effects

Burner acoustics

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The flame response can be characterized in terms of a describing function

\[ \mathcal{F}(\omega, u') = \frac{\dot{Q}/\overline{Q}}{u'/\overline{u}} = \frac{\text{Relative heat release fluctuation}}{\text{Relative velocity fluctuation}} \]

\[ \mathcal{F}(\omega_r, |u'|) = G(\omega_r, |u'|) e^{i\varphi(\omega_r, |u'|)} \]
Flame Describing Function

\[ \mathcal{F} (\omega_r, |u'|) = G(\omega_r, |u'|) e^{i\varphi(\omega_r, |u'|)} \]
The system is unstable if \( \omega_i > 0 \).

The complex roots of the dispersion relation \( \mathcal{H}(\omega) = 0 \) characterize the stability of the system.
Type 1 trajectory in state space

Positive growth rate indicates that small perturbations are amplified
The limit cycle is obtained when the growth rate vanishes (negligible damping) or when the growth rate equals the damping rate (finite damping)
Type 2 trajectory in state-space

When the perturbation level exceeds a certain amplitude threshold the growth rate becomes positive ((perturbations are amplified) )
Positive growth rate contours in a “burner depth - velocity fluctuations amplitude” diagram

1/4 wave
3/4 wave
5/4 wave

Triggering
Mode switching

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Frequency shift during growth of oscillation

Suitable estimation of the limit cycle amplitude and of the frequency shift during the growth of oscillation
Prediction of the limit cycle amplitudes and of hysteresis

Cavity size $L$ progressively increased

Cavity size $L$ progressively decreased
Mode switching takes place when combustion is perturbed by blowing the flame collection with a lateral air blowing.
General view of combustion system


Combustion dynamics

Lecture 6b

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Combustion instabilities prediction: Recent progress based on Flame Describing Function

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Feedback resonant loop

Acoustics
Culick et al. (2001) Proceedings RTO/VKI

\[ p'(x, t) = \overline{p} \sum_{i=1}^{\infty} \eta_n(t) \psi_n(x) \]

Combustion dynamics
Candel et al. (1996) In ‘Unsteady combustion’

\[ \mathcal{F} = \frac{\dot{Q}' / \dot{\bar{Q}}}{u'/\bar{u}} = G e^{i\omega \tau} \]

Schlieren images of a conical flame subjected to acoustic modulation
Gas turbines and aerojet engines

Many experiments during thermoacoustic instabilities indicate that the main nonlinearity results from the flame oscillation:

- **Nonlinear flame**
  \[
  \frac{\dot{Q}'}{\dot{\bar{Q}}} \approx \mathcal{O}(1)
  \]

- **Linear acoustic**
  \[
  \frac{p'}{p_{\text{mean}}} \approx \mathcal{O}(0.01)
  \]

**Linear treatment of acoustic with wave equation**

\[
\nabla^2 p' - \frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} = 0
\]

**Need of nonlinear treatment of flame / flow interaction**

\[
\mathcal{F}(\omega, |u'|) = \frac{\dot{Q}'}{\dot{\bar{Q}}} \frac{u'}{\bar{u}}
\]

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Objectives

(1) Provide modeling elements to assess thermoacoustic instabilities
(2) Develop a nonlinear stability analysis based on the Flame Describing Function, a nonlinear extension of Flame Transfer Function concepts
(3) Show how to anticipate several nonlinear phenomena often observed in combustors
(4) Analyze limitations of the FDF framework
Outline

1. Experimental configuration
2. Linear stability analysis with Flame Transfer Function (FTF)
3. Nonlinear analysis with Flame Describing Function (FDF)
4. Nonlinear modeling results
5. Current issues using FDF framework
6. Conclusions
Flexible configuration offering possibilities to analyze combustion instabilities with and without confinement tube
Close-up views of the burner

Piston and feeding manifold

Perforated plate
420 holes of ID 2 mm OR
189 holes of ID 3 mm

Piston

Feeding manifold

Top view
During unstable operation, after ignition and transient growth, oscillations of flow variables reach a limit cycle.

For a given flow operating condition, stability depends on the feeding manifold $L_1$ and flame confinement $L_2$ lengths.

Typical transition to self-sustained oscillation

During unstable operation, after ignition and transient growth, oscillations of flow variables reach a limit cycle;
Typical flame oscillations

**Stable regime**
- Feeding manifold \((L_1)\) 0.25 m
- Confinement tube \((L_2)\) 0.1 m
- No flame motion
- SPL = 80 dB

**Unstable regime**
- Feeding manifold \((L_1)\) 0.29 m
- Confinement tube \((L_2)\) 0.1 m
- Cyclic oscillation of the flame at \(f = 750\) Hz accompanied by radiation of sound at the same peak frequency

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Example of stable and unstable regimes

Régime de combustion stable
Equilibre stable non-oscillant
$L=25\text{ cm}$
The oscillation frequencies at limit cycles lie close to, but do not always match, the eigenmodes of the combustor.
Exploration as a function of feeding manifold length $L_1$

The oscillation frequencies at limit cycles lie close to, but do not always match, the eigenmodes of the combustor.
Exploration as a function of feeding manifold length $L_1$

The oscillation frequencies at limit cycles lie close to, but do not always match, the eigenmodes of the combustor.
Self-sustained instabilities for different confinement tubes $L_2$

Bands of instabilities also depend on flame confinement. The system with the long confinement tube is always unstable. Larger deviations of limit cycle oscillation frequency from acoustic mode are observed for increased confinement tube lengths.

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Outline

1. Experimental configuration
2. Linear stability analysis with Flame Transfer Function (FTF)
3. Nonlinear analysis with Flame Describing Function (FDF)
4. Nonlinear modeling results
5. Current issues using FDF framework
6. Conclusions
Different modeling approaches

\[ M(\omega)X = F \]

- **A** (Acoustic Analysis)
  - No unsteady flame: \( F = 0 \)
  - Acoustic modes and modal structure
  - No information on unstable modes

- **B** (Linear Stability analysis)
  - Flame Transfer Function (FTF): \( F = F(\omega) \)
  - Initial oscillation frequency and exponential growth

- **C** (Nonlinear stability analysis)
  - Flame Describing Function: \( F = F(\omega, |u'|) \)
  - Instability frequencies and growth rate evolution
  - No information at limit cycle, limited to the prediction of linearly unstable modes
  - Limit cycle amplitude, frequency shift, hysteresis, mode switching, triggering
A – Acoustic analysis: $M(\omega)X = 0$

Boundary and matching conditions:

- $u_{1,0}' = 0$
- $p_{2,L2}' = 0$
- $S_2 u_{2,0}' = S_1 u_{1,L1}'$
- $p_{2,0}' = p_{1,L1}'$

Solution:

$X = X_0 \exp(-i\omega t)$, $\omega$ is a real number $\omega = 2\pi f$
Comparison between predictions and measurements

Frequencies measured at limit cycles are compared to the eigenmodes solutions of $\text{det}(M) = 0$

Limit cycle frequencies lie close but are shifted with respect to the acoustic predicted modes

The stable band between $L_1=0.23$ and $0.26$ m is not predicted

Switch between the different acoustic mode remains unexplained
B - Linear stability analysis: \( M(\omega)X = F(\omega) \)

Flame Transfer Function (FTF)

\[
F(\omega) = \frac{\dot{Q}'/\dot{\bar{Q}}}{u'/\bar{u}} = G(\omega_r)e^{i\varphi(\omega_r)}
\]

Solution: \( X = X_0 \exp(-i\omega t) \)

\( \omega \) is now a complex number

\[
\omega = \omega_r + i\omega_i
\]

\( \omega_r = 2\pi f \): Angular oscillation frequency

\( \omega_i \): Growth rate: instability when \( \omega_i > 0 \)
FTF determination

\[ \mathcal{F} = \frac{I'_OH/\bar{I}_OH}{u'/\bar{u}} \approx \frac{\dot{Q}'/\bar{Q}}{u'/\bar{u}} \]

Instability bands are determined by the phase of the FTF \( \omega_i > 0 \)
\[ \pi < \varphi < 2\pi \text{ modulo } 2\pi \]
Comparison between predictions with FTF and measurements

Oscillation frequencies at limit cycle (almost) lie within instability bands

Amplitudes of oscillation remain unknown
Outline

1. Experimental configuration
2. Linear stability analysis with Flame Transfer Function (FTF)
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C - Nonlinear stability analysis: $M(\omega)X = F(\omega, |u'|)$

Flame Describing Function (FDF)

$F(\omega, |u'|) = \dot{Q}' / \ddot{Q} = G(\omega_r, |u'|)e^{i\varphi(\omega_r, |u'|)}$

Solution: $X = X_0 \exp(-i\omega t)$

$\omega = \omega_r + i\omega_i$

$\omega_r = \omega_r(|u'|)$: Angular oscillation frequency

$\omega_i = \omega_i(|u'|)$ Growth rate: instability when $\omega_i > 0$

$\rho_1, c_1, T_1, k_1$ $\rho_2, c_2, T_2, k_2$

$0 \quad L_1 \quad z_1 \quad L_2 \quad z_2$

$G$

$\varphi$

$\omega_r$

$\omega_r$

$6\pi$

$4\pi$

$2\pi$

$0$

$0$

$\dot{Q}' / \ddot{Q}$

$\omega_r$

$|u'|$

$u'/\bar{u}$

$G(\omega_r, |u'|)e^{i\varphi(\omega_r, |u'|)}$

$\omega$ is a complex number that depends on the input level

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Describing Function (DF) has been used to designate the nonlinear saturation associated with flashback.

The Flame Describing Function has been devised to include the nonlinear dependence of the phase lag.

Possibility to determine limit cycle levels. The method is limited to linearly unstable modes.

Possibility to determine limit cycle levels. The method encompasses linearly unstable modes and nonlinearly unstable modes.
FDF determination

The FTF is determined for a single input level

\[ u' \xrightarrow{\mathcal{F}(\omega_r)} \dot{Q}' \]

The FDF is determined by a set of FTF measured for increasing input levels

\[ u' \xrightarrow{\mathcal{F}(\omega_r, |u'|)} \dot{Q}' \]
Outline

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Growth rates are calculated separately for the 3 modes using the following procedure:

\[ F = 0 \quad F = F(\omega_r, |u'_1|) \quad F = F(\omega_r, |u'_2|) \quad F = F(\omega_r, |u'_n|) \]

These growth rates and frequencies depend on the fluctuation amplitude:

\[ \omega_r = \omega_r \left( \frac{u'}{\bar{u}} \right) \]
\[ \omega_i = \omega_i \left( \frac{u'}{\bar{u}} \right) \]

This procedure is repeated for each length \( L_1 \) of the feeding manifold.
Stability analysis for mode 1:
Analysis of a linearly unstable mode

Growth rates $\omega_i = \omega_i(|u'|, L_1)$ are calculated for the first eigenmode of the combustor.

Regions where $\omega_i$ is negative are linearly stable.
Linearly unstable mode: Positive growth rate for infinitesimally small amplitude.

Limit cycle is reached when $\omega_i$ equals zero. The method yields the limit cycle oscillation level and oscillation frequency.
Nonlinearly unstable mode: Negative growth rate for small perturbation amplitudes, but positive values above a certain threshold. Limit cycle is reached when $\omega_i = 0$;
Stability analysis for mode 2 and 3

Mode 2 is linearly unstable for most of the feeding manifold lengths.

Mode 3 is linearly unstable for most of the feeding manifold lengths.
Bifurcation diagram

Obtained by superposition of instability bands calculated for each mode separately

Instability bands of different modes can overlap

This diagram can be used to determine:

A. Limit cycles
B. Hysteresis
C. Switching and triggering
Limit cycle (A) and hysteresis (B) predictions

$L_1 = 0.15\, \text{m} \rightarrow L_1 = 0.54\, \text{m}$
Limit cycle (A) and hysteresis (B) predictions

$L_1 = 0.15 \text{ m} \rightarrow L_1 = 0.54 \text{ m}$

Acoustic eigenmodes

Feeding manifold $L_1$ (m)

Sound pressure level (dB)

Frequency (Hz)

Stable

Unstable

$M_2$

Amplitude $\bar{u}/\bar{u}$

Feeding manifold $L_1$ (m)
Limit cycle (A) and hysteresis (B) predictions

$L_1 = 0.15 \text{ m} \rightarrow L_1 = 0.54 \text{ m}$
Limit cycle (A) and hysteresis (B) predictions

$L_1 = 0.54 \text{ m} \rightarrow L_1 = 0.15 \text{ m}$
Limit cycle (A) and hysteresis (B) predictions

$L_1 = 0.54 \text{ m} \quad \rightarrow \quad L_1 = 0.15 \text{ m}$
$L_1 = 0.54 \text{ m} \rightarrow L_1 = 0.15 \text{ m}$
C. Analysis of mode switching and triggering

Mode switching

Change of frequency and oscillation level in the absence of external perturbation

Mode triggering

Change of frequency and oscillation level due to an external perturbation of finite amplitude


It is shown here that these phenomena can be explained within the Flame Describing Function (FDF) framework.
Analysis of mode triggering

The system is set where mode 2 is linearly unstable and mode 1 nonlinearly unstable.

A: Unstable – 2 limit cycles
B: Stable – No limit cycle
C: Unstable – 1 limit cycle

In this case, the limit cycle level reached by mode 2 is close but lower than the threshold level to trigger mode 1: use of an external perturbation is needed.

Mode triggering

FDF calculation prediction

Switch 2\textsuperscript{nd} (1313 Hz) → 1\textsuperscript{st} (447 Hz)

Experiment

Triggering assisted with an external pulse

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The system is set where mode 3 is linearly unstable and mode 2 nonlinearly unstable. In this case, the limit cycle level reached by mode 3 is close but slightly higher than the threshold level to trigger mode 2: the system naturally switches to mode 2.

A : Unstable – 2 limit cycles
B : Stable – No limit cycle
C : Unstable – 1 limit cycle
Natural mode switching

FDF calculation prediction

Switch 3$^{rd}$ (769 Hz) $\rightarrow$ 2$^{nd}$ (465 Hz)

Experiment

Switch naturally to mode 2 during growth

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Validation of FDF framework

Unconfined laminar flames:
Noiray et al. (2008) JFM 615

Confined laminar flames:
Boudy et al. (2011) JEGTP 133
Boudy et al. (2011) PCI 33

Possibility to anticipate:
- Limit cycle amplitudes
- Limit cycle frequencies (including frequency shift)
- Hysteresis
- Triggering
- Mode switching

Confined Turbulent swirling flame:
Palies et al. (2011) C&F 158
Outline

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The amplitude and frequency of the limit cycle is not always fixed at a certain value.
Analysis of cases featuring multiple frequencies

Experiments indicate that multiple unstable modes may coexist for some conditions. One of such example is presented here.

Stable limit cycle

Galloping limit cycle

This problem is often evoked, Sterling (1993), Moeck et al. (2010), Lamraoui et al. (2011), Kabiraj et al. (2011).

It is shown here that some of these features can be explained through the nonlinear flame dynamics within the Flame Describing Function (FDF) framework.
Analysis of a configuration with two simultaneously unstable modes

- Single or main frequency
- Secondary frequency

![Diagram showing a configuration with two unstable modes](image.png)

Frequency (Hz) vs. Feeding Manifold $L_1$ (m)

- Second mode
- Side band

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Analysis of a configuration with two simultaneously unstable modes

- Single or main frequency
- Secondary frequency

![Diagram with labels: L1, L2, M1, M2, HW, L1 = 0.58 m, Side band, Frequency (Hz), Feeding Manifold L1 (m)]
Pressure measurements for $L_1 = 0.58$ m

Mode 3

Mode 2

Frequency (Hz)

Time (s)

$|p'|$

$|p'(t - 2\pi)|$

$-800$ $0$ $800$

$-800$ $0$ $800$

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Overlap of mode 3 linearly unstable and mode 2 nonlinearly unstable. The objective is to delineate cases where mode switching (single frequency at limit cycle) or simultaneous modes oscillations (galloping limit cycle with two frequencies) occur.

Analysis of the bifurcation diagram in the region where two simultaneously unstable modes were observed.

Overlap of mode 3 linearly unstable and mode 2 nonlinearly unstable. The objective is to delineate cases where mode switching (single frequency at limit cycle) or simultaneous modes oscillations (galloping limit cycle with two frequencies) occur.
Growth rate trajectories

Mode switching

(a) $L_1 = 0.54\ m - L_2 = 0.1\ m$

The product of the slopes of the growth rates is negative

Two modes sustained

(b) $L_1 = 0.52\ m - L_2 = 0.1\ m$

The product of the slopes of the growth rates is positive
Conclusions

FDF framework allows predictions of instability frequency and amplitude during thermoacoustic self-sustained oscillations.

When modes overlap different nonlinear phenomena can be anticipated leading to hysteresis, triggering and mode switching, which are well retrieved by predictions.

Current efforts aim at predicting self-sustained oscillations featuring multiple frequencies. FDF calculations allow to consider situations where one unstable mode takes over or two modes coexist.

Generalization of the FDF framework to predict variable amplitude limit cycles is in progress.
Recent EM2C publications on this topic


Combustion dynamics control concepts
Control techniques

- Acoustic losses at the boundaries
- Acoustic gain of the combustion process
- Acoustic energy balance positive: potential thermoacoustic coupling

Balance of the acoustic energy within the system

Acoustic response of the combustor

Frequency
First strategy: increase losses by inserting liners, Helmholtz resonators, quarter wave cavities perforated bias flow systems…

**Control techniques**

- Acoustic losses at the boundaries
- Acoustic gain of the combustion process

**Balance of the acoustic energy within the system**

**Acoustic damping**

**Gain of the process**

**Acoustic response of the combustor**

**Frequency**
Control techniques

Balance of the acoustic energy within the system

Second strategy: reduce the gain by modifying the flame dynamics. The present method belongs to this group.

Acoustic response of the combustor

Frequency

Acoustic damping

Gain of the process

Acoustic losses at the boundaries

Acoustic gain of the combustion process
Reactive flow simulation
Compressible flow solver AVBP:

- 4 million cells, 3D calculations
- No flame thickening
- 1 step Arrhenius kinetics
- Modulated velocity inlet
- Non reflecting outlet
- Periodic lateral boundaries
- TTGC scheme

Acoustic excitation ($f = 650$Hz)

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3D simulation of a system equipped with a dynamic phase converter

Dynamical control of instabilities

- A novel concept developed to control flame dynamics relies on mode conversion
- A Dynamical Phase Converter (DPC) demonstrated experimentally and numerically
- Technology adapted to practical configurations (patent)


Top: coherent flame response to incoming perturbations. Bottom: response of the system equipped with the DPC
Active control concepts

Flow → Combustion

Acoustic Feedback

Controller
Active methods: theoretical studies during the 50’s.
Tsien (1952), Marble (1953), Crocco and Cheng (1956)...
Sensitive time lag model was introduced to analyze rocket instabilities and their control.

Frank Marble and H.S. Tsien in China


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• **Further studies in the 1990’s:** additional demonstrations, actuator developments, low-order modeling of control, control algorithms (self-tuning, state feedback, robust control, adaptive and self-adaptive control…),

• **More recent work:** actuators and sensors for practical applications, scale-up and application in real systems, control modeling, multidimensional simulation
Adaptive control of combustion instabilities

**Active Control of a Premixed Prevaporized Combustor**

Schematic Active Control of the Siemens model Vx4.3A heavy duty gas turbine

Adapted from Hermann (2001)
Numerical simulation of active control with low order models

Difficulty in modeling complex nonlinear dynamics and in describing actuator effects

Useful as a guide for controller development

Block diagrams representation of active control systems

Representation of combustor coupled to an external control loop

Transfer function description of the system

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Adaptive control system

Transfer function representation of adaptive control

$H$ : combustion

$G$ : acoustic feedback

$W$ : LMS filter

$S_1, S_2$ : actuator transfer function and secondary path between actuator and sensor
Less well documented

Important requirements in terms of computer resources

Difficulties in actuator description and in coupling the flow solver to the controller
Objective: Reduce vortex driven instabilities in large solid propellant solid rocket motors.

Diminish the pressure oscillation by injecting a liquid oxidizer through an actuator. This oxidizer reacts with the surrounding hot gases. If properly phased this will reduce the oscillation.


Three mechanisms leading to vortex driven instabilities in solid propellant rocket motors

(1) Vortices shed by baffle

(2) Vortices shed by propellant edge

(3) Vortices shed by turning flow originating from propellant

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\[ mT = \frac{l}{kU} + \frac{l}{c-U} + \Delta t \]

Number of vortices  Acoustic period  Convective time  Acoustic time  Delay of conversion from convective to acoustic mode

\[ \Delta t = \alpha T \]

\[ \frac{m - \alpha}{f} = \frac{l}{U} \left[ \frac{1}{k} + \frac{M}{1-M} \right] \]

\[ f = \frac{kU}{l} \frac{m - \alpha}{1 + kM} \]
Model scale model of segmented solid rocket engine

Contours of pressure fluctuations with respect to frequency and mean flow velocity

Without adaptive control

Controller operating

Contours of pressure fluctuations with respect to frequency and mean flow velocity
Multidimensional simulation of active control

Actuator description by distributed source terms

\[ \rho^{n+1} = \rho^n + \delta t \omega_s \]
\[ (\rho u)^{n+1} = (\rho u)^n + (\delta t \omega_s) u_s \]
\[ (\rho v)^{n+1} = (\rho v)^n + (\delta t \omega_s) v_s \]
\[ (\rho E)^{n+1} = (\rho E)^n + (\delta t \omega_s) (e + U^2) \]
There is a timestepping mismatch between flow-solver and controller.
$f_{\text{sierra}} = 4.1 \text{ MHz}$

$\frac{1}{2} f_{\text{control}} = 20 \text{ kHz}$

This mismatch rapidly deteriorates the simulation.

The problem is suppressed by placing low pass filters at the controller input and output.
Controller switched on

Pressure signal at the nozzle entrance

Initial level

Final level

Controller switched on

Actuator signal
Resonant vortex shedding before control (elevated level of pressure fluctuation)

Vortex shedding under controlled operation (pressure fluctuation level is reduced by controller)

Combustion dynamics
Lecture 7b

S. Candel, D. Durox, T. Schuller

CentraleSupélec
Ecole Centrale Paris, EM2C lab, CNRS
Université Paris-Saclay

Princeton summer school, June 2016
A Novel Strategy for Passive Control of Combustion Instabilities through modification of flame dynamics

Nicolas Noiray, Daniel Durox, Thierry Schuller and Sébastien Candel

EM2C Laboratory, CNRS - Ecole Centrale Paris, 92295 Châtenay-Malabry, FRANCE
• **Thermoacoustic instabilities** cause serious problems in a wide range of combustion industrial applications

• **Causes**: Resonant coupling between flames and burner acoustics

• **Consequences**:
  - Structural vibration (sometimes destruction)
  - Heat fluxes enhancement to the boundaries
  - Flame extinction

• **Solutions**:
  - Active control
  - Passive Control

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Passive control techniques

Balance of the acoustic energy within the system

Acoustic response of the combustor

Acoustic losses: boundary conditions

Acoustic gain induced by the flame

Acoustic energy balance positive: Potential thermoacoustic coupling

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Passive control techniques

First strategy: modify the losses
Helmholtz resonator, perf. screen

Balance of the acoustic energy within the system

Acoustic losses: boundary conditions

Acoustic gain induced by the flame

Acoustic response of the combustor

Frequency
Passive control techniques

Second strategy: modify the gain by acting on the flame dynamics

Balance of the acoustic energy within the system

Acoustic losses: boundary conditions

Acoustic gain induced by the flame

Acoustic response of the combustor
I. Experimental configuration
II. Dynamic phase convertor principle
III. Forced flow response
   – Numerical simulations
   – Experimental results
IV. Unforced operation
V. Conclusions
Experimental setup

Burner diameter $D$ : 70 mm
Adjustable burner depth $L$ from 90 to 750 mm
Unconfined flame configuration

Eq. Ratio : $\Phi = 0.86$
Mass flow rate : $\dot{m} = 5.4 \times 10^{-3}$ kg s$^{-1}$
Thermal power : 14.4 kW

Perforated plate
400 holes
hole radius 1 mm
holes spacing 3 mm

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Depending on the burner size $L$, the combustion regime is either stable or unstable.
Oscillation cycle ($f=530$ Hz) for a typical unstable combustion regime

*the SPL exceeds 110 dB, 40 cm away from the flames*
Dominant frequency of the acoustic pressure radiated by the flames under unstable operation (>100 dB).

The dashed lines in the lower part of the diagram correspond to the eigenfrequencies of the cavity (quarter wave type).

**Burner size influence**

![Graph showing the relationship between burner size and sound pressure level](image)

- **Stable**
- **Unstable**

**Table:**

<table>
<thead>
<tr>
<th>Burner size L (m)</th>
<th>Frequency (Hz)</th>
<th>Sound pressure level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Make use of the **multiple flame configuration** and try to **avoid the coherent response** of the flames to acoustic perturbations.

**Problem**: the **acoustic wavelengths are much larger than the flame dimensions** (acoustic perturbations propagate at the sound speed) so it is not possible to decouple the individual flames from each other with an acoustic-based device.

**Transform the acoustic perturbations** which impinge the flames to **hydrodynamic perturbations** (vortices) which propagate at the mean velocity (much lower than the sound speed).

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Create a shear layer by adding a constriction, an orifice plate or a sudden expansion in the injectors.

Steady state, identical flames

Instead of suffering the coherent vortex shedding (possible mechanism yielding thermo-acoustic instability) it is here used to suppress the resonant coupling.
The phase difference between the two types of flames is

\[ \Delta \varphi = 2\pi f \frac{\Delta l}{U_{cv}} \]

where \( U_{cv} \) is the vortices velocity and \( \Delta l \) is the stagger distance.

In order to achieve an opposite motion (\( \Delta \varphi = \pi \)), \( \Delta l \) has to be defined as

\[ \Delta l = \frac{U_{cv}}{2f} \]

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Convection velocity

Forced flow experiments:
confined jet - LDV centerline measurements

Numerical LES simulations:
- code AVBP (Cerfacs), 1.7 M cells,
- compressible flow, 3D, pulsed velocity inlet, non-reflecting outlet boundary,
- TTGC scheme, $f=1000$ Hz, channel diameter and bulk velocity: $D=2\text{mm}$, $U=2\text{m/s}$

Q-criterion isosurface
Dynamic phase converter forced flow simulations and experiments
Acoustic excitation
($f = 650\text{Hz}$)

Reactive flow simulation
Compressible LES solver AVBP:

- 4 million of cells, 3D
- Non-thickened flame
- 1 step reaction
- Pulsed velocity inlet
- Non reflecting outlet boundary
- Periodic boundary conditions
- TTGC scheme
Color contours: Reaction rate

Gray contours: Axial fluctuating velocity

\( \lambda = 52 \text{ cm} \)

Inlet condition:

\[
\begin{align*}
\bar{u} &= 0.65 \text{ m s}^{-1} \\
\nu &= 650 \text{ Hz} \\
\nu' &= 0.1 \bar{u} \cos(2\pi ft)
\end{align*}
\]

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Velocity measurements

\[ \Delta l = 4 \text{ mm} \]
\[ U_{cv} = 5.3 \text{ m s}^{-1} \]

Nominal frequency

\[ f = \frac{U_{cv}}{2\Delta l} \approx 660 \text{ Hz} \]

-DPV Injector

-LDV - beams cross points

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Classic injector

DPC equipped injector
Classic injector

DPC equipped injector
Unforced operation

Classic injector
- Injector including constrictions at the same axial level (not staggered)

Dynamic phase converter (DPC)

Nominal frequency
\[ f = \frac{U_{cv}}{2\Delta l} = 1215 \text{ Hz} \]

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Features of the DPC

Advantages:
- Simple sizing method,
- Low requirement in terms of space and mass,
- Minor modification of the injection geometry,
- Broad band effectiveness,

Drawbacks:
- A trade-off between additional head loss (which depends on the constrictions dimensions) and vortices strength has to be considered to alter as less as possible the steady state performance
- Eigenmodes far from the nominal frequency may arise but could be suppressed with additional damping devices

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A new passive control strategy was developed. A hydrodynamic instability is used to suppress a thermo-acoustic instability.

The dynamic phase converter was successfully tested numerically and experimentally:

- under forced flow operation
- without forcing

The configuration features small scale laminar flames but the principle could be transferred to larger turbulent flames.
Swirling flame combustion dynamics
Swirl is used in jet engines systems to stabilize combustion

It serves to anchor the flame in modern lean premixed gas turbines

It is exploited in a variety of other combustion processes

Swirling flame dynamics constitutes a central issue in many applications
Combustion stabilization and swirl

- Stabilization relies on a central recirculation zone (CRZ) formed by hot combustion products which continuously initiate the reaction process.

- Swirling flames are more compact than flames anchored on a bluff body allowing a notable reduction in the chamber size.

- However, swirling combustors often develop self-sustained oscillations which have serious consequences.

- There are many other dynamical issues which arise in practical systems and deserve fundamental investigations.
Objectives

- Examine the interaction between a swirler and axial acoustic waves
- Determine effects of this interaction and obtain the flame response in terms of a describing function
- Use the describing function in the analysis of a generic system comprising a single injector
Confined swirling flame

Upstream manifold

Injector and swirler

Flame tube

Upstream manifold

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Experimental flame describing function

\[ \Phi = 0.7 \quad U_b = 2.67 \text{ m s}^{-1} \]

Distributions of phase average volumetric heat release
Examine interactions between swirlers and axial acoustic waves

Acoustics in a duct

\[ u' \]

\[ [\quad \quad \quad \quad \quad \quad \quad] \]

Acoustics in duct with swirler

\[ u' \]

\[ [\quad \quad \quad \quad \quad \quad \quad] \]

Determine

- Flow field induced on the downstream side of the swirler
- Phase velocities and disturbance amplitudes

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The interaction of entropy fluctuations with turbine blade rows; a mechanism of turbojet engine noise

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(Communicated by Sir William Hawthorne, F.R.S. – Received 23 November 1976)
Based on the theory of Marble & Cumpsty (1977):

\[ u_1' = \frac{A}{\rho c} \exp i\omega \left( \frac{x}{c} - t \right) + \frac{AR}{\rho c} \exp i\omega \left( -\frac{x}{c} - t \right) \]

Incident and reflected acoustic wave

\[ u_2' = \frac{A}{\rho c} \exp i\omega \left( \frac{x}{c} - t \right) \]

Acoustic mode

\[ v_2' = B \exp i\omega \left( \frac{x}{u_2} - t \right) \]

Convective (vorticity) mode
(A) Actuator disk analysis

\[ u_2' = \frac{A}{\rho c} \exp \left( \frac{x}{c} - t \right) \]

\[ v_2' = \frac{A}{\rho c} \tan \bar{\theta}_2 \exp \left( \frac{x}{\bar{u}_2} - t \right) \]

\[ R = 0 \quad \text{Reflection coefficient} \]
\[ T = 1 \quad \text{Transmission coefficient} \]

\[ B = \frac{A}{\rho c} \tan \bar{\theta}_2 \]

\[ \text{Acoustic mode} \]
\[ \text{Convective (vorticity) mode} \]

Acoustic-convective mode conversion in an airfoil cascade.
(B) Filtered velocity signals

\[ v'_2 = u'_2 \tan \bar{\theta}_2 \]

\[ \bar{\theta}_2 = 25^\circ \]

\[ f = 60 \text{ Hz} \quad u'_\infty = 1 \text{ m s}^{-1} \]

\[ f = 100 \text{ Hz} \quad u'_\infty = 1 \text{ m s}^{-1} \]
Axial and azimuthal velocities are measured with LDV.

Cross-spectral density analysis provides the phase and corresponding velocity.

\[
\begin{align*}
U_b &= 2.67 \text{ m s}^{-1} \\
f &= 60 \text{ Hz} \\
f &= 100 \text{ Hz} \\
u'/U_b &= 0.5 \\
d &= 22 \text{ mm} \\
L &= 50 \text{ mm}
\end{align*}
\]
Mode conversion at the swirler generates
- an axial acoustic wave
- an azimuthal velocity fluctuation in the 3D case or a transverse velocity fluctuation in the 2D case.

This corresponds to a convective vorticity mode

There are important consequences on the flame dynamics
Determine effects of this interaction and obtain the flame response in terms of a describing function.

Global view of the experiment:

- **1 - Flame tube**
- **2 - Injector**
- **3 - Swirler**
- **4 - Upstream manifold**

Measurements:

- \( U_b = 2.67 \text{ m s}^{-1}, \text{ (Flame A)} \)
- \( U_b = 4.13 \text{ m s}^{-1}, \text{ (Flame B)} \)
- \( S = 0.55 \) (Swirl number)
- \( \phi = 0.7 \) (Equivalence ratio)
The flame response can be characterized in terms of a transfer function

\[ F(\omega) = \frac{\dot{Q}' / \bar{Q}}{u' / \bar{U}} \]

But the flame transfer function (FTF) only provides the linear growth rate in the analysis of instabilities,
It is more informative to use the flame describing function

\[ \mathcal{F}(\omega, u') = \frac{\dot{Q}'/\bar{Q}}{u'/\bar{U}} \]

The Flame Describing Function (FDF) extends the transfer function concept to the nonlinear case.

In the FDF the flame response depends on the frequency and amplitude of the incident perturbation.

Nonlinear analysis

The FDF can be used to predict
Limit cycle amplitudes and frequencies (including frequency shift)
Nonlinear triggering and mode switching


Linear analysis provides an indication on growth rates but yields no information on limit cycle amplitudes and frequency shifting during instability growth.

Linear analysis also gives no indication on nonlinear triggering and mode switching and misses many of the complexities observed in practice.

$L$ is the size of the upstream manifold.
Experimental determination of the Flame Describing Function

$$\mathcal{F}(\omega, u') = \frac{\dot{Q}'/\bar{Q}}{u'/\bar{U}}$$

- **Flame tube**
- **Injector**
- **Swirler**
- **Upstream manifold**
- **Hotwire**
- **Loudspeaker**

**Diagram Notes:**
- Hotwire is located on the upstream side of the swirler.
- A loudspeaker is at the base of the burner.

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Operating Point B: \( U_b = 4.16 \text{ m/s} \)

The characteristic frequencies of the gain response scale with a Strouhal number based on the bulk velocity

It is interesting to examine the maximum and minimum responses of the flame

The combined dynamics of swirler and turbulent swirling flames.
Effect of mode conversion process on flame dynamics

Two mechanisms combine to define the flame response to incident perturbations:

- Swirl number fluctuations
- Vortex rollup of the flame sheet

\[ \frac{v'}{v} = \frac{u'}{u} \exp(i\phi) \]
\[ \frac{S'}{S} = \frac{v'}{v_2} - \frac{u'}{u_2} \]
Velocity components at the injector exhaust and swirl number fluctuations


The combined dynamics of swirler and turbulent swirling flames.

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Velocity components at the injector exhaust and swirl number fluctuations

$\frac{v'_{x2}}{\overline{v}_{x2}}$, $\frac{v'_{\theta2}}{\overline{v}_{\theta2}}$

$\frac{S'}{\overline{S}}$

Phase $[\degree]$ $f = 90 \text{ Hz}$

The combined dynamics of swirler and turbulent swirling flames.
**Swirling flames dynamics**

Swirl number fluctuations *act at the base of the flame and influence the flame angle*

![Diagram showing swirl number fluctuations](image1.png)

Vortex rollup *acts at the extremity of the flame*

![Diagram showing vortex rollup](image2.png)

Depending on the frequency, these mechanisms interfere constructively or destructively to determine the transfer function gain.
Flame dynamics can also be investigated with Large Eddy Simulations (LES).

Mesh: 6 millions cells

**AVBP** LES flow solver

Time step: $1.25 \times 10^{-7}$ s

Subgrid model: WALE

Thickened Flame Model (thickening factor $F=3.3$)

Phase locked averaging over 9 periods of modulation

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Experimental and simulated flame dynamics

Experiment

\[ \dot{Q}(Wm^{-3}) \]

Simulation

\[ \dot{Q}(Wm^{-3}) \]

Flame transfer function modeling

The flame transfer function can be deduced from a perturbed level set equation

\[
\frac{\partial G_1}{\partial t} + (v_0 + S_D n) \cdot \nabla G_1 = -v_1 \cdot \nabla G_0
\]  

(1)

Kinematic equation for a perturbed swirling flame:

\[
\frac{\partial G_1}{\partial t} + (v_0 + S_{T_0} n) \cdot \nabla G_1 = -v_1 \cdot \nabla G_0 + S_{T_1} |\nabla G_0|
\]

(1) (2)

The flame motion is controlled by:
(1) Velocity fluctuations \( v_1 \) (described by Schuller et al. 2003)
(2) Fluctuations in turbulent burning velocity \( S_{T_1} \) (2)
Swirling flame transfer function

Kinematic equation for a perturbed swirling V-flame:

$$\frac{\partial G_1}{\partial t} + (v_0 + S_{T_0} n) \cdot \nabla G_1 = -v_1 \cdot \nabla G_0 + S_{T_1} |\nabla G_0|$$

1. FTF of a laminar V-flame submitted to convective disturbances

$$\mathcal{F}^s_{th}(\omega) = \mathcal{F}^V_{th} \times \left[ 1 - \frac{S_{T_1}/S_{T_0}}{v'_x/\bar{v}_x} \right]$$

2. Fluctuations of the turbulent burning velocity $S_{T_1}/S_{T_0}$ result from swirl number oscillations modeled in terms of the incident perturbations $v'_x$ and $v'_\theta$

3. Azimuthal disturbances convected by the mean flow downstream the swirler feature a phase lag at the burner outlet with respect to axial perturbations, which is a function of the distance to the swirler outlet

Effects of turbulent burning velocity

$$\frac{S_{T_1}}{S_{T_0}} = \chi \frac{v'_\theta}{\bar{v}_\theta} + \zeta \frac{v'_x}{\bar{v}_x}$$

$$\frac{v'_\theta}{\bar{v}_\theta} = \frac{v'_x}{\bar{v}_x} \exp(i\phi)$$
Transfer function of swirling flames

\[ \frac{v_{\theta 2}}{v_{\theta 2}^2} = \frac{v_{z 2}}{v_{z 2}^2} \exp(i\phi) \]

\[ F_{th}^s(\omega) = F_{th}^v(\omega) \left[ 1 - (\zeta + \chi \exp(i\phi)) \right] \]

FTF of a V flame

Swirl number fluctuations

Flame transfer function modeling

Flame A, $U_b = 2.67 \text{ m/s}$

Good agreement between theory and experiment

Minimum and maximum locations are well retrieved

The phase is slightly overestimated in the model
Use the describing function to analyze a generic system comprising a single injector.

1. Flow

2. Combustion

3. Acoustics

Predict combustion system instabilities
Determine amplitudes and frequencies at limit cycles.
Anticipate mode switching, triggering, hysteresis…

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Four flame tube lengths $l_3$
- $l_3 = 100$ mm
- $l_3 = 150$ mm
- $l_3 = 200$ mm
- $l_3 = 400$ mm

Three upstream manifold lengths $l_1$
- short: $l_1 = 117$ mm
- medium: $l_1 = 181$ mm
- long: $l_1 = 245$ mm

Two operating points
- A: $U_b = 2.67$ m/s
- B: $U_b = 4.13$ m/s
The flame response is represented by the flame describing function:

\[ \mathcal{F}(\omega, u') = \frac{\dot{Q}'/\ddot{Q}}{u'/U} \]

When this is combined with a network description of the system acoustics one obtains a nonlinear dispersion relation:

\[ D(\omega, u') = 0 \]

The complex roots of this relation depend on the amplitude level:

\[ \omega = \omega_r(u') + i\omega_i(u') \]

When the growth rate is greater than the damping rate the perturbation grows:

\[ \omega_i(u') > \alpha \]

The limit cycle is obtained when:

\[ \omega_i(u') = \alpha \]
Combining the balance of mass and the balance of energy

\[ \frac{1}{\gamma} \frac{d \ln p}{dt} + \nabla \cdot \mathbf{u} = \frac{1}{\rho c_p T} \dot{q} \]

Linearizing around the mean state assuming a low Mach number and isobaric combustion region

\[ \frac{1}{\rho_0 c_0^2} \frac{\partial p'}{\partial t} + \nabla \cdot \mathbf{u}' = \frac{\gamma - 1}{\rho_0 c_0^2} \dot{q}' \]

Integrating over a control volume V with no pressure jump across the flame

\[ \int_V \left[ \frac{1}{\rho_0 c_0^2} \frac{\partial p'}{\partial t} + \nabla \cdot \mathbf{u}' \right] dV = \int_V \left[ \frac{\gamma - 1}{\rho_0 c_0^2} \dot{q}' \right] dV \]

Assuming that the flame is compact with respect to the wavelength

\[ \int_S \mathbf{u}' \cdot \mathbf{n} dS = \frac{\gamma - 1}{\rho_0 c_0^2} \int_V \dot{q}' dV \]

The difference between the volumetric flow rates is determined by fluctuations in heat release:

\[ S_3 u'_3 - S_2 u'_2 = \frac{\gamma - 1}{\rho_0 c_0^2} \dot{Q}' \]
The determinant of this matrix must vanish \( D(\omega, u') = \text{Det}[M] = 0 \)

Roots of this dispersion relation \( D(\omega, u') \) depend on the geometrical characteristics and on the flame response.

\[
K = K(\omega, u')
\]

Convention \( a' = \tilde{a} e^{-i\omega t} \), \( \omega = \omega_r + i\omega_i \), \( \omega_r = 2\pi f \)
Results of instability analysis

Time traces of pressure and heat release rate for flame A with the short upstream manifold

Stable case

Unstable case
Results: flame A

Frequency - Growth rate trajectories for stable and *unstable* cases

Medium upstream manifold – Flame tube $l_3 = 100$ mm

Trajectory is on the left of the gray zone indicating that the system is stable as observed during experiments for this configuration

Long upstream manifold – Flame tube $l_3 = 400$ mm

Trajectory is on the right of the gray zone indicating that the system is unstable as observed during experiments for this configuration. Limit cycle: $f = 102$ Hz – $u'/U_b = 0.78$


Modal analysis of chamber acoustics

Rectangular cavities
Cylindrical cavities
Annular cavities
It is now useful to examine the modal response of cavities. We will successively consider rectangular and cylindrical enclosures. For simplicity one assumes rigid wall or pressure release boundary conditions. More general situations may also be handled with modal concepts but they are not considered in this study.

A rectangular chamber with rigid walls

We first consider a rectangular chamber bounded by rigid walls
Harmonic disturbances in the cavity may be written in the form

\[ p(x, t) = \Psi(x)e^{-i\omega t} \]

\( \Psi(x) \) satisfies the Helmholtz equation and rigid wall boundary conditions

\[ \nabla^2 \Psi + k^2 \Psi = 0 \]
\[ \partial \Psi / \partial n = 0 \quad \text{on} \quad S \]

The eigenfunctions which satisfy this boundary value problem and the corresponding eigenvalues form an infinite set of solutions.

For a rectangular cavity one may search the eigenfunctions by making use of a factored form

\[ \Psi_n(x) = X(x)Y(y)Z(z) \]
When this expression is substituted in the Helmholtz equation one obtains

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} + k^2 = 0$$

The method of separation of variables indicates at once that each of the three terms which appear on the left side of this equation must be constant. It is convenient to write these constants as

$$-k_x^2, \quad -k_y^2, \quad -k_z^2$$

respectively so that

$$X'' + k_x^2 X = 0, \quad Y'' + k_y^2 Y = 0, \quad Z'' + k_z^2 Z = 0$$

The constants appearing in these equations are related by

$$k^2 = k_x^2 + k_y^2 + k_z^2$$
Consider now the boundary conditions for the function $X(x)$.

These conditions are obtained by specifying that

$$\frac{\partial \Psi}{\partial n} = 0 \quad \text{on } x = 0 \text{ and on } x = l_x$$

which yield

$$\left( \frac{dX}{dx} \right)_{x=0} = 0, \quad \left( \frac{dX}{dx} \right)_{x=l_x} = 0$$

The solution of $X'' + k_x^2 X = 0$

which satisfies the boundary condition at $x=0$ has the form

$$X(x) = a \cos k_x x$$

The other boundary condition is satisfied if

$$\sin k_x l_x = 0$$

This requires that $k_x = n_x \pi / l_x$.
Similar considerations finally yield

\[ \Psi_{n_x n_y n_z}(x) = A \cos \frac{n_x \pi x}{l_x} \cos \frac{n_y \pi y}{l_y} \cos \frac{n_z \pi z}{l_z} \]

and the corresponding eigenvalues takes the form

\[ k^2_{n_x n_y n_z} = \pi^2 \left[ \left( \frac{n_x}{l_x} \right)^2 + \left( \frac{n_y}{l_y} \right)^2 + \left( \frac{n_z}{l_z} \right)^2 \right] \]

Each mode is specified by a set of three integer indices. The corresponding eigenfrequencies are of the form

\[ \omega^2_{n_x n_y n_z} = c^2 \pi^2 \left[ \left( \frac{n_x}{l_x} \right)^2 + \left( \frac{n_y}{l_y} \right)^2 + \left( \frac{n_z}{l_z} \right)^2 \right] \]
and the resonance frequencies of the cavity are given by

\[ f_{n_x n_y n_z} = \frac{c}{2} \left[ \left( \frac{n_x}{l_x} \right)^2 + \left( \frac{n_y}{l_y} \right)^2 + \left( \frac{n_z}{l_z} \right)^2 \right]^{1/2} \]

Application

A rectangular chamber is filled with hot gases at a temperature \( T = 2000 \text{ K} \)

\[ l_x = l_y = 0.10 \text{ m}, \quad l_z = 0.20 \text{ m} \]

The mixture is characterized by a specific heat ratio \( \gamma = 1.4 \) and a gas constant \( r = \mathcal{R}/W = 287 \text{ J/kg K} \)

Calculate the eigenfrequencies corresponding to the first few modes \((1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1)\) and \((1,1,1)\).
The speed of sound in the cavity is:
\[ c = \left[ (1.4)(287)(2000) \right]^{1/2} = 896.4 \text{ m/s} \]

The eigenfrequencies are given by
\[ f_{n_x n_y n_z} = \frac{c}{2} \left[ \left( \frac{n_x}{l_x} \right)^2 + \left( \frac{n_y}{l_y} \right)^2 + \left( \frac{n_z}{l_z} \right)^2 \right]^{1/2} \]

\[ f_{0,0,1} = \frac{c}{2l_z} = 2241 \text{ Hz} \]
\[ f_{1,0,0} = f_{0,1,0} = \frac{c}{2l_x} = 4480 \text{ Hz} \]
\[ f_{1,0,1} = f_{0,1,1} = \left( \frac{c}{2l_z} \right)(5)^{1/2} = 5011 \text{ Hz} \]
\[ f_{1,1,0} = \left( \frac{c}{2l_x} \right)(2)^{1/2} = 6338 \text{ Hz} \]
\[ f_{1,1,1} = \left( \frac{c}{2l_z} \right)(9)^{1/2} = 6723 \text{ Hz} \]
A cylindrical cavity with rigid walls

The chamber has a radius $a$ and a length $L$

$$\Psi(r, \theta, z) = R(r) \Theta(\theta) Z(z)$$

Substituting this expression

$$\frac{R''}{R} + \frac{1}{r} \frac{R'}{R} + \frac{1}{r^2} \frac{\Theta''}{\Theta} + \frac{Z''}{Z} + k^2 = 0$$

Applying the method of separation of variables one finds that

$$Z'' + k_z^2 Z = 0, \quad \Theta'' + n^2 \Theta = 0$$

$$R'' + \frac{1}{r} R' + (k^2 - k_z^2 - \frac{n^2}{r^2}) R = 0$$

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\[ k_{\perp}^2 = k^2 - k_z^2 \]

With this definition the radial equation becomes
\[ R'' + \frac{1}{r} R' + \left( k_{\perp}^2 - \frac{n^2}{r^2} \right) R = 0 \]

The solution of this set of problems now proceeds as follows. First consider the longitudinal equation and the relevant boundary conditions
\[ Z'' + k_z^2 Z = 0 \]
\[ \left( \frac{dZ}{dz} \right)_{z=0} = 0, \quad \left( \frac{dZ}{dz} \right)_{z=L} = 0 \]

The function \( Z \) which satisfies this problem is of form
\[ Z(z) = \cos k_z z, \quad k_z = q\pi / L \]
Next let us examine the azimuthal equation.

\[ \Theta'' + n^2 \Theta = 0 \]

The solution of this equation must be periodic with respect to the azimuthal angle

\[ \Theta(\theta) = \Theta(\theta + 2\pi) \]

The general solution of this problem takes the form

\[ \Theta(\theta) = Ce^{in\theta} + De^{-in\theta} \]

Finally consider the radial problem

\[ R'' + \frac{1}{r} R' + \left( k_{\perp}^2 - \frac{n^2}{r^2} \right) R = 0, \quad \left( \frac{dR}{dr} \right)_{r=a} = 0 \]
The general solution of the radial differential equation may be written in terms of Bessel functions

\[ R(r) = AJ_n(k_\perp r) + BY_n(k_\perp r) \]

Since the Bessel function \( Y_n \) is singular as \( r=0 \) one deduces that the coefficient \( B \) vanishes.

The boundary condition on the rigid cylinder yields

\[ J'_n(k_\perp a) = 0 \]

Consider the roots of the following equation

\[ J'_n(\alpha_{mn}) = 0 \]

The radial wave numbers corresponding to these roots are of the form

\[ k_{\perp mn} = \alpha_{mn}/a \]
It is sometimes more convenient to express the radial wavenumbers in terms of the roots of the following characteristic equation

\[ J'_n(\pi \beta_{mn}) = 0 \]

The wavenumbers then take the form

\[ k_{\perp mn} = \frac{\pi \beta_{mn}}{a} \]

The modes of the closed cylindrical cavity take the following general form

\[ \Psi_{mnq}(r, \theta, z) = J_n(k_{\perp mn} r) \cos \frac{q \pi z}{L} (ae^{in\theta} + be^{in\theta}) \]

and the corresponding eigenfrequencies are given by

\[ \left( \frac{\omega_{mnq}}{c} \right)^2 = k_{\perp mn}^2 + k_z^2 \]
or more explicitly

$$\omega_{mnq} = c \left[ \left( \frac{\pi \beta_{mn}}{a} \right)^2 + \left( \frac{q \pi}{L} \right)^2 \right]^{1/2}$$

The frequencies associated with the cavity modes may be cast in the simple form

$$f_{mnq} = \frac{c}{2} \left[ \left( \frac{\beta_{mn}}{a} \right)^2 + \left( \frac{q}{L} \right)^2 \right]^{1/2}$$
A liquid rocket engine has a length \( L = 1 \) m and a radius \( a = 0.3 \) m. The gas temperature inside the chamber is \( T = 3000 \) K and the gases have the following properties:

\[
\gamma = 1.3, \quad r = 460 \text{ J/kg K}
\]

Determine the first few eigenfrequencies by considering that the rocket chamber behaves like a rigid enclosure.

The sound velocity in the chamber is

\[
c = \left[ (1.3)(460)(3000) \right]^{1/2} = 1339.4 \text{ m/s}
\]

The eigenfrequencies are given by

\[
f_{mnq} = \frac{c}{2} \left[ \left( \frac{\beta_{mn}}{a} \right)^2 + \left( \frac{q}{L} \right)^2 \right]^{1/2}
\]

Consider first the purely longitudinal modes characterized by \( m = 0 \) and \( n = 0 \). Calculate the various frequencies as a home work problem.
Modal identification in annular configurations

Harmonic modes are governed by a Helmholtz equation

\[
\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{R^2} \frac{\partial^2 p}{\partial \theta^2} - c^2 \frac{\partial^2 p}{\partial z^2} = 0
\]

Purely azimuthal modes

\[
p_n = a \exp(in\theta - i\omega_n t) + b \exp(-in\theta - i\omega_n t)
\]

\[
\omega_n = \frac{nc}{R} \quad f_n = n \frac{c}{\rho}
\]

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A standing mode in an annular chamber

The nodal line is horizontal
Mixed mode

Purely longitudinal modes

\[ p_m = \psi_m(z) \exp(-i\omega_m t) \]
\[ \psi_m(z) = \cos(k_m z) \]
\[ k_m l = (m - \frac{1}{2})\pi \]
\[ f_m = (2m - 1)(\frac{c}{4l}) \]

Mixed mode

\[ f_{mn} = \left[n^2 \left(\frac{c}{\mathcal{P}}\right)^2 + (2m - 1)^2 \left(\frac{c}{4l}\right)^2\right]^{1/2} \]
\[ f_{11} = \left[\left(\frac{c}{\mathcal{P}}\right)^2 + \left(\frac{c}{4l}\right)^2\right]^{1/2} \]
\[ f_{10} = \frac{c}{4l} \]
In these expressions one has to take into account that the sound velocity is different near the chamber backplane and at a distance from this plane. It is then reasonable to use two sound velocities so that:

\[ f_{mn} = \left[ n^2 \left( \frac{c}{\mathcal{P}} \right)^2 + (2m - 1)^2 \left( \frac{\bar{c}}{4l} \right)^2 \right]^{1/2} \]

Then:

\[ f_{10} = \frac{\bar{c}}{4l} \quad f_{11} = \left[ \left( \frac{c}{\mathcal{P}} \right)^2 + \left( \frac{\bar{c}}{4l} \right)^2 \right]^{1/2} \]

One may use:

- \( c = 820 \text{ m s}^{-1} \)
- \( \bar{c} = 500 \text{ m s}^{-1} \)
- \( l = 0.5 \text{ m} \)
- \( \mathcal{P} = 1.1 \text{ m} \)

This gives:

\[ f_{10} = 250 \text{ Hz} \quad f_{11} = 786 \text{ Hz} \]
Modal identification in the MICCA combustor

The mode 1L0A approximately corresponds to $m=1, n=0$

The mode 1L1A approximately corresponds to $m=1, n=1$

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Acoustic mode analysis

A rectangular chamber with rigid walls
A cylindrical cavity with rigid walls
Annular systems
Modal structures in annular systems

Harmonic modes are governed by a Helmholtz equation

\[
\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{R^2} \frac{\partial^2 p}{\partial \theta^2} - c^2 \frac{\partial^2 p}{\partial z^2} = 0
\]

Purely azimuthal modes

\[
p_n = a \exp(in\theta - i\omega_nt) + b \exp(-in\theta - i\omega_nt)
\]

\[
\omega_n = \frac{nc}{R} \quad f_n = n\frac{c}{\rho}
\]
Purely longitudinal modes

\[ p_m = \psi_m(z) \exp(-i \omega_m t) \]

\[ \psi_m(z) = \cos(k_m z) \]

\[ k_m l = (m - \frac{1}{2}) \pi \]

\[ f_m = (2m - 1) \left( \frac{c}{4l} \right) \]

Mixed modes

\[ f_{mn} = \left[ n^2 \left( \frac{c}{P} \right)^2 + (2m - 1)^2 \left( \frac{c}{4l} \right)^2 \right]^{1/2} \]

\[ f_{11} = \left[ \left( \frac{c}{P} \right)^2 + \left( \frac{c}{4l} \right)^2 \right]^{1/2} \]

\[ f_{10} = \frac{c}{4l} \]
Annular systems combustion dynamics
Annular combustors are used in many practical systems like jet engines and gas turbines.

In these devices combustion dynamics oscillations may arise from azimuthal modes.

Because the diameter is the largest dimension, these modes occur at the lowest frequencies and they are less well damped than the longitudinal modes.

This type of coupling raises scientific problems and technical issues.
Consider annular systems equipped with multiple swirling injectors and examine their ignition dynamics.

Characterize instabilities in annular systems with multiple swirling injectors.

Examine instabilities in annular systems with multiple matrix injectors.
Overview of swirling flame dynamics research

<table>
<thead>
<tr>
<th>Single injector systems</th>
<th>Annular systems with multiple injectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
<td>Theory</td>
</tr>
<tr>
<td>Simulations</td>
<td>Simulations</td>
</tr>
<tr>
<td>Experiments</td>
<td>Experiments</td>
</tr>
</tbody>
</table>

- **Single injector systems**
  - Theory
  - Simulations
  - Experiments
  - Large number of investigations

- **Annular systems with multiple injectors**
  - Theory
  - Simulations
  - Experiments
  - Relatively large number
  - A few recent simulations
  - Very few model scale experiments

---

**References**

Consider annular systems equipped with multiple swirling injectors and examine their ignition dynamics.
Five microphones Pr1 to Pr5
An intensified high speed CCD camera

Experimental setup
Detailed view of a swirler

Diagram:
- $D_a = 0.05 \text{ m}$
- $D_m = 0.35 \text{ m}$
- Five microphones Pr1 to Pr5
- An intensified high speed CCD camera
- 3.5 m from the center of the annular chamber
- 8 premixed Air-C3H8 injectors
- Radial swirlers
- quartz tubes
- Swirlers
- 16 injectors
- Plenums
- Pr1 Pr2 Pr3 Pr4 Pr5
- Intensified high speed camera

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Experimental setup

Five microphones Pr1 to Pr5
An intensified high speed CCD camera
High speed imaging

Frame rate
\[ f_c = 1/6000 \text{s} \]

Exposure time
\[ \tau = 16.6 \mu\text{s} \]

Operating conditions

Bulk velocity
\[ U_b = 11 \text{ m.s}^{-1} \]

Equivalence ratio
\[ \phi = 0.76 \]

Thermal power
\[ P_{th} = 40 \text{ kW} \]

Pth = 40kW, Ub = 12.2m/s, phi = 0.76
EM2C CNRS–ECP
supported by ANR and Sncema

Pth = 81kW, Ub = 24.5m/s, phi = 0.76
EM2C CNRS–ECP
supported by ANR and Sncema
Formation of a hot pocket of burnt gases

Propagation as an arch

Flame front merging

Steady combustion Regime established

Increase of the bulk Velocity by 100%

Decrease of the ignition delay by 23%
Using the jump conditions across the flame one can link the cold flow velocity to the burnt gas velocity:

\[ \mathbf{v}_u = \mathbf{v}_b + \left( \frac{\rho_u}{\rho_b} - 1 \right) S_d \mathbf{n} \]

One obtains an equation which features the burnt gas velocity:

\[ \frac{\partial G}{\partial t} + \mathbf{v}_b \cdot \nabla G = \frac{\rho_u}{\rho_b} S_d |\nabla G| \]

This equation can be used to calculate the flame motion. This requires models for the burnt gas velocity and for the flame displacement speed.
Comparison between experimental data and simulation

Initial flame contour obtained from experiments

Propagation dynamics is well retrieved. Propagation delay is correctly predicted.
Full level set simulation

\[ \tau_m = 0.2 v_0^{-0.38} \]
Ignition sequence of an annular multi-injector combustor

Objectives:


Contents

- Background
- Annular systems with multiple swirling injectors (MICCA 2)
- Annular systems with multiple matrix injectors (MICCA 3)
- Summary points

MICCA 2

Swirling injectors

MICCA 3

Matrix injectors
Modal identification

The mode 1L0A approximately corresponds to $m=1$, $n=0$

The mode 1L1A approximately corresponds to $m=1$, $n=1$
Longitudinal mode ($f=252$ Hz)

Microphone signals

Power spectral density
Longitudinal mode ($f=252$ Hz)

First azimuthal mode (1A)

The pressure field is a sum of two waves rotating in the counterclockwise and clockwise directions:

\[ p(\theta, t) = a \exp(i\theta - i\omega t) + b \exp(-i\theta - i\omega t) \]

\[ s = \frac{|a| - |b|}{|a| + |b|} \]

- \( s = -1 \): rotating mode in the clockwise direction
- \( s = 0 \): standing mode
- \( s = 1 \): rotating mode in the counterclockwise direction

This spin ratio differs from that introduced by Evesque et al. (2003).

Pressure signal reconstruction and spin ratio determination

\[ p(\theta, t) = a \exp(i\theta - i\omega t) + b \exp(-i\theta - i\omega t) \]

The wave amplitudes \( a \) and \( b \) can be determined from microphone data

\[
\begin{pmatrix}
  p_1 \\
  p_2 \\
  \vdots \\
  p_n \\
  p
\end{pmatrix}
\begin{pmatrix}
  \exp(i\theta_1) \exp(-i\theta_1) \\
  \exp(i\theta_2) \exp(-i\theta_2) \\
  \vdots \\
  \exp(i\theta_n) \exp(-i\theta_n)
\end{pmatrix}
\begin{pmatrix}
  a \exp(-i\omega t) \\
  b \exp(-i\omega t)
\end{pmatrix}
= (M^* M)^{-1} M^* P
\]

\[ s = \frac{|a| - |b|}{|a| + |b|} \]

Symbols: experimental data, Continuous lines: reconstructed signals

Standing mode

Rotating mode
Continuous switching from a quasi-spinning mode to a standing mode

\[ p(\theta, t) = a \exp(i\theta - i\omega t) + b \exp(-i\theta - i\omega t) \]

\[ s = \frac{|a| - |b|}{|a| + |b|} \]

\( s = +1 \) or \(-1\): spinning
\( s = 0 \): standing

The spin ratio

The spin ratio varies as a function of time. This may be characterized by a probability density function.

This function is close to a Gaussian.

On average, the counterclockwise direction of rotation is favored.
The spin ratio is close to zero

Standing mode

<table>
<thead>
<tr>
<th>Mic 1</th>
<th>Mic 2</th>
<th>Mic 3</th>
<th>Mic 4</th>
<th>Mic 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>150</td>
<td>250</td>
<td>-100</td>
<td>100</td>
</tr>
</tbody>
</table>

Pressure (Pa)

Time (s)
Standing mode

This mode is coupled by the 1A1L mode at a frequency $f=792$ Hz

Rotating mode

The spin ratio is close to one

Pr 1
Pr 2
Pr 3
Pr 4
Pr 5

ψ = 0
ψ = π/2
ψ = π
ψ = 3π/2
A calculated rotating mode in a gas turbine combustor

38.36000 ms


38.360 ms


Long injection tubes, stabilization by a bluff body, low swirl. Variable spacing

Heat release rate fluctuations (standing azimuthal mode)

Pressure fluctuation distribution in the chamber
Heat release rate fluctuations (standing azimuthal mode)

Pressure fluctuation distribution in the chamber

Heat release rate fluctuations (standing azimuthal mode)

Pressure fluctuation distribution in the chamber
To better understand what determines the structure of the azimuthal modes, it is interesting to work on an annular chamber operating with simpler flames than turbulent flames. This is done here by making use of multipoint injectors formed by perforated plates.
Laminar conical flames stabilised on a single matrix burner

MICCA3 with laminar matrix injectors

- Sixteen injectors
- Matrix burner
- Waveguide outlet
- Cylindrical concentric quartz tubes
- Combustion chamber
- Chamber backplane
- Upstream plenum

Laminar conical flames stabilised on a single matrix burner
The experimental setup consists of an annular chamber designed for studying micro-scale thermo-acoustic instabilities. The chamber is made of quartz and has a thickness of 6 mm, providing an optical access to the combustion region. The chamber is equipped with 16 perforated plates, each comprising 89 holes with a diameter of 2 mm on a square mesh of 3 mm. The propane/air mixture delivered by a premixing unit flows through the feeding lines, which are composed of a straight inner quartz tube and an outer quartz tube closed at its extremity. A microphone is flush mounted perpendicular to the wave duct, at 170 mm from the chamber backplane, supporting the injectors. The chamber walls are made of quartz allowing optical access to the combustion region.

The microphone signals are used to measure pressure fluctuations in the plenum. The delay in the chamber and in the plenum is determined accurately to study the sensitivity of the delay to the microphone signals. The distance between the zone of interest and the microphone position defines a time lag that is negligible compared to the period of the instabilities observed in the present experiment. The delay to MC8 fixed on waveguides (Fig. 2) is correspondingly limited to 0.08 rad. In the present study (2 ms), this time has to be determined accurately to study the sensitivity of the delay in the chamber and in the plenum. It is therefore interesting to also establish a possible phase shift between pressure signals in the chamber and the heat release rate records but also to establish a possible phase shift between pressure signals in the chamber and the heat release rate records.

The inner and outer quartz tubes are 300 mm and 400 mm respectively. In the present experiments, the two cylindrical walls have the same length. The distance between the feeding line and the injector constitutes the chamber backplane. The diameters of the inner and outer quartz tubes are 0.3 m and 0.4 m, respectively. The height of the inner tube is 0.2 m.

The experimental setup includes eight pressure taps (MC) and plenum microphone taps (MP) in the plenum (Fig. 2) or in eight other locations in the upstream plenum. The pressure taps are fixed on the external walls of the chamber and on the backplane, supporting the injectors, which are cooled by a water flow. High speed cameras are used to capture images of the combustion region, providing a full visualization of the flame. The propane/air mixture delivered by a premixing unit is fed to the chamber through the feeding lines.

The diameters of the inner and outer quartz tubes are 0.3 m and 0.4 m, respectively. The height of the inner tube is 0.2 m.
Part III - Experimental results and comparison with model

7.3 Annular chamber modes characterization

7.3.1 Stability map of the annular combustor

Under lean conditions ($\phi < 0.7$), the flames are long, quiet and strongly detached from the matrix injectors and the system is essentially stable. When the equivalence ratio is increased, the flames get closer and closer to their respective injectors and as soon as a few flames attach to their burners three or four, the annular chamber exhibits strong longitudinal thermo-acoustic oscillations. For an equivalence ratio $\phi > 0.8$ and bulk velocities $u_b > 1 \text{ m.s}^{-1}$, the flames are all attached to their injectors and the combustor is always unstable. For most of the flow conditions, a complex low frequency chugging mode is observed, but, by setting the gas and air flow rates at certain values, some distinct oscillation modes can be found. The chugging mode is not examined in this chapter.

![Diagram showing stability map of annular combustor](image)

**Figure 7.4:** Unstable modes observed in the annular combustion chamber as a function of equivalence ratio $\phi$ and bulk flow velocity $u_b$. Symbols show the boundaries of the different oscillation regimes. Specific unstable modes are observed inside the domains colored in grey. The chugging mode is plotted only for illustrative purposes because the limits of this mode could not be clearly identified.

The different unstable modes are mapped in Fig. 7.4 as a function of the equivalence ratio $\phi$ and bulk flow velocity $u_b$. The distinct zones are obtained by setting the air and gas flow rates at a condition where the combustor becomes unstable. Then the air or gas flow rate is increased or decreased until the instability switches to the chugging mode. Four limit conditions are therefore obtained and represented by symbols which are linked by dotted lines to highlight the domains corresponding to the different unstable regimes. Due to hysteresis of the system, the chugging mode can also be found in regions corresponding to the four other modes. The "slanted" and spinning modes are particularly sensitive to thermal conditions and can only be observed when the combustor has been running for about ten minutes. An unusual "slanted" mode is manifested in the form of a stable azimuthal mode with small amplitudes in

![Diagram showing unstable modes](image)
Standing mode
Standing mode

\[ v_0 = 2.12 \text{ m s}^{-1} \phi = 1.11 \]

Close to the nodal line, the flames move with a small amplitude of vibration. At 90° from this line, the flames oscillate vigorously, and they are blown-off on their periphery.
Standing azimuthal mode

\[ v_0 = 2.12 \text{ m s}^{-1}, \phi = 1.11 \]

Phase average of the oscillation, from the images recorded by the intensified high speed camera at 12500 fr/s. Images are plotted in false color.
Standing azimuthal mode

High speed film: 12500 frames/s

Injector close to the nodal line
Standing azimuthal mode

High speed film: 12500 frames/s

Injector at 90° from the nodal line
Standing azimuthal mode

High speed movie: 12500 frames/s

Injector between the nodal line and the orthogonal location
In the combustion chamber

**Pressure (Pa)**

**PM1** / **MeanPM1**

*Filtered between 450 and 550 Hz*

*Filtered between 900 and 1100 Hz*
Modal eigenfunctions

- Plenum
- Quartz tube

300 K → 1500 K

300 K → 1500 K

Modal eigenfunctions

- 0L-1A Plenum mode at 482 Hz (azimuthal)
- 1L-0A Chamber mode at 969 Hz (longitudinal)

Acoustic modes of the experimental setup obtained with a Helmholtz finite element solver. Three modes are represented from top to bottom: Helmholtz mode at 354 Hz, 0L-1A Plenum mode at 482 Hz, and 1L-0A Chamber mode at 969 Hz. The curves show the modal pressure dynamics represented in Fig. 7.3.
Spinning mode

\[ \phi = 0.96 \text{ and } u_b = 1.49 \text{ m s}^{-1} \]

Pressure signals recorded by microphones in the chamber (top) and in the plenum (bottom) for the spinning mode.
Direct imaging of unstable regime

Standard video (25 frames/s) of the combustor under spinning mode oscillation

C₃H₈ - Air  \( f = 450 \) Hz

\[ \phi = 1.14 \ \text{and} \ \dot{u}_b = 1.33 \text{m.s}^{-1} \]
Pressure and photomultiplier signals corresponding to a spinning mode for a bulk velocity $u_b = 1.49$ m.s$^{-1}$ and an equivalence ratio $\phi = 0.96$. In black: microphone signals MC1 and MC7 in the chamber. In red: photomultipliers signals H1 and H7.
Spinning mode

The nodal lines feature an angular shift which can be explained (see lecture tomorrow morning)
Images are recorded by an Intensified CMOS camera.
The frame rate is set at 12500 Hz.
Each phase of the sequence is averaged over 1000 instantaneous images.
Some reflections on the transparent walls are visible in the neighborhood of the two sides of the image.
The slanted mode

Time evolution of the light intensity for the 8 injectors located on the same side with respect to the line of symmetry, from injector 14 up to injector 5.

While the pressure signals are in phase in the plenum, the light intensity signals are out of phase.
The flame describing function is determined in a single injector configuration that is modulated externally by a driver unit.

Flame response on a single matrix burner versus the relative amplitude of an axial perturbation, at the frequency of 450 Hz.
The slanted mode

Figure 7: Time evolution of the brightness for 8 injectors located on the same side with respect to the line of symmetry, from injector 14 up to injector 5.

(a) Measured heat release rate signals

(b) Reconstructed heat release rate signals
Summary points

- The annular system with swirling injectors features various types of thermo-acoustic oscillations.

- Analysis of pressure signals indicates a continuous switching between standing and spinning modes. The greatest probability corresponds to the standing mode.

- A well established spinning mode is observed in the annular burner equipped with matrix injectors.

- The nodal lines in the plenum and chamber feature an angular shift.
In the slanted pattern case the acoustic field comprises an axial mode and a standing azimuthal mode which have coinciding frequencies. This combination produces a pressure pattern with a maximum amplitude of oscillation on one side of the annulus while the amplitude is minimal on the other side. Using the flame describing function one can explain the time shift in the flame motions at the various injectors which takes the form of a wave sweeping the different injectors. It is shown that the phase shift evolution in the light intensity of the different injectors is a direct result of the nonlinear response of these elements when they are subjected to large velocity oscillations.
Conclusions

- Many difficult issues in combustion dynamics of swirling flames and annular systems
- Substantial progress in the fundamental understanding
- New experiments and computational tools to examine complex annular configurations
- The right time to combine expertise and start new collaborations to work on these problems collectively
Recent publications


Recent publications


Recent publications


Nonlinear interaction between the precessing vortex core and acoustic oscillations

Figure 15: Spectral distributions of photomultiplier (a) and microphone (b) signals, $\phi = 0.69$, $u_b = 9.9$ m.s$^{-1}$. One vertical half of the photomultipliers field of view is masked, and the signal has been normalized by the mean value. The microphone is located outside of the flame tube. $f_i$, $f_a$, and $f_h$ denote interaction, acoustic and helical-mode frequency, respectively.

$$f_i = f_h - f_a$$

Figure 19: Phase-averaged images of the light intensity deviation from the mean distribution in the burner gathered from a top view at the interaction frequency (254 Hz).

Combustion dynamics
Lecture 10b

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Ecole Centrale Paris, EM2C lab, CNRS
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Princeton summer school, June 2016

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A hysteresis phenomenon leading to spinning or standing azimuthal instabilities in an annular combustor.

Kevin Prieur, Daniel Durox, Thierry Schuller et Sébastien Candel

MICCA 2 is equipped with 16 matrix injectors
The Dual mode region is where the spinning and standing regions overlap. The hatched area is where the chugging mode is manifested. The blank area corresponds to the lean extinction limits of the chamber.
High-speed snapshots of the full annular combustion chamber during a spinning mode cycle ($T = 2$ ms). The mode is rotating in a counterclockwise direction and the maximum luminosity reached by each flame during the cycle is the same for all burners.
High-speed images of the full annular combustion chamber during a standing mode cycle (T = 2 ms). The nodal line (dashed line) is fixed between two burners. The thickness of the curved line gives an indication on the oscillation level. The flames next to the nodal line have a lower amplitude of oscillation than those situated near the anti-nodal region.
Left: hysteresis cycles for operating conditions indicated in Fig.\ref{fig:carte_stab_zoom}. Arrows indicate the path followed during the experiment. When $\phi$ increases from $\numrange{0.9}{1.1}$ a spinning mode arises. When $\phi$ is diminished from $\numrange{1.2}{1.1}$ a standing mode prevails. Right: zoom on the loop for $\phi$ ranging between $\num{1.03}$ and $\num{1.13}$. 
State space maps of the pressure signals ($t_{max} = 0.4 \text{ s}$) recorded by microphones MP2 and MP3 in the plenum separated by a 90° angular shift.
Multiple longitudinal and azimuthal modes are observed in the annular configuration in regions which in general do not overlap.

Spinning and standing modes with stable limit cycles are observed for the same flow operating conditions in a limited « Dual mode » domain.

The oscillation arising in this « Dual mode » region depends on the path taken to reach the operating point. If $\phi$ is increased, with the same air mass flow rate, from lean conditions to the target value, a spinning mode is obtained. If $\phi$ is decreased from rich conditions, a standing mode is manifested at the target conditions.

The spinning and standing modes do not switch from one to the other but instead when a mode arises, it is locked on.

The chugging oscillation observed just outside the region of azimuthal instability contains information that can be used to predict the azimuthal mode structure that will be established in the « Dual mode » domain.

Conclusions
It would be interesting to see if these observations can be explained by recent theoretical analysis like (Ghirardo et al., 2015) which allow the coexistence of both spinning and standing modes for the same operating conditions. This is however not straightforward since the theory relies on a nonlinear time invariant relationship between heat release rate and pressure fluctuation in the chamber.

Computational Flame Dynamics

- Historical perspective (RANS, DNS, LES)
- Laminar flame dynamics
- Large eddy simulation of turbulent flames
- Ignition of annular combustors
- Annular systems azimuthal instabilities
Trends in computational flame dynamics

1960

CFD beginnings
(Computational Fluid Dynamics)

1970

Numerical combustion

1960

New CFD
(Computational Flame Dynamics)

2000
Simulation is crucial to the development of advanced combustion concepts.

- **Direct simulation**: All temporal and spatial scales are calculated.
- **Large eddy simulation**: Large scales are calculated, small scales are represented by subgrid scale models.
- **Reynolds average Navier-Stokes equations**: Equations are averaged and Reynolds stresses and turbulent fluxes are modeled.

**Laboratory** → **Applications**
Ozone decomposition flame

\[ O_3 + M \rightarrow O + O_2 + M \]
\[ O + O_3 \rightarrow 2O_2 \]

©S. Candel, 2013

1954

H₂/air flame
Flame vortex interactions

1987

Reynolds average simulation of turbulent ducted flame

1988

Direct simulation of ignition

1993

Reactive shear layer
Flashback

1998

Combustion acoustic coupling

2000


2002

Turbulent premixed combustion

2006

Transverse acoustic coupling

2007

IC engine LES

2010

Transcritical combustion
2011
Calculated rotating mode in an annular combustor

2012
Multiple cryogenic jets under transverse modulation

2013
Ignition dynamics of an annular combustor

2015
Triggered combustion instability of a full rocket engine (42 injectors)
(1) Much of the current modeling effort in combustion is carried out in the LES framework.

Over two hundred publications per year featuring these two keywords:

- «Large Eddy Simulation»
- «Combustion»
Direct simulation of premixed flames is feasible if...

\[ N \Delta x > l \]
\[ \Delta x = l_k \]
\[ N > (Re_l)^{3/4} \]

A minimum of \( n \) points is used to discretize the flame.

\[ \delta = n \Delta x \]
\[ \frac{N}{n} > \frac{l}{\delta} \]
\[ (N/n)^2 > Re_l Da \]
This limits direct simulation to low Reynolds and Damköhler numbers.

\[
N = 1000 \quad N^3 = 10^9 \\
n = 20 \\
\text{Re}_l < N^{4/3} = 10^4 \\
\text{Re}_l\text{Da} < (N/n)^2 = 2500
\]

If one chooses \( \text{Re}_l = 250 \)
then \( \text{Da} < 10 \)

In simulations of turbulent flames at Damköhler numbers greater than unity (typical of combustion conditions where the chemical time is short compared to the mechanical time) the Reynolds number can only take moderate values.
This restricts the domain accessible to direct simulation

Direct numerical simulation of premixed turbulent combustion.
Combustion dynamics of flames interacting with equivalence ratio perturbations

Conical flame perturbed by equivalence ratio modulations

Inverted flame perturbed by equivalence ratio modulations


The nonlinear response of inverted « V »-flames submitted to equivalence ratio nonuniformities.
Combustion dynamics of flames interacting with equivalence ratio perturbations

Conical flame perturbed by equivalence ratio modulations

Inverted flame perturbed by equivalence ratio modulations

The nonlinear response of inverted « V »-flames submitted to equivalence ratio nonuniformities.
(a-e) Methane mass fraction distributions during a cycle. \( \phi(t) = \phi_0 + \phi_1 \sin \omega t \)

\[ f = 375 \, \text{Hz}, \quad \frac{\phi'}{\bar{\phi}} = 0.1, \quad \phi_0 = 0.8 \]

(f) Relative heat release and equivalence ratio perturbation
Calculation of flame dynamics corresponding to the Dynamic Flame Converter
(2) It is hoped that LES will allow to analyze practical applications.
The future clearly lies in Large Eddy Simulations

Direct simulation

All scales are calculated

Power spectral density

Wavenumber

£¿

DNS

Large Eddy Simulation

Large scales are calculated

Small scales are modeled

Power spectral density

Wavenumber

£¿

LES

$k_c \approx \frac{1}{\Delta}$
In direct simulation, dissipative scales and flame must be resolved on the grid!

In large eddy simulations the small scales are modeled but the grid is too rough to resolve the flame:

- The flame is replaced by a thin front (d)
- The flame is artificially thickened (e)
- The flame is spatially filtered (f)
LES methods are now widely used to examine combustion dynamics and instabilities. LES naturally describes the flame motion induced by the large scales. One effective model in premixed combustion relies on the artificial thickening of the flame so that it can be calculated on a relatively coarse grid.

This method, originally proposed by Bracco and O’Rourke in a different context, was later explored by Thibaut and Candel (1998) in a simulation of oscillations in a dump configuration.

Combined with a subgrid scale efficiency function the flame thickening method (FTM) has been extensively exploited to investigate combustion dynamics in premixed gas turbine combustors.


Ignition of an annular combustor

Experiment

Simulation
