Turbulent Combustion
Experiments and Fundamental Models
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Premixed DNS
PROCI 31, 1291

Bell, Day,
Driscoll
“corrugated”
premixed DNS
PROCI 31, 1299

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Outline for the week

**Mon:** Physical concepts
faster mixing, faster propagation, optimize liftoff, flame surface density, reaction rate, PDF

**Tues:** Kilohertz PLIF, PIV measurements of flame structure - to assess models

**Wed:** Non-Premixed and Premixed flames - measurements, models
gas turbine example

**Thurs:** Partially premixed flames - and some examples

**Fri:** Future challenges: Combustion Instabilities (Growl), Extinction
Outline for Monday = Physical concepts

Motivation - premixed is the way of the future – low NOx, CO, soot

What problems are important?

Background what does turbulence do?
what do we need to model and to measure? turbulent burning velocity, flame surface density, reaction rate

Turbulent reaction rate

Probability density function helps to model turbulent reaction rate

Fuel air mixing and Flame stability
Good references


Turns, S., An Intro to Combustion, McGraw Hill, 2000


Peters N. Laminar flamelet concepts in turbulent combust. Proc Comb Inst 21 1986

C. K. Law and C. J. Sung, Structure, aerodynamics, and geometry of premixed flamelets, PECS, 26, 4-6, 2000, 459-505RS

Barlow, Turbulent Nonpremixed Flame workshop website: http://www.ca.sandia.gov/TNF/


Lockwood, F, Naguib, A, k-epsilon model of jet flame, Comb Flame 24, 109, 1975


Motivation: Premixed is the way of the future - low NOx, CO, soot

GE-TAPS in Michigan High Pressure GT Combustor

premixed flame difficult to anchor
Temme, Driscoll, Combust. Flame 161, 958

Tim Lieuwen – equivalence ratio oscillations
PROCI 27, 1809
Motivation - challenges

Most practical problems are “Partially-Premixed”

Partially-Premixed = ER varies in space from 0 to \( \infty \), a point sometimes sees premixed, sometimes non-premixed flamelets

Stratified Premixed = ER varies in space, reactants are within flam. Limits see: Masri, PROCI 35, 1115, Driscoll Comb Flame 162, 2808

No dependable model of flame blowout, combustion instabilities or turbulent burning velocity at large turbulence level

- Need “robust” LES submodel: flamelets: Bray / Flame surface density progress variable (Moin, Pitsch, Ihme)
  thickened: Poinset (TFM)
  distributed: Menon(LEM), Pope (PDF)
  pyrolysis chemistry (?)

- No measurements of boundaries of regimes - when are flamelet models appropriate?
- Premixed turbulent combustion is more difficult than non-premixed turbulent combustion, not just a mixing problem, also wave propagation
Motivation: What problems do we want to solve?

1. Premixed: Engines: IC engine & HCCI, industrial burners, premixed GT

2. Non-premixed: jet, jet in cross flow, jet in co-flow, jet in swirl flow

3. Partially-premixed: gas turbine, afterburner, base of lifted jet

4. Canonical experiments for model assessment:
   - non-premixed: piloted jet flame (Sandia flame D)
   - premixed: Bunsen (high-Re), premixed jet, low-swirl, spherical
Motivation— kilohertz imaging

Cam Carter (AFRL)
Tonghun Lee (UIUC)
10 kilohertz CH
Reaction layer
Applied Optics B 116: 515

Steinberg, Driscoll
Michigan kHz PIV
eddies passing
through flame
Comb Flame 156, 2285
Best current models?

### PREMIXED LES

<table>
<thead>
<tr>
<th>Rank</th>
<th>Method</th>
<th>Description</th>
<th>React Rate</th>
<th>Authors</th>
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<tr>
<td>1</td>
<td>FSD</td>
<td>Flame surface density, also called F-TACLES = tabulated chemistry LES</td>
<td>FSD eqn w flamelet state relns</td>
<td>Bray, Vervisch Veynante, Fureby Ihme,</td>
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<tr>
<td>2</td>
<td>FPV</td>
<td>Flamelet progress variable</td>
<td>flamelet state relns</td>
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<td>3</td>
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<td>4</td>
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<td>Swami, Huh</td>
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<td>5</td>
<td>G Eqn</td>
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<td>G-Eqn</td>
<td>Pitsch, Bai</td>
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<td>LEM</td>
<td>Linear eddy model</td>
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<td>Menon</td>
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### NON PREMIXED LES

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<td>Steady strained laminar flamelet Z eqn with scalar dissipation rate</td>
<td>strained flamelet</td>
<td>Peters, Pitsch</td>
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<td>FPV</td>
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<td>flamelet state relns</td>
<td>Moin</td>
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<td>flamelet</td>
<td>Bilger, Masri Pope</td>
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<tr>
<td>4</td>
<td>PDF</td>
<td>PDF transport method</td>
<td>parcels</td>
<td></td>
</tr>
</tbody>
</table>
References – Premixed models

**FSD = Flame surface density LES models F-TACLES**
Mercier, Veynante, PROC 15, 1259, Hawkes, Cant, Comb Flame 126, 1617
See, Ihme, PROC 15, 125, Duwig, Flow, Turb Comb 79, 433

**FPV = Flamelet progress variable**
Pierce, Moin, J. Fluid Mech 504-73, Chen, Ihme Comb Flame 160, 2896
Lamouroux, Ihme, Comb Flame 161, 2120

**TFM = Thickened flamelet model**
Selle, Poinsot, Comb Flame 137, 489, Esclapez, Cuenot, PROC 15, 3133
De, Acharya, Comb Sci Technol 181, 1231

**CMC = Conditional Moment Closure**
Amzin, Swaminathan, Comb Sci Tech 184, 1743, Amzin Cant, Comb Sci Tech 187, 1705

**G-Equation**
Knudsen, Pitsch, Comb Flame 159, 242, Nogenmyr, Comb Flame 156, 25

**LEM = Linear Eddy Model**
Srinivasan, Menon, Flow Turb Comb 94, 237, Sankaran, Menon, PROC 30, 575
Motivation: DNS examples (at low Reynolds numbers)

R. Sankaran, E. Hawkes, Jackie Chen) & T. Lu, C. K. Law
Premixed DNS
PROCI 31, 1291

Bell, Day, Driscoll
“corregated” premixed DNS
PROCI 31, 1299

DNS flame in rectangular duct - L. Vervisch, A. Poludnenko,
Bunsen, V-flame, jet in cross-flow J.H. Chen
IC engine, Gas turbine, industrial burner:

RANS = KIVA (FSD/coherent flamelet), Fluent (empirical)
DNS  - 3-D, complex chemistry

Bell, Proc Natl Acad Sciences 102, 29 10006–10011

Aspden J. Fluid Mech 680, 287
Flame in a duct – periodic boundary conditions
Sees broken reactions
Claims to see distributed reactions
Background – what does turbulence do?

**Faster mixing** - large turbulent diffusivity, shorter flames, smaller combustors

**Faster propagation** - large turbulent burning velocity avoids blowout

**Optimize liftoff height** - keep flame away from walls but do not blowout

**Reduce Pollutants** - less NOx if velocities are large (for short residence times) and fuel-air mixing is fast and lean

**Avoid growl** - combustion oscillations in gas turbines, rockets, due to poor flame anchoring

**Challenge:** to model the turbulent reaction rate for non-premixed, premixed and partially-premixed combustion what are the best current models, and how good are they?
Background

turbulent kinetic energy

\[ k = \left( \frac{1}{2} \right) \left( \overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right) = \left( \frac{3}{2} \right) \overline{u'^2} \quad \text{if isotropic} \]

Favre (density-weighted) average

\[ \tilde{u} = \frac{\rho \overline{u}}{\overline{\rho}} = \frac{(\overline{\rho} + \rho')(\overline{u} + u')}{\overline{\rho}} = \overline{\rho} \frac{\overline{u}}{\overline{\rho}} + \frac{\rho' u'}{\overline{\rho}} \quad \text{so:} \]

\[ \tilde{u} = \overline{u} + \frac{\rho' u'}{\overline{\rho}} \]

replacing mean quantities in Navier Stokes Equations with their Favre averages removes all the density fluctuations - this removes one unknown (\( \rho' \)) but it adds one new unknown: turbulent mass flux: \( \frac{\rho' u'}{\overline{\rho}} \) which we determine using:

\[ \frac{\rho' u'}{\overline{\rho}} = -D_T \frac{\partial \overline{\rho}}{\partial x} \quad \text{Prandtl’s gradient diffusion assumption} \]
Gradient diffusion assumption - relates Favre avg to time average

$$\tilde{u} = \bar{u} + \frac{\rho' u'}{\bar{\rho}}$$

Favre average gets rid of one unknown ($\rho'$) but introduces another one: $\frac{\rho' u'}{\bar{\rho}}$

$$\rho' u' = -D_T \frac{\partial \bar{\rho}}{\partial x}$$

Gradient Diffusion assumption

Suppose $\frac{\partial \bar{\rho}}{\partial y}$ and $v'$ are positive

Small element will carry a low density from A to the higher density region at B. Therefore fluid at B will see a low density fluctuation, thus $\rho'$ is negative at B

So:

$$\rho' = -L \frac{\partial \bar{\rho}}{\partial y}$$

where $L = $ integral scale

Thus $\rho'v' = -L v' \frac{\partial \bar{\rho}}{\partial y}$ taking time average $\Rightarrow$ eqn above

We also proved that

$$D_T = v' L$$
Mixing: Turbulent viscosity = diffusivity is large

Since molecular viscosity $\frac{\mu}{\rho} = \text{(speed of sound)} \ (\text{mean free path})$

Prandtl suggested: $\frac{\mu_T}{\rho} = (u' L)$  \( L = \text{integral scale} \)

Dissipation rate of TKE $\varepsilon = \frac{u'^3}{L}$  \( \text{see turbulence text by Tennekes} \)

Since $u' = k^{1/2}$ then $\varepsilon = \frac{k^{3/2}}{L}$  or  $L = \frac{k^{3/2}}{\varepsilon}$

Since we said above $\mu_T = \rho (u') (L)$, then: $\mu_T = c_\mu \ \bar{\rho} \ \frac{k^2}{\varepsilon}$

How do we compute $k$ and $\varepsilon$? Prandtl suggested the $k$ - equation
How to compute turbulence level $k$ and dissipation rate $\varepsilon$?

$$
\bar{\rho} \tilde{u} \frac{\partial k}{\partial x} + \bar{\rho} \tilde{v} \frac{\partial k}{\partial y} = \frac{\partial}{\partial y} \left( \mu_T \frac{\partial k}{\partial y} \right) + \mu_T \left( \frac{\partial \tilde{u}}{\partial y} \right)^2 - \bar{\rho} \varepsilon
$$

$k$-epsilon equations

$$
\bar{\rho} \tilde{u} \frac{\partial \varepsilon}{\partial x} + \bar{\rho} \tilde{v} \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y} \left( \mu_T \frac{\partial \varepsilon}{\partial y} \right) + C_\varepsilon C_D \bar{\rho} k \left( \frac{\partial \tilde{u}}{\partial y} \right)^2 - \frac{C_\varepsilon \bar{\rho} (\varepsilon)^2}{C_D k}
$$

convection = diffusion + creation - destruction

Example: Grid in a Wind tunnel $\bar{\rho}, \tilde{u}$ are constant, $\frac{\partial}{\partial y} = 0$ at $x > 0$ behind grid

Given: at $x = 0$ (grid) $k = k_0 = 10 \text{ m}^2/\text{s}^2$ and $\varepsilon = \varepsilon_0 = 1 \text{ (m/s)}^3/\text{m}$ then integrate:

$$
\tilde{u} \frac{\partial k}{\partial x} = -\varepsilon \quad \text{and} \quad \tilde{u} \frac{\partial \varepsilon}{\partial x} = - \frac{C_\varepsilon (\varepsilon)^2}{C_D k}
$$

Solve on MATLAB
Non – premixed turbulent flames

Mixture fraction \((Z)\) = mass fraction of H-atoms

\[
Z = \frac{(Y_H - Y_{H,2})}{(Y_{H,1} - Y_{H,2})}
\]

\(Y_H\) = mass fraction of H atoms contained in all molecules at a point
\(Y_{H,1}\) = mass fraction of H atoms entering in stream 1
\(Y_{H,2}\) = mass fraction of H atoms entering in stream 2 = 0 for our examples

Ex. Suppose a jet of methane CH4 is surrounded by a large stream of pure O2. Stream 1 = methane, \(Y_{H,1} = \frac{4}{16} = 0.25\), Stream 2 = O2, so \(Y_{H,2} = 0.0\)

At some point P downstream suppose we have 2 moles \(\text{H}_2\text{O}\) and 1 mole \(\text{CO}_2\). At that point,

\[
Y_H = \frac{4 \text{ g}}{\left[44 \text{ g} + 2(18 \text{ g})\right]} = 0.05
\]

\[
Z = \frac{(Y_H - Y_{H,2})}{(Y_{H,1} - Y_{H,2})} = \frac{(0.05 - 0)}{[0.25 - 0]} = 0.20
\]
Conservation equations for, $Z$, mass fractions of $H_2$ and $O_2$

Now consider a planar 2-D jet of $H_2$ surrounded by a stream of $O_2$.

\[
\bar{\rho} \, \bar{u} \, \frac{d\bar{Y}_{H_2}}{dx} + \bar{\rho} \, \bar{v} \, \frac{d\bar{Y}_{H_2}}{dy} = \bar{\rho} \, D_T \, \frac{d^2 \bar{Y}_{H_2}}{dy^2} + \bar{\omega}_{H_2}
\]

\[
\bar{\rho} \, \bar{u} \, \frac{d\bar{Y}_{O_2}}{dx} + \bar{\rho} \, \bar{v} \, \frac{d\bar{Y}_{O_2}}{dy} = \bar{\rho} \, D_T \, \frac{d^2 \bar{Y}_{O_2}}{dy^2} + \bar{\omega}_{O_2}
\]

\[
\bar{\rho} \, \bar{u} \, \frac{d\bar{Y}_{H_2O}}{dx} + \bar{\rho} \, \bar{v} \, \frac{d\bar{Y}_{H_2O}}{dy} = \bar{\rho} \, D_T \, \frac{d^2 \bar{Y}_{H_2O}}{dy^2} + \bar{\omega}_{H_2O}
\]

The rate at which $H_2O$ is formed is related to rate $H_2$ is consumed:

\[
H_2 + \frac{1}{2} O_2 \rightarrow H_2O \quad \bar{\omega}_{H_2O} = \left(-\frac{18}{2}\right) \bar{\omega}_{H_2} \quad \bar{\omega}_{O_2} = \left(\frac{16}{2}\right) \bar{\omega}_{H_2}
\]

Inserting these into the above and put in terms of $Z$ to get:

\[
\bar{\rho} \, \bar{u} \, \frac{d\bar{Z}}{dx} + \bar{\rho} \, \bar{v} \, \frac{d\bar{Z}}{dy} = \bar{\rho} \, D_T \, \frac{d^2 \bar{Z}}{dy^2}
\]

Mixture fraction Conservation eqn has no source term
Flamelet idea: turbulent flame is filled with wrinkled, strained flamelets

State relation - relates mass fraction of $O_2$ at any point to mixture fraction $Z$

For our methane mixing into $O_2$ problem, for lean combustion at any equivalence ratio $\phi$:

$$\text{CH}_4 + (2/\phi) \text{ O}_2 \rightarrow \text{CO}_2 + 2 \text{ H}_2\text{O} + [(2/\phi)-2] \text{ O}_2$$

Fast Chemistry

The gas mixture at any point that resulted from lean combustion must be the mixture in the oval. Assuming fast chemistry. How to relate $\phi$ to $Z$?

For the mixture in the oval, $Z = 4 / [44 + 2(18) + (2/\phi-2)32] / Y_{H,1}$, where $Y_{H,1} = 0.25$

Solve this for $\phi$ to get: $\phi = 4Z / (1-Z)$

Now what is $Y_{O_2}$ for the mixture in the oval? It is

$$Y_{O_2} = [(2/\phi)-2] 32 / \{44 + 36 + [(2/\phi)-2] 32\} = (4 - 4\phi) / (4 + \phi)$$

Insert into this the above formula for $\phi$ to get:

$$Y_{O_2} = (1- 5 Z)$$

(state relation for lean combustion, and $Z$ only varies from 0 to 0.25)
State relations for unstrained non-premixed flamelets

If you know the instantaneous mixture fraction \( (Z) \) at a point \( P \), you use these state relations to look up instantaneous temperature and mass fractions at \( P \).
Probability density function - used to define a mean value

\[ P(c) \, dc = \text{probability that } c \text{ lies in the range between } c - dc/2 \text{ and } c + dc/2 \]

\[
\bar{Y}_{CO}(\bar{Z}, \bar{Z}'^2) = \int_0^1 Y_{CO}(Z) \, P(Z, \bar{Z}, \bar{Z}'^2) \, dZ
\]

State relation = Mass fraction of CO Conditioned on Z

At each point in the flame, we solve conservation equations to get the mean mixture fraction and variance in mixture fraction \( \bar{Z}'^2 \)

Idea: you only have to solve conservation equations for \( \bar{Z} \) and \( \bar{Z}'^2 \) and use above integral to get other mean values; you avoid solving more conservation equations for each variable
Assume PDF (Z) to be a Beta function for non-premixed flames

\[ P(x; \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \, du} \]

Here \( x = Z \) = mixture fraction. \( \alpha \) and \( \beta \) are related to mean and variance of \( Z \).
Importance of state relations

We could solve partial differential equations for every variable, but then we would need source terms for each. Instead use Schvab-Zeldovich approach:

Assume turbulent diffusivity of mass ($D_T$) equals that of momentum ($v_T$) and that of heat ($\alpha_T$)

- Premixed flames: solve only the differential equation for non-dimensional temperature, called reactedness (this is the energy eqn) and its variance

- Non-premixed flames: solve only the differential equations for mean mixture fraction (the Z equation) and its variance

- Then use state relations to compute mean values of $\rho$, $T$, $Y_i$ and turbulent reaction rates, using:

$$\bar{Y}_{CO} (\bar{Z}, \bar{Z}''^2) = \int_0^1 Y_{CO}(Z) \cdot P(Z, \bar{Z}, \bar{Z}''^2) dZ$$

- Where to get the state relations? From equilibrium chemistry, or from strained flamelet (non-equil) chemistry – we will discuss
Turbulent reaction rate $\overline{\omega}$ - the biggest unknown

The conservation equation for time-averaged CO mass fraction is:

$$
\bar{\rho} \, \bar{u} \, \frac{d\bar{Y}_{CO}}{dx} + \bar{\rho} \, \bar{v} \, \frac{d\bar{Y}_{CO}}{dy} = \bar{\rho} \, \alpha_T \, \frac{d^2 \bar{Y}_{CO}}{dy^2} + \bar{\omega}_{CO}
$$

kg/s/m$^3$ CO produced = ?

Turbulence-chemistry interaction – why do we need PDFs?

Reaction rate depends on the joint probability that:
sufficient fuel, sufficient O$_2$ and sufficient temperature are simultaneously present

You could ignore interactions and say the reaction rate is:

$$
\overline{\omega}_{CO} = \bar{Y}_{fuel} \, \bar{Y}_{O2} \, \exp\left(-\frac{E}{R\bar{T}}\right)
$$

That would be WRONG. Suppose a glob of pure reactants (cold) and a glob of pure products (hot) oscillate over point P, as shown:
The actual temperature at P is 300 K when the reactants are present = no reaction

When products are present at P, temperature is 2100 K but no reactants = no reaction

If you use the time-average temperature at P, which is 2400/2 = 1200 K, and the time-average fuel-air ratio at P and plug into the above Arrhenius eqn, you compute a large reaction rate at P! But reaction rate is nearly zero! This is totally wrong.

Reaction occurs at P only when the thin line between the reactants and products is on P; then you have simultaneously the proper fuel, O\textsubscript{2} and temperature.

The correct reaction rate of CO is the following, which has the PDF in it:

$$\overline{\dot{\omega}}_{\text{CO}} (\bar{Z}, \bar{Z}''^2) = \int_0^1 \dot{\omega}_{\text{CO}} (Z) \ P (Z, \bar{Z}, \bar{Z}''^2) \ dZ$$
Turbulent reaction rate of a premixed flame - proportional to FSD

For a premixed turbulent flame, the turbulent reaction rate at any point, in kg/sec reactants consumed/volume, is needed because it is the source term in the mean conservation of energy equation. Reaction rate is given by:

\[ \bar{\omega}_R = \rho_R \ S_L \ \Sigma \]

Proof: Consider a wrinkled flame that at time \( t_1 \) moves normal to itself at speed \( S_L \) to new position at time \( t_2 \)

The volume/sec of reactants overtaken by the wave = (area of wave \( A_T \))
Times the (distance moved /sec) of each segment of the wave
distance moved / sec = \( S_L \)  laminar burning velocity

mass/sec of reactants overtaken by the wave = \( \rho_R \) (volume/sec overtaken)
So: mass/sec of reactants overtaken = \( \rho_R \ S_L \ A_T \)

But: \( \bar{\omega} = \text{mass/sec/volume} = \rho_R \ S_L \ (A_T /\text{volume}) = \rho_R \ S_L \ \Sigma \)

See review of Driscoll, Prog Energy Comb Sci 34, 91
Premixed flame reaction rate depends on flame surface density

\[ \Sigma = \lim_{\Delta x \to 0} \frac{A_f}{(\Delta x)^3} = \lim_{\Delta x \to 0} \frac{P_f}{(\Delta x)^2} \]

\(A_f\) = average surface area of a premixed turbulent flame inside a small 1 mm\(^3\) interrogation box

\(P_f\) = average perimeter of flame boundary inside a 1 mm\(^2\) box in laser sheet

\(=\) average perimeter of flame when it is inside the box, which is approximately \(\Delta x\), times the probability that it is inside box

FSD is a Gaussian-like function in space
Physical meaning of FSD = flame surface density

Suppose flame is not wrinkled
But oscillates - right to left and back

a. perimeter of flame inside of interrogation box, when flame is inside box = approx. $\Delta x$

b. fraction of time flame is inside box is: $\Delta x / \delta_T$

Time avg perimeter in box is $a \times b = (\Delta x)^2 / \delta_T$

Average FSD = avg perimeter / $(\Delta x)^2$ so:

Average FSD = approx. $1 / \delta_T$ (= typically 0.2 mm$^{-1}$)

FSD should be independent of box size $\Delta x$
How to measure FSD?

Image the flame boundary - using PLIF of OH or Mie scattering

Binarize the signal: green = 1, blue = 0

Canny edge detection - to obtain coordinates of a continuous contour, infinitely thin, fit to the flame boundary

Determine the average perimeter of this contour in the 1 mm$^3$ interrogation box; vary the box location

Resulting value of FSD must be independent of interrogation box size

$\Sigma = \frac{\text{[area of flame when it is in box] \ (prob. flame is in box)}}{(\Delta x)^3}$

$\Sigma = \text{approx. } \frac{((\Delta x)^2 \ (\Delta x / \delta_T))}{(\Delta x)^3}$ so it should be independent of $\Delta x$

$\delta_T = \text{brush thickness}$
Turbulence increases propagation speed - of a premixed flame

Damköhler first concept- moderate turbulence increases flame area by wrinkling

Consider this thin wrinkled flamelet; its wrinkled area is $A_T$, and the area of the straight dotted line is $A_L$.

Each point on the wrinkled line propagates normal to the wrinkled line at a speed $S_L$, so the mass per second of reactants overtaken by the wrinkled line is $\rho_R S_L A_T$.

→ So larger wrinkled area = more reactants consumed /sec

The time-averaged wave is the dotted line; it propagates to left at $S_T$ so the mass/sec of reactants dotted line overtakes = $\rho_R S_T A_L$

Equating these two mass flow rates, we get:

$$\frac{S_T}{S_L} = \frac{A_T}{A_L}$$

Turbulent flames propagate faster because they have more wrinkled surface area to consume the reactants.
What is total wrinkled flame area $A_T$?

$A_T = \text{Wrinkled flame area} = \text{area/volume integrated over the entire volume of the flame brush}$

$$A_T = \int \Sigma dV = \iint \Sigma d\eta \, d\xi \, W$$

Area / volume

normal
tangential
to brush

$\eta$
$\xi$

$W$
First definition of turbulent burning velocity $S_T$

Bray: assumed that thin flamelets propagate at $S_L$ normal to themselves

Contribution of turbulent burning velocity due to thin flamelets is:

$$\frac{S_{T,F}}{S_L} = \frac{A_T}{A_L}$$

Where:

$$A_T = \int \sum dV = \iint \sum d\eta\, d\xi\, W$$

And $A_L = W\, L$ so:

$$S_{T,F} = S_L\, I_0 \, \frac{1}{L} \, \iint \sum d\eta\, d\xi$$

To measure $S_{T,F}$, measure $\Sigma$ everywhere and integrate it over the flame brush
Second definition of turbulent burning velocity $S_T$

$S_{T,GC} = \text{Global consumption speed}$

$S_{T,GC} = \frac{\dot{m}_R}{\rho_R A_T} = \text{mass flow reactants} / (\text{density reactants}) \ (\text{area of} \ \bar{C}={0.5} \ \text{surface})$

How to measure $S_{T,GC}$?

From the triangle drawn:

$$\sin \alpha = \frac{d}{2} \left[ h^2 + \left(\frac{d}{2}\right)^2 \right]^{-1/2} = S_T / U_o$$

so:

$$S_T = U_o \left(\frac{d}{2}\right) \left[ h^2 + \left(\frac{d}{2}\right)^2 \right]^{-1/2}$$

large burning velocity $S_T = \text{short flame}$
Suppose the flame wrinkles into two Bunsen cones, where $\alpha$ is the cone half-angle. Similar to a Bunsen burner, the velocity normal to the wave must be $S_L$, and the velocity normal to the cone is $(U_2 - S_L) \sin \alpha$, so equating these gives:

$$\sin \alpha = S_L / (U_2 - S_L)$$

and we define $u' = (U_2 - S_L)$

the cone has a radius of $L/4$ and height $h$, so:

$$\sin \alpha = (L/4)[h^2 + (L/4)^2]^{-1/2}$$

Equating these (and neglecting $L/4$ wrt $h$) yields:

$$h = (u' L) / (4 S_L)$$

Now the surface area of a cone is

$$A_T = \pi/4 (L/2)^2 \left[\left(h/(L/2)\right)^2 + 1\right]^{1/2}$$

and the area of the base of the cone is

$$A_L = \pi/4 (L/2)^2$$

so:

$$\frac{S_T}{S_L} = \frac{A_T}{A_L} = \left[1 + c_1 \left(\frac{u'}{S_L}\right)^2\right]^{1/2}$$

Predicted turbulent burning velocity

see Kuo, Turb Combustion
Damkohler’s second concept - small eddies increase thermal diffusivity

\[ S_L \sim \sqrt{\alpha \ RR} \]

\[ S_T \sim \sqrt{\left(\alpha + \alpha_T\right) \ RR} \]

\[ \alpha_T = c_2 \ u' L \]

\[ \frac{S_T}{S_L} = \sqrt{1 + c_2 \ \frac{u'L}{\nu}} \quad \text{where} \quad \frac{u'L}{\nu} = Re_T \]

If eddies get inside preheat zone, we assume turbulent flames propagate faster because eddies create larger thermal diffusivity \( \alpha_T \)

Turbulent diffusivity = velocity fluctuation times integral scale \( L \); \( \nu = \) kinematic viscosity

Turbulent flames propagate faster because turbulence diffuses the heat upstream to preheat the reactants faster than laminar flames
Turbulent burning velocity at “extreme” turbulence levels

\[ S_{T,GC} = \frac{\dot{m}_R}{\rho_R A_T} \]

Global consumption speed

Difference is Damköhler second idea - increased thermal diffusivity

\[ S_{T,F} = S_L I_0 \frac{1}{L} \iiint \Sigma \, d\eta \, d\xi \]

Contribution from Flame surface density (wrinkling)

previous turbulence levels

extreme turbulence levels

Wabel, Skiba, Driscoll PROCI 36
Turbulent burning velocity – what do we know?

1. Six major canonical geometries for premixed turbulent flames
   Bunsen, jet, low-swirl, V, spherical and swirl (Gas Turbine)

2. Burning velocity formula is different for each

3. Residence time \( (x/U) \) is important, bunsen flame tip becomes more wrinkled than flame base, spherical flame gets more wrinkled in time

4. Reactant temperature, Reynolds number, Karlovitz number are important

5. Role of integral scale different for each geometry - can we correlate burning velocity with Reynolds number \( \text{Re}_T \)?
Turbulence Causes Faster Mixing = shorter flame length

Consider a non-premixed turbulent jet flame. Suppose we simplify by saying the fuel from the fuel tube stays within the cylinder shown. Air enters at an entrainment velocity $U_e$ that is perpendicular to the cylinder wall.

Measurements show that the eddies rotate to cause the entrainment velocity $U_e$ to be proportional to $|c_1 U_F - U_A| \left(\rho_F / \rho_A\right)^{1/2}$.

The length of the flame $L_f$ is where the mass/sec of fuel, divided by the mass/sec of entrained air, equals the stoichiometric fuel-air ratio $f_s$, which is 0.055 for methane.

$$\rho_F U_F \left(\pi \frac{d_F^2}{4}\right) / \left(\rho_A U_e \pi d_F L_f\right) = f_s$$

(H. Rehab, J Fluid Mech 345, 357)
Combine the above relations to eliminate $U_e$ and solve for $L_f$ to give:

\[
\frac{L_f}{d_F} = \frac{c_2}{f_S} \left( \frac{\rho_F}{\rho_A} \right)^{1/2} \frac{U_F}{|c_1 U_F - U_A|}
\]

Length of a jet flame with co-axial air

If there is no co-flow ($U_A$ is zero) the turbulent eddies at the edge of the cylinder are created by the jet velocity $U_F$ so $U_e$ is prop. to $U_F$ and the above reduces to:

\[
\frac{L_f}{d_F} = \frac{c_3}{f_S} \left( \frac{\rho_F}{\rho_A} \right)^{1/2}
\]

Length of a jet flame with NO co-axial air

$H_2-O_2$ has large $f_s = \text{short}$

If there is strong co-flow air velocity, as in a gas turbine engine or rocket, then We neglect $c_1 U_F$ with respect to $U_A$ so the above reduces to:

\[
\frac{L_f}{d_F} = \frac{c_2}{f_S} \left( \frac{\rho_F}{\rho_A} \right)^{1/2} \frac{U_F}{|U_A|}
\]

Length of a jet flame with STRONG co-axial air

larger air velocity = shorter
Flame Liftoff and Blowout

Cases of interest:
- Fuel jet with no coflow air (flame base is partially premixed)
- Fuel jet with cold coflow air (base is partially premixed)
- Fuel jet w very hot coflow air (base is not a flame, is auto ignition)
- Fuel jet in hot cross flow (base is not a flame, is auto ignition)
- Fuel jet w swirled air (base is partially premixed)
- Cavity stabilized flame (premixed in shear layer at top of cavity)

DNS of Mizobuchi, Takeno
red= rich premix, blue = lean premix green = non-premix
PROCI 30, 2005
Consider a fuel jet issuing into air (initially non-premixed) with

\[ U_F = \text{jet exit velocity} \quad Z_s = \text{stoichiometric mixture fraction} = 0.055 \text{ for methane} \]

\[ U_s = \text{axial velocity of gas along the stoichiometric contour}; \text{ theory says:} \]

\[ U_s = \text{constant} \quad \text{along the stoichiometric contour} \text{ and this constant is} \quad U_F Z_s \]

Why? See Kuo, conservation equations for mixture fraction \((Z)\) and \((u/U_F)\) are identical for a jet flame

Shear layer at fuel/air boundary is \textit{premixed} upstream of lifted flame base

\[ S_{\text{base}} = \text{turbulent burning velocity} \quad \text{– propagation speed of flame base against incoming reactants} \]
Stability criterion  (VanTiggelen, Comb Flame 10, p. 59)

Two requirements
For flame to be stable:

\[ U_s = S_{\text{base}} \]
\[ \frac{dS_{\text{base}}}{dx} > \frac{dU_s}{dx} \]

Experiments show: The propagation speed \( S_{\text{base}} \) decreases in the negative-x direction since strain rate on the flame increases as you approach the fuel tube.

In last slide we argue that \( U_s \) is constant in x-direction.

notice that the slope \( dS_{\text{base}}/dx \) > \( dU_s/dx \) in this plot.
Flame normally is at location A where the two curves cross \( (U_s = S_{\text{base}}) \)

Suppose the flame base was perturbed to move downstream to location B
Since \( \frac{\delta S_{\text{base}}}{\delta x} > \frac{\delta U_s}{\delta x} \), it follows that \( S_{\text{base}} > U_s \) at location B, this means
That the flame will propagate upstream at speed \( (S_{\text{base}} - U_s) \) until it reaches A

This flame is stable – a perturbation causes it to return to its original position!
Scaling Analysis – for Liftoff height, Blowout velocity jet flame

we said: \( U_s = S_{\text{base}} \) and \( U_s = U_F Z_s \)

now we need a formula to estimate \( S_{\text{base}} \)

Define \( \text{Karlovitz number} = \text{nondim strain rate} = \frac{(U_s / x)}{(S_L / \alpha)} \)

Assume: \( S_{\text{base}} = S_L [1 + Ka]^{-2} \) if \( Ka \) is large, \( S_{\text{base}} \) is small

\( S_L = \text{laminar burning velocity, stoichiometric} \)

Liftoff height \( h = x \) and we said: \( U_s = U_F Z_s \)

Combine above, liftoff height is:

\[
h = \left\{ c_1 \frac{U_F}{(S_L^2/\alpha)} \right\} F
\]

where function \( F \) is nearly constant

Larger jet velocity \( U_F \) – higher liftoff height
Blowout velocity of a jet flame

There is no stoichiometric location downstream of \( x = L_f \) = mixing length to stoich.

So when liftoff height (\( h \)) of flame base exceeds \( (L_f) - \) blowout occurs

\[
h = L_f \quad L_f = c_2 \, d_F \, Z_s^{-1} \quad \text{from previous slide}
\]

\[
h = c_1 \, U_F / (S_{L,0}^2 / \alpha) \quad \text{from previous slide}
\]

Combine these three equations and solve for \( U_F \)

Blowout fuel velocity: \[
U_F = c_3 \, d_F / (S_L^2 / \alpha) / F
\]

RESULT: Liftoff and blowout formulas agree with measurements!
Don’t make fuel injector diameter \( d_F \) too small.
Co-flowing air is de-stabilizing

Co-flow air flows at velocity $U_A$, as shown below. Co-flow adds a velocity of $U_A$ to $U_s$, at every point along the stoichiometric contour (red line).

Since $U_s = S_{\text{base}}$, the flame base must propagate against a larger velocity, so you must reduce the fuel velocity to avoid blowout.

But co-flow air promotes shorter flames, faster mixing (see previous slide).

Feikema, Chen, Driscoll, Combust. Flame 80, 1990
Swirl is both STABILIZING and promotes faster mixing

so all gas turbine designs employ swirl
swirl creates internal recirculation = low gas velocities

Much larger fuel Velocity achieved with swirl

Feikema, Chen, Driscoll, Combust. Flame 80, 1990
Scramjets use wall cavities to stabilize the flame

Air enters across upper side of shear layer

Fuel and hot products from recirculation zone enter across lower side of shear layer

Flame exists along stochiometric contour
1. Liftoff height \( h \) where propagation speed \( S_{\text{base}} \) = gas velocity \( U_g \)
2. Temperature at base is elevated due to hot recirculation zone
3. Too much cold fuel into RZ lowers temperatures, flame speed
4. When liftoff height \( h \) exceeds \( L_{\text{RZ}} \) = “rich blowout”
5. Stable if \( \partial S_{\text{base}} / \partial x > \partial U_s / \partial x \)
Base of a lifted flame is a “triple flame”

How fast does a triple flame propagate?

![Diagram showing a triple flame with rich premixed flame, non-premixed flame, and lean premixed flame.]

- $U_G =$ gas velocity at base (0.4 m/s)
- $U_P =$ gas velocity in undisturbed flow (2 m/s)
References - Flame blowout

Vanquickenborne L, van Tiggelen A. Combust Flame 1966;10, p. 59


Driscoll, J. F. Correlation and Analysis of Blowout Limits of Flames in High-Speed Airflows, J. Propulsion Power 21, 6,1035, 2005

Mizobuchi, Takeno, PROCI 30, p. 611, 2005