Internal Combustion Engines
I: Fundamentals and Performance Metrics

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Course Length: 9 hrs
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Short course outline:

Internal Combustion (IC) engine fundamentals and performance metrics, computer modeling supported by in-depth understanding of fundamental engine processes and detailed experiments in engine design optimization.

Day 1 (Engine fundamentals)

- Hour 1: IC Engine Review, Thermodynamics and 0-D modeling
- Hour 2: 1-D modeling, Charge Preparation
- Hour 3: Engine Performance Metrics, 3-D flow modeling

Day 2 (Computer modeling/engine processes)

- Hour 4: Engine combustion physics and chemistry
- Hour 5: Premixed Charge Spark-ignited engines
- Hour 6: Spray modeling

Day 3 (Engine Applications and Optimization)

- Hour 7: Heat transfer and Spray Combustion Research
- Hour 8: Diesel Combustion modeling
- Hour 9: Optimization and Low Temperature Combustion
1-D compressible flow

Mass conservation:

\[ g = 1 \quad \frac{dMg}{dt}_{\text{system}} = 0 \]

cv fixed

\[ 0 = \int_{cv} \left\{ \frac{\partial (\rho A)}{\partial t} + \nabla \cdot (\rho AV) \right\} dx \]

1. \[ \frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho AV)}{\partial x} = 0 \]

Momentum conservation:

2. \[ \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} + 2fV^2 / D = 0 \]

Energy conservation:

3. \[ \frac{\partial e}{\partial t} + V \frac{\partial e}{\partial x} = \dot{q} + 2fV^3 / D - \frac{P}{\rho A} \frac{\partial (VA)}{\partial x} \]

Reynolds Transport Equation

\[ \frac{dMg}{dt}_{\text{system}} = \frac{d}{dt} \int \rho g d\forall + \frac{d}{dt} \int_{cv} \rho g d\forall + \int_{cs} \rho g \mathbf{V}_{rel} \cdot \mathbf{n} dA \]

dx

Supplementary:

4. \[ P = \rho RT \]

5. \[ e = c_v T \]

\[ f = \tau_w / \rho V^2 / 2 \]

\[ \dot{Q} = \dot{q} \rho A dx \]

5 unknowns U: \( \rho, V, e, P, \) and \( T \)

5 equations for variation of flow variables in space and time
In 1-D models friction factors are used to account for losses at area change or bends by applying a friction factor to an “equivalent” length of straight pipe.

Flow losses

Apply experimentally or numerically determined Loss Coefficient to equivalent straight pipe:

\[ \Delta P = C_p \rho V^2 / 2 \]
1-D Modeling Codes

1-D codes (e.g., GT-Power, AVL-Boost, Ricardo WAVE) predict wave action in manifolds. At high engine speed valve overlap can improve engine breathing, \( \Rightarrow \) inertia of flowing gases can cause inflow even during compression stroke.

Variable Valve Actuation (VVA) technologies, control valve timing to change effective compression ratio (early or late intake valve closure), or exhaust gas re-induction (re-breathing) to control in-cylinder temperatures.

Residual gas left from the previous cycle affects engine combustion processes through its influence on charge mass, temperature and dilution.
Numerical solution

To integrate the partial differential equations:
Discretize domain with step size, $\Delta x$
Time marches in increments of $\Delta t$ from initial state $U_i^0$: $\rho_i^n, V_i^n, e_i^n, P_i^n$, and $T_i^n$

$$t = n\Delta t \quad n = 0, 1, 2, 3, \ldots$$

$$\frac{\partial U(x, t)}{\partial x} = \frac{\Delta U(x_i, n \cdot dt)}{\Delta x_i} = \frac{U_{i+1}^n - U_i^n}{\Delta x_i}$$

$$\frac{\partial U(x, t)}{\partial t} = \frac{\Delta U(x_i, n \cdot dt)}{\Delta t} = \frac{U_{i+1}^n - U_i^n}{\Delta t}$$

Considerations of stability require the Courant-Friedrichs-Levy (CFL) condition

$$\Delta t \leq \min (\Delta x_i / (|V_i^n| + c_i^n))$$
Analytical solutions – Method of Characteristics

R: right-running wave  \[ \text{slope} \frac{dx}{dt} = V + c \]

L: left-running wave  \[ \text{slope} \frac{dx}{dt} = V - c \]

P: particle-path  \[ \text{slope} \frac{dx}{dt} = V \]

All points continuously receive information about both upstream and downstream flow conditions from both left and right-running waves. These waves originate from all points in the flow.
**Analytical solutions** – Method of Characteristics

**R**: right-running wave

slope \( \frac{dx}{dt} = V + c \)

**L**: left-running wave

slope \( \frac{dx}{dt} = V - c \)

**P**: particle-path

slope \( \frac{dx}{dt} = V \)

\( \Delta t \leq \min(\Delta x_i / (|V_i^n| + c_i^n)) \)

**R; L; P;** are called Characteristic Lines in the flow
Along R: \( dP + \rho c dV = F dt \)  
Along L: \( dP - \rho c dV = G dt \)  
Along P: \( d\rho - \frac{dp}{c^2} = H dt \)

\[ F, G, H = \text{Functions of } (\frac{\dot{q}}{f}, \ln A/\Delta x) \]

The discrete versions are:
\[ (P_4 - P_R) + (\rho c)_R (V_4 - V_R) = F_R \Delta t \]
\[ (P_4 - P_L) - (\rho c)_L (V_4 - V_L) = G_L \Delta t \]
\[ (\rho_4 - \rho_P) - \left( \frac{1}{c^2} \right)_P (P_4 - P_P) = H_P \Delta t \]

3 equations to solve for \( \rho_4, V_4 \) and \( P_4 \)

Note: from Gibbs’ equation
\[ dS = \frac{c_p}{\rho} \left( \frac{dP}{c^2} - d\rho \right) = \frac{c_p}{\rho} H dt \]
**Lagrange ballistics**

Flow velocities in IC engine cylinders are usually $<<$ than the speed of sound. Lagrange ballistics shows that cylinder pressure and density is the same at all points within the combustion chamber.

\[
\begin{align*}
L: & \quad P_4 = P_L + (\rho c)_L (0 - V_L) \\
R: & \quad P_4 = P_R - (\rho c)_R (V_{piston} - V_R) \\
P: & \quad \rho_4 = \rho_P + \left(\frac{P_4 - P_P}{c^2}\right)_P
\end{align*}
\]

Pressure increases by $dP$ each wave reflection ($dV<0$) in order to alternately ensure that the flow meets the boundary conditions: $V=0$ at head, and $V=V_{piston}$ at piston.

Order of magnitude analysis of $L$, $R$, and $P$: gives

\[
dP \sim \rho c dV \quad \text{and} \quad \frac{d\rho}{\rho} \sim \frac{dV}{c}
\]

For $dV<<c$ relative density change is small—density and pressure changes only in time.
Steady Compressible flow – A review

Gibbs \[ Tds = dh - dp / \rho \]

Energy \[ dh = -V dV \]

Euler \[ dP = -\rho V dV \]

\[ \rho AV = \text{Const} \Rightarrow \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \]

\[ \frac{dA}{A} = (M^2 - 1) \frac{dV}{V} \]

\[ \frac{dA}{A} = \frac{(1 - M^2)}{\rho V^2} dP \]

Area-velocity relations

for \( M < 1 \)

Subsonic nozzle \( dA < 0 \)

Subsonic diffuser \( dA > 0 \)

Supersonic diffuser \( dA < 0 \)

Supersonic nozzle \( dA > 0 \)

from \( \rho AV \Rightarrow dV > 0 \)

from Euler \( dP < 0 \)

kinetic energy

pressure recovery

Traffic flow behaves like a supersonic flow!
Isentropic nozzle flows

\[
\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2
\]

\[
\frac{P_0}{P_1} = (1 + \frac{\gamma - 1}{2} M_1^2)^{\frac{\gamma}{\gamma - 1}}
\]

Ex. Flow past throttle plate

Choked flow for \( P_2 < 53.5 \text{ kPa} = 40.1 \text{cmHg} \)

\[ P_0 \rightarrow \psi \rightarrow P_1 \]

\[ \dot{m} \]

Manifold pressure, \( P_1 \text{ cmHg} \)

0.528

\[ \frac{P}{P_0} \]

reservoir

ambient

0

1

\[ P_b \]

Hour 2: 1-D modeling, Charge Preparation

Anderson, 1990

PCI-1-2, 2018
Model passages as compressible flow in converging-diverging nozzles

\[ \dot{m} = \rho A \dot{V} = \frac{P}{RT} A \frac{V}{c} \sqrt{\gamma RT} \]

\[ = P_0 \sqrt{\frac{\gamma}{RT_0}} A M (P / P_0) / (T / T_0)^{-1/2} \]

With \( M=1 \): Fliegner’s formula

\[ \dot{m}_{M=1} = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \sqrt{\frac{\gamma}{RT_0}} P_0 A^* \]

Area Mach number relations

\[ \frac{A}{A^*} = \frac{1}{M} \left\{ \frac{2}{\gamma + 1} (1 + (\gamma - 1) M^2) \right\}^{\frac{\gamma+1}{2(\gamma-1)}} \]

\[ \frac{A}{A^*} = \left( \frac{P}{P_0} \right)^{\frac{1}{\gamma-1}} \left( 1 - \left( \frac{P}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right) \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}} \left( \frac{\gamma+1}{\gamma-1} \right)^{1/2} \]
Application to turbomachinery

Fliegner’s Formula:

\[
\dot{m} = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \sqrt{\frac{\gamma}{RT_0}} P_0 A^* \\
\]

“Corrected mass flow rate”

A measure of effective flow area

Variable Geometry Compressor/
turbine performance map

- Increased speed
- Choked flow
- Reduced flow passage area

\[ \frac{P_0}{P} \]

Total/static pressure ratio

\[ \frac{1}{0.528} = 1.89 \]
Turbocharging

Pulse-driven turbine was invented and patented in 1925 by Büchi to increase the amount of air inducted into the engine.
- Increased engine power more than offsets losses due to increased back pressure
- Need to deal with turbocharger lag

Improved
• Fuel economy
• Torque
• Power density
Turbocharging

Purpose of turbocharging or supercharging is to increase inlet air density,
- increase amount of air in the cylinder.

Mechanical supercharging
- driven directly by power from engine.

Turbocharger - connected compressor/turbine
- energy in exhaust used to drive turbine.

Supercharging necessary in two-strokes for effective scavenging:
- intake P > exhaust P
- crankcase used as a pump

Some engines combine engine-driven and mechanical (e.g., in two-stage configuration).

Intercooler after compressor
- controls combustion air temperature.
Turbocharging

Energy in exhaust is used to drive turbine which drives compressor

Wastegate used to by-pass turbine
Charge air cooling after compressor further increases air density - more air for combustion
Regulated two-stage turbocharger

Duplicated Configuration per Cylinder Bank

GT-Power R2S Turbo Circuit

HP TURBINE
Compressor Bypass
Regulating Valve
LP TURBINE
LP Stage Bypass
LP stage Turbo-Charger with Bypass
HP stage Turbo charger
Regulating valve
EGR Cooler
EGR Valve
Charge Air Cooler
Compressor Bypass
Charge Air Cooler
Compressor Bypass
EGR Cooler
EGR Valve
**Automotive compressor**

Centrifugal compressor typically used in automotive applications

Provides high mass flow rate at relatively low pressure ratio ~ 3.5

Rotates at high angular speeds
- direct coupled with exhaust-driven turbine
- less suited for mechanical supercharging

Consists of:
- stationary inlet casing,
- rotating bladed impeller,
- stationary diffuser (w or w/o vanes)
- collector - connects to intake system
Air at stagnation state \( P_{0,in} \) accelerates to inlet pressure, \( P_1 \), and velocity \( V_1 \).

Compression in impeller passages increases pressure to \( P_2 \), and velocity \( V_2 \).

Diffuser between states 2 and out, recovers air kinetic energy at exit of impeller producing pressure rise to, \( P_{out} \) and low velocity \( V_{out} \)

\[
\eta_c = \frac{(T_{out-isen} - T_{in})}{(T_{out} - T_{in})}
\]

\[
P_3 = P_{out}
\]

\[
P_0 = P_{0,in}
\]

\[
V_1^2/2c_p
\]

\[
P_2
\]

\[
P_1
\]

Note: use exit static pressure and inlet total pressure, because kinetic energy of gas leaving compressor is usually not recovered.
Compressor maps

Work transfer to gas occurs in impeller via change in gas angular momentum in rotating blade passage.

Surge limit line
- reduced mass flow due to periodic flow reversal/reattachment in passage boundary layers.
Unstable flow can lead to damage.

Pressure ratio evaluated using total-to-static pressures since exit flow kinetic energy is not recovered.

Speed/pressure limit line

Non-dimensionalize blade tip speed (~ND) by speed of sound.

At high air flow rate, operation is limited by choking at the minimum area point within compressor.

Supersonic flow

Shock wave

Heywood, Fig. 6-46
Compressor selection

To select compressor, first determine engine breathing lines. The mass flow rate of air through engine for a given pressure ratio is:

\[
m_{\text{intake}} = \left[ \frac{\eta_{\text{vol}} \times D \times N \times P_{\text{ref}}}{2 \times R \times T_{\text{ref}}} \right]
\]

Where:
- \( m_{\text{intake}} \) = Physical mass flow of air through engine (mass/time)
- \( \eta_{\text{vol}} \) = Volumetric efficiency (unitless)
- \( D \) = Displacement of engine per cycle (length³/cycle)
- \( N \) = Engine speed (rev/time)
- \( P_{\text{ref}} \) = Reference pressure (psi) = IMP = PR * atmospheric pressure (no losses)
- \( R \) = Gas constant for air (length*force / mass*temperature)
- \( T_{\text{ref}} \) = Reference temperature (Rankin) = IMT = Roughly constant for given Speed
Engine breathing lines

Engine Breathing Lines
1.4L Diesel, Air-to-Air AfterCooled, Turbocharged

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Torque Peak</th>
<th>Rated</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horsepower</td>
<td>48</td>
<td>69</td>
<td>hp</td>
</tr>
<tr>
<td>BSFC</td>
<td>0.377</td>
<td>0.401</td>
<td>lb/hp-hr</td>
</tr>
<tr>
<td>A/F</td>
<td>23.8</td>
<td>24.5</td>
<td>none</td>
</tr>
</tbody>
</table>
Compressor maps

GM 1.9L diesel engine

- Efficiency (T/T)
- Corrected Air Flow (kg/s)
- Pressure Ratio (t/t)

Key:
- 35000
- 40000
- 50000
- 70000
- 90000
- 110000
- 130000
- 150000
- 170000
- 180000
- 190000
Automotive turbines

Naturally aspirated:
\[ P_{\text{intake}} = P_{\text{exhst}} = P_{\text{atm}} \ \text{(5-7-8-9-1)} \]

Boosted operation:

Negative pumping work:
\[ P_7 < P_1 \] – but hurts scavenging

\[
\dot{W}_t = \dot{m} (h_{\text{in}} - h_{0,\text{out}})
\]

\[
\dot{W}_t = \dot{m} c_p T_{\text{in}} \eta_t \left\{ 1 - \left[ \frac{P_{0,\text{out}}}{P_{\text{in}}} \right]^{\frac{\gamma - 1}{\gamma}} \right\}
\]

P-V diagram showing available exhaust energy
- turbocharging, turbocompounding, bottoming cycles and thermoelectric generators further utilize this available energy

\[ P_{\text{intake}} \quad P_{\text{exhst}} \quad P_{\text{amb}} \]

Reitz, 2007
Turbochargers

Radial flow – automotive; axial flow – locomotive, marine

\[ P_0 = P_{0,\text{in}} \]

\[ P_1 \]

\[ P_2 \]

\[ P_{03} \]

\[ P_3 = P_{out} \]

\[ \eta_t = \frac{(T_{out} - T_{in})}{(T_{out-isen} - T_{in})} \]

\[ m_{\text{corrected}} = m_g \frac{\sqrt{T_3}}{\sqrt{T_0}} \frac{p_3}{p_0} \]

\[ N_{\text{corrected}} = \frac{N}{\sqrt{T_3}} \frac{T_0}{T_0} \]
Hour 2: 1-D modeling, Charge Preparation

Matching

Centering the Engine Map on the Compressor Map for Optimum Performance

The flow characteristics of rotary turbomachines and reciprocating engines are not ideally suited to operate in tandem.

• Automotive engines
  - wide speed, load and flow range
  - positive displacement
  - discontinuous flow

• Turbochargers
  - high mass flow, with high design point efficiency.
  - narrow range
  - continuous flow no defined displacement

\[
\frac{\dot{m}_{\text{fuel}}}{\dot{m}_{\text{air}}} = \left[ 1 + \frac{Cp_g \cdot T_3}{Cp_a \cdot T_1} \right] \left[ 1 + \frac{m_{\text{fuel}}}{m_{\text{air}}} \right] \left( \eta_t \cdot \eta_c \cdot \eta_{\text{mech}} \right) \left[ 1 - \frac{p_4}{p_3} \right] \left( \frac{\gamma_g - 1}{\gamma_a - 1} \right)
\]

\[\dot{W}_t = \dot{W}_c\]

Heywood, 1988

DERB 2nd. Ed.-SAE
Summary

1-D models/codes based on thermodynamic models are available, and they are very useful for understanding charge preparation and engine breathing.

But, 1-D models require calibration against engine or theoretical data.

Turbocharging increases overall engine efficiency by using available energy in exhaust and by reducing pumping work.
References

1-2:3,7-9,11-14 J. D. Anderson, Modern Compressible Flow (With Historical Perspective), McGraw-Hill (2nd or 3rd Edition), 1990.


