Lecture 13

The Turbulent Burning Velocity

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One of the most important unresolved problems in premixed turbulent combustion is that of the turbulent burning velocity.

This statement implies that the turbulent burning velocity is a well-defined quantity that only depends on local mean quantities.

The mean turbulent flame front is expected to propagate with that burning velocity relative to the flow field.

Gas expansion effects induced at the mean front will change the surrounding flow field and may generate instabilities in a similar way as flame instabilities of the Darrieus-Landau type are generated by a laminar flame front (cf. Clavin, 1985).

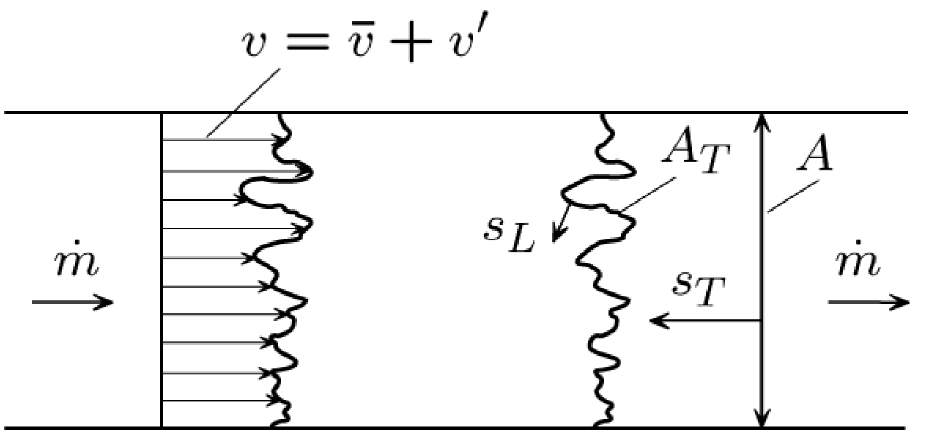
Damköhler (1940) was the first to present theoretical expressions for the turbulent burning velocity.

He identified two different regimes of premixed turbulent combustion which he called large scale and small scale turbulence.

We will identify these two regimes with the corrugated flamelets regime and the thin reaction zones regime, respectively.

Damköhler equated the mass flux through the instantaneous turbulent flame surface area A_T with the mass flux through the cross sectional area A, using the laminar burning velocity s_L for the mass flux through the instantaneous surface and the turbulent burning velocity s_T for the mass flux through the cross-sectional area A as

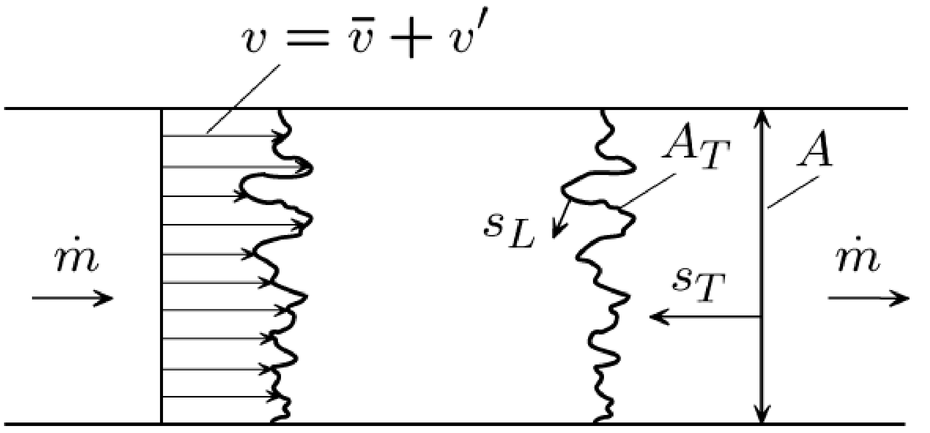
$$\dot{m} = \rho_u s_L A_T = \bar{\rho}_u s_T A$$



The burning velocities s_L and s_T are defined with respect to the conditions in the unburnt mixture and the density ρ_u is assumed constant.

From that equation it follows

$$\frac{s_T}{s_L} = \frac{A_T}{A}$$



Since only continuity is involved, averaging of the flame surface area can be performed at any length scale Δ within the inertial range.

If Δ is interpreted as a filter width one obtains a filtered flame surface area $\hat{A}_T$

$\dot{m} = \rho_u s_L A_T = \bar{\rho}_u s_T A$ then also implies $s_L A_T = \hat{s}_T \hat{A}_T = s_T A$

This shows that the product $\hat{s}_T \hat{A}_T$ is inertial range invariant, similar to the dissipation in the inertial range of turbulence.

As a consequence, by analogy to the large Reynolds number limit used in turbulent modeling, the additional limit of the ratio of the turbulent to the laminar burning velocity for large values of v'/s_L is the backbone of premixed turbulent combustion modeling.

For large scale turbulence, Damköhler (1940) assumed that the interaction between a wrinkled flame front and the turbulent flow field is purely kinematic.

Using the geometrical analogy with a Bunsen flame, he related the area increase of the wrinkled flame surface area to the velocity fluctuation divided by the laminar burning velocity

$$\frac{A_T}{A} \sim \frac{v'}{s_L}$$

Combining

$$\frac{s_T}{s_L} = \frac{A_T}{A} \qquad \text{and} \qquad \frac{A_T}{A} \sim \frac{v'}{s_L}$$

leads to

$$s_T \sim v'$$

in the limit of large v'/s_L, which is a kinematic scaling.

We now want to show that this is consistent with the modeling assumption for the *G*-equation in the corrugated flamelets regime.

For small scale turbulence, which we will identify with the thin reaction zones regime, Damköhler (1940) argued that turbulence only modifies the transport between the reaction zone and the unburnt gas.

In analogy to the scaling relation for the laminar burning velocity

$$s_L \sim \left(\frac{D}{t_c}\right)^{1/2}$$

where t_c is the chemical time scale and D the molecular diffusivity, he proposes that the turbulent burning velocity can simply be obtained by replacing the laminar diffusivity D by the turbulent diffusivity D_t:

$$s_T \sim \left(\frac{D_t}{t_c}\right)^{1/2}$$

while the chemical time scale remains the same.

Thereby it is implicitly assumed that the chemical time scale is not affected by turbulence.

This assumption breaks down when Kolmogorov eddies penetrate into the thin reaction zone. This implies that there is an upper limit for the thin reaction zones regime which was identified as the condition $\mathrm{Ka}_\delta = 1$.

Combining

$$s_L \sim \left(\frac{D}{t_c}\right)^{1/2} \qquad \text{and} \qquad s_T \sim \left(\frac{D_t}{t_c}\right)^{1/2}$$

the ratio of the turbulent to the laminar burning velocity becomes

$$\frac{s_T}{s_L} \sim \left(\frac{D_T}{D}\right)^{1/2}$$

Since the turbulent diffusivity D_T is proportional to the product $v'\ell$, and the laminar diffusivity is proportional to the product of the laminar burning velocity and the flame thickness ℓ_F one may write

$$\frac{s_T}{s_L} \sim \left(\frac{D_T}{D}\right)^{1/2}$$

as

$$\frac{s_T}{s_L} \sim \left(\frac{v'}{s_L}\frac{\ell}{\ell_F}\right)^{1/2}$$

showing that for small scale turbulence the burning velocity ratio not only depends on the velocity ratio v'/s_L but also on the length scale ratio ℓ/ℓ_F.

There were many attempts to modify Damköhler's analysis and to derive expressions that would reproduce the large amount of experimental data on turbulent burning velocities.

By introducing an adjustable exponent n, where $0.5 < n < 1.0$

$$s_T \sim v' \quad \text{and} \quad \frac{s_T}{s_L} \sim \left(\frac{v'}{s_L}\frac{\ell}{\ell_F}\right)^{1/2}$$

may be combined to obtain expressions of the form

$$\frac{s_T}{s_L} = 1 + C\left(\frac{v'}{s_L}\right)^n$$

This includes the limit $v' \to 0$ for laminar flame propagation where $s_T = s_L$.

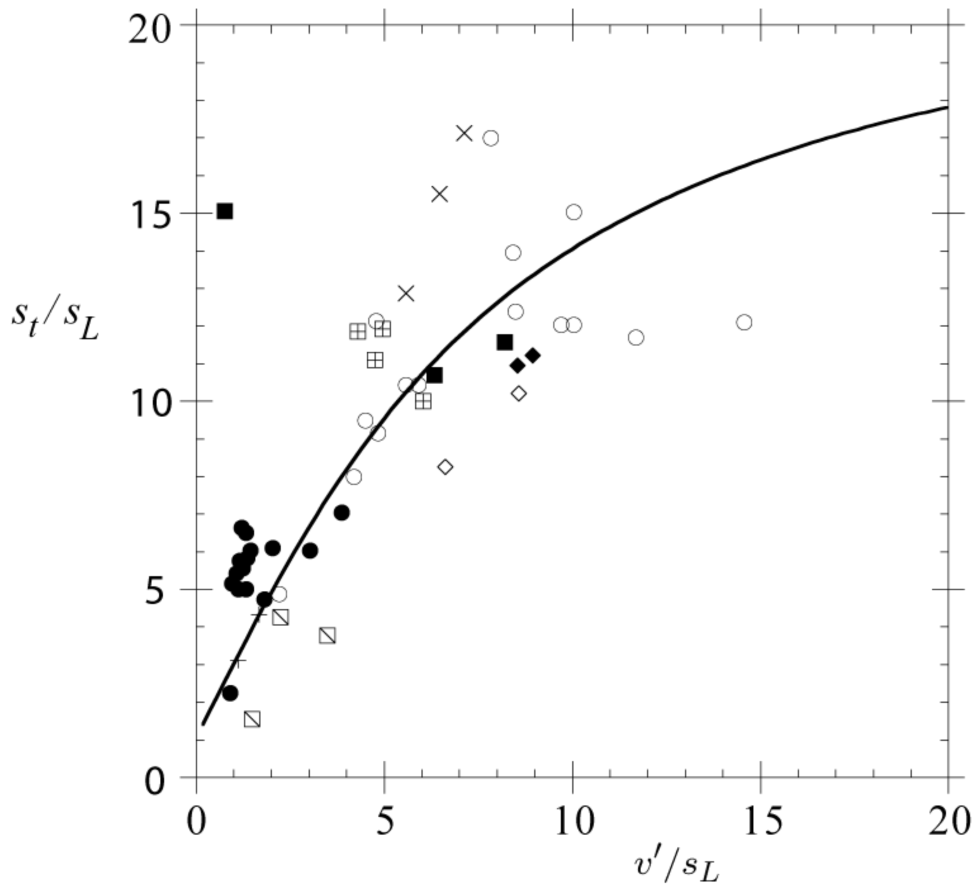
The constant C is expected to depend on the length scale ratio ℓ/ℓ_F.

By comparison with experiments the exponent *n* is often found to be in the vicinity of 0.7 (cf. Williams ,1985).

Attempts to justify a single exponent on the basis of dimensional analysis, however, fall short even of Damköhler's pioneering work who had recognized the existence of two different regimes in premixed turbulent combustion.

There is a large amount of data on turbulent burning velocities in the literature.

Correlations of this material, mos presented in terms of the burning velocity ratio s_T/s_L plotted as a function of v'/s_L, called the burning velocity diagram.

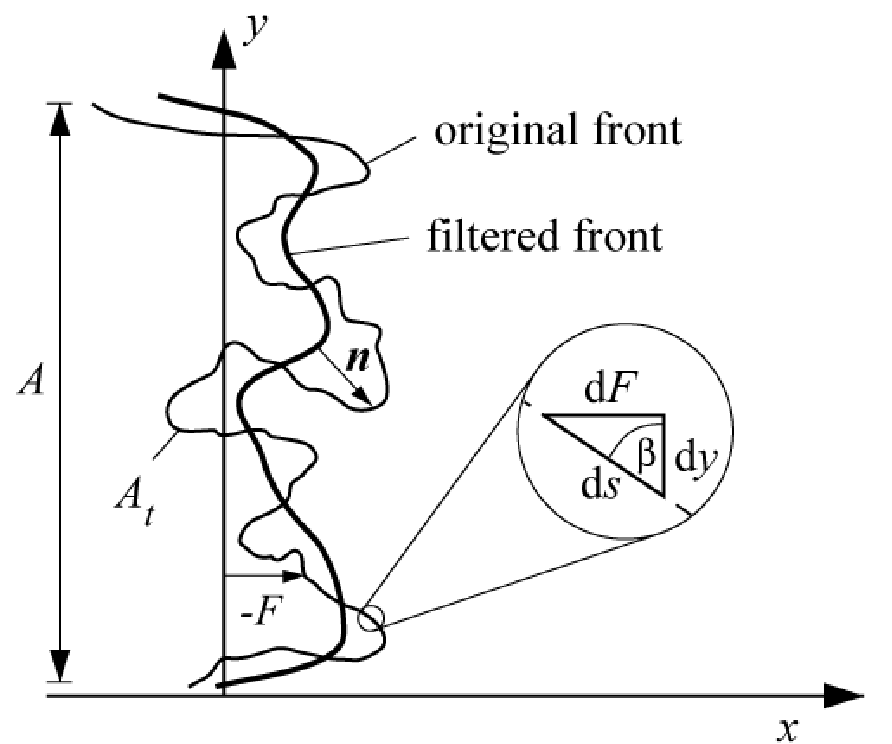


When experimental data from different authors are collected in such a diagram, they usually differ considerably.

In the review articles by Bray (1990) and Bradley (1992) the many physical parameters that affect the turbulent burning velocity are discussed.

A Model Equation for the Flame Surface Area Ratio

It was stated previously that the mean gradient $\bar{\sigma} = \overline{|\nabla G|}$ represents the flame surface area ratio.



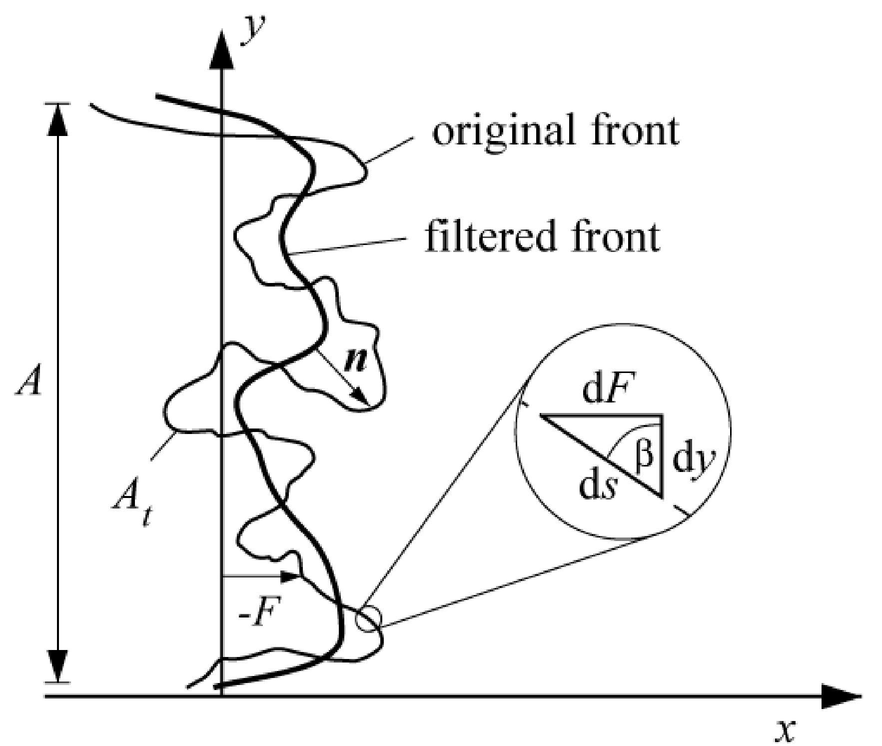
In the two-dimensional illustration the instantaneous flame surface area A_T is identified with the length of the line G=G_0.

The blow-up shows that a differential section dS of that line and the corresponding differential section dy of the cross sectional area A are related to each other by

$$\frac{\mathrm{d}S}{\mathrm{d}y} = \frac{1}{|\cos\beta|}$$

On the other hand, in two dimensions the gradient σ is given by

$$\sigma = \left(1 + \left(\frac{\partial F}{\partial y}\right)^2\right)^{1/2}$$



It can be seen that $\partial F/\partial y = \tan\beta$

which relates σ to the angle β as

$$\sigma = \frac{1}{|\cos\beta|}$$

and therefore

$$\sigma = \left(1 + \left(\frac{\partial F}{\partial y}\right)^2\right)^{1/2}$$

the differential flame surface area ratio is equal to the gradient σ: $\frac{\mathrm{d}S}{\mathrm{d}y} = \sigma$

We therefore expect to be able to calculate the mean flame surface area ratio from a model equation for the mean gradient $\bar{\sigma} = \overline{|\nabla G|}$

There remains, however, the question whether this is also valid for multiple crossings of the flame surface with respect to the *x*-axis.

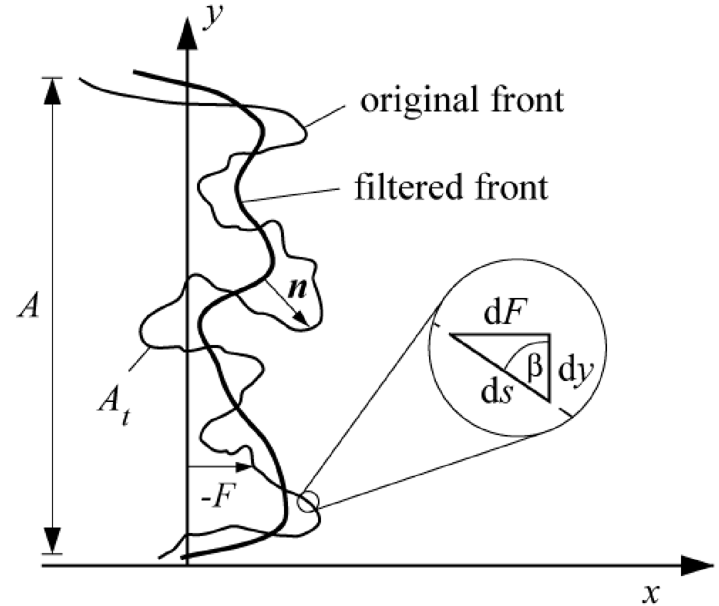
To resolve this conceptual difficulty one may define a filtered flame surface by eliminating large wave-number contributions in a Fourier representation of the original surface, so that in a projection of the original surface on the filtered surface no multiple crossings occur.

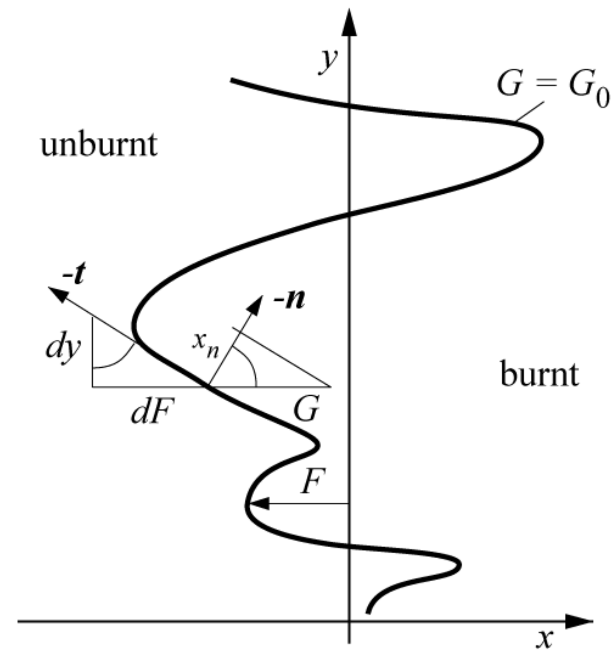
This is also shown in the illustration.

The normal co-ordinate x_n on the filtered surface then corresponds to x in a previous figure repeated here.

This shows that the ansatz Assuming a single valued function of x is again applicable.

A successive filtering procedure can then be applied, so that the flame surface area ratio is related to $\bar{\sigma} = \overline{|\nabla G|}$ at each level of filtering.





Within a given section dy

$$\frac{\mathrm{d}S}{\mathrm{d}y} = \sigma$$

is then replaced by

$$\frac{\mathrm{d}S^{\nu}}{\mathrm{d}S^{\nu+1}} = \frac{\sigma^{\nu}}{\sigma^{\nu+1}}$$

where ν is an iteration index of successive filtering.

The quantities $\mathrm{d}S^0$ and σ^0 correspond to the instantaneous differential flame surface area dS and the respective gradient σ, and $\mathrm{d}S^1$ and σ^1 to those of the first filtering level.

At $\bar{\sigma} = \overline{|\nabla G|}$ iteration one has $\mathrm{d}S^1/\mathrm{d}S^2=\sigma^1/\sigma^2$ and so on.

At the last filtering level for $\nu \to \infty$ the flame surface becomes parallel to the y-co-ordinate, so that $dS^{\infty} = dy$ and $\sigma^{\infty} = 1$.

Canceling all intermediate iterations we obtain again

$$\frac{dS}{dy} = \sigma$$

This analysis assumes that the original flame surface is unique and continuous.

We now want to derive a modeled equation for the flame surface area ratio in order to determine the turbulent burning velocity.

An equation for σ can be derived from

$$\rho\frac{\partial G}{\partial t} + \rho \boldsymbol{v} \cdot \boldsymbol{\nabla} G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

For illustration purpose we assume constant density and constant values of s_L^0 and D.

Applying the Nabla-operator to both sides of

$$\rho \frac{\partial G}{\partial t} + \rho \boldsymbol{v} \cdot \boldsymbol{\nabla} G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

multiplying this with $-\boldsymbol{n} = \boldsymbol{\nabla} G / \sigma$

one obtains

$$\frac{\partial \sigma}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \sigma = -\boldsymbol{n} \cdot \boldsymbol{\nabla} \boldsymbol{v} \cdot \boldsymbol{n}\, \sigma + s_L^0 (\kappa\sigma + \boldsymbol{\nabla}^2 G) + D\, \boldsymbol{n} \cdot \boldsymbol{\nabla} (\kappa\sigma)$$

The terms on the l.h.s. of this equation describe the unsteady change and convection of σ.

The first term on the r.h.s accounts for straining by the flow field which amounts to a production of flame surface area.

$$\frac{\partial \sigma}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \sigma = -\boldsymbol{n} \cdot \boldsymbol{\nabla} \boldsymbol{v} \cdot \boldsymbol{n}\, \sigma + s_L^0 (\kappa \sigma + \boldsymbol{\nabla}^2 G) + D\, \boldsymbol{n} \cdot \boldsymbol{\nabla} (\kappa \sigma)$$

The second term on the r.h.s. containing the laminar burning velocity has a similar effect as kinematic restoration has in the variance equation.

The last term is proportional to *D* and its effect is similar to that of scalar dissipation in the variance equation.

In order to derive a model equation for the mean value of σ we could, in principle, take the appropriate averages.

There is, however, no standard two-point closure procedure of such an equation, as there is none for deriving the ε-equation from an equation for the viscous dissipation.

Therefore another approach was adopted in Peters (1999):

The scaling relations between $\bar{\sigma}$, $\tilde{k}$, $\tilde{\varepsilon}$, $\widetilde{G''^2}$ were used separately in both regimes to derive the quation from a combination of the $\tilde{k}$, $\tilde{\varepsilon}$, $\widetilde{G''^2}$ -equations.

The resulting equations contain the local change and convection of $\bar{\sigma}$, a production term by mean gradients and another due to turbulence.

However, each of them contains a different sink term:
In the corrugated flamelets regime the sink term is proportional to $s_L^0\,\bar{\sigma}$
and in the thin reaction zones it is proportional to $D\bar{\sigma}^3$

Since proportionality between the turbulent burning velocity and $\bar{\sigma}$ is valid only in the limit of large values of v'/s_L, it accounts only for the increase of the flame surface area ratio due to turbulence, beyond the laminar value

$$\bar{\sigma} = |\nabla \widetilde{G}| \quad \text{for} \quad v' \to 0$$

We will therefore simply add the laminar contribution and write

$$\bar{\sigma} = |\nabla \widetilde{G}| + \bar{\sigma}_t$$

where $\bar{\sigma}_t$ now is the turbulent contribution to the flame surface area ratio.

The resulting model equation for the unconditional quantity $\bar{\sigma}_t$ from Peters (1999), that covers both regimes, is written as

$$
\begin{aligned}
\bar{\rho}\frac{\partial \bar{\sigma}_t}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\bar{\sigma}_t \quad = \quad & \boldsymbol{\nabla}_{||}\cdot(\bar{\rho}D_t\boldsymbol{\nabla}_{||}\,\bar{\sigma}_t) \\
& + c_0\bar{\rho}\frac{(-\widetilde{\boldsymbol{v}''\boldsymbol{v}''}) : \boldsymbol{\nabla}\tilde{\boldsymbol{v}}}{\tilde{k}}\,\bar{\sigma}_t + c_1\bar{\rho}\frac{D_t(\boldsymbol{\nabla}\widetilde{G})^2}{\widetilde{G''^2}}\,\bar{\sigma}_t \\
& - c_2\bar{\rho}\frac{s_L^0\bar{\sigma}_t^2}{(\widetilde{G''^2})^{1/2}} - c_3\bar{\rho}\frac{D\bar{\sigma}_t^3}{\widetilde{G''^2}}.
\end{aligned}
$$

The terms on the r.h.s. represent the local change and convection.

$$
\begin{aligned}
\bar{\rho}\frac{\partial\bar{\sigma}_t}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\bar{\sigma}_t \;&=\; \boldsymbol{\nabla}_{||}\cdot(\bar{\rho}D_t\boldsymbol{\nabla}_{||}\,\bar{\sigma}_t) \\
&\quad + c_0\bar{\rho}\frac{(-\widetilde{\boldsymbol{v}''\boldsymbol{v}''}) : \boldsymbol{\nabla}\tilde{\boldsymbol{v}}}{\tilde{k}}\,\bar{\sigma}_t + c_1\bar{\rho}\frac{D_t(\boldsymbol{\nabla}\tilde{G})^2}{\widetilde{G''^2}}\,\bar{\sigma}_t \\
&\quad - c_2\bar{\rho}\frac{s_L^0\bar{\sigma}_t^2}{(\widetilde{G''^2})^{1/2}} - c_3\bar{\rho}\frac{D\bar{\sigma}_t^3}{\widetilde{G''^2}}.
\end{aligned}
$$

The second term models production of the flame surface area ratio due to mean velocity gradients.

The constant $c_0 = c_{\varepsilon 1} - 1 = 0.44$ originates from the ε-equation Eq.

The last three terms represent turbulent production, kinematic restoration and scalar dissipation of the flame surface area ratio, respectively, and correspond to the three terms on the r.h.s of

$$
\frac{\partial\sigma}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\sigma = -\boldsymbol{n}\cdot\boldsymbol{\nabla}\boldsymbol{v}\cdot\boldsymbol{n}\,\sigma + s_L^0(\kappa\sigma + \boldsymbol{\nabla}^2 G) + D\,\boldsymbol{n}\cdot\boldsymbol{\nabla}(\kappa\sigma)
$$

Wenzel (1998, 2000) has performed DNS of the constant density *G*-equation in an isotropic homogeneous field of turbulence (cf. also Wenzel (1997), which show that the strain rate at the flame surface is statistically independent of σ and that the mean strain on the flame surface is always negative.

This leads to the closure model

$$\overline{-\boldsymbol{n}\cdot\boldsymbol{\nabla}\boldsymbol{v}\cdot\boldsymbol{n}\,\sigma} = c_1\,\frac{v'}{\ell}\,\bar{\sigma}$$

The modeling constant c_1 was determined by Wenzel (2000) as $c_1=4.63$.

In order to determine the remaining constants c_2 and c_3 we consider again the steady planar flame.

In the planar case the convective term on the l.h.s. and the turbulent transport term on the r.h.s. of

$$
\begin{aligned}
\bar{\rho}\frac{\partial \bar{\sigma}_t}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\bar{\sigma}_t \;=\; & \boldsymbol{\nabla}_{||}\cdot(\bar{\rho}D_t\boldsymbol{\nabla}_{||}\,\bar{\sigma}_t) \\
& + c_0\bar{\rho}\frac{(-\widetilde{\boldsymbol{v}''\boldsymbol{v}''}):\boldsymbol{\nabla}\tilde{\boldsymbol{v}}}{\tilde{k}}\,\bar{\sigma}_t + c_1\bar{\rho}\frac{D_t(\boldsymbol{\nabla}\tilde{G})^2}{\widetilde{G''^2}}\,\bar{\sigma}_t \\
& - c_2\bar{\rho}\frac{s_L^0\bar{\sigma}_t^2}{(\widetilde{G''^2})^{1/2}} - c_3\bar{\rho}\frac{D\bar{\sigma}_t^3}{\widetilde{G''^2}}.
\end{aligned}
$$

vanish and since the flame is steady, so does the unsteady term.

The production term due to velocity gradients, being in general much smaller than production by turbulence, may also be neglected.

In terms of conditional quantities defined at the mean flame front, using the definition

$$\ell_{F,t} = \left. \frac{(\widetilde{G''^2}(\boldsymbol{x},t))^{1/2}}{|\nabla \widetilde{G}|} \right|_{\widetilde{G}=G_0}$$

for the flame brush thickness, the balance of turbulent production, kinematic restoration and scalar dissipation leads to the algebraic equation

$$c_1 \frac{D_t}{\ell_{F,t}^2} - c_2 \frac{s_L^0}{\ell_{F,t}} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - c_3 \frac{D}{\ell_{F,t}^2} \frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} = 0.$$

In the limit of a steady state planar flame the flame brush thickness is proportional to the integral length scale.

In that limit we may therefore use:

$$\ell_{F,t} = b_2 \ell$$

Therefore

$$c_1 \frac{D_t}{\ell_{F,t}^2} - c_2 \frac{s_L^0}{\ell_{F,t}} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - c_3 \frac{D}{\ell_{F,t}^2} \frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} = 0.$$

$$\downarrow$$

$$c_1 \frac{D_t}{\ell^2} - c_2 b_2 \frac{s_L^0}{\ell} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - c_3 \frac{D}{\ell^2} \frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} = 0$$

This covers two limits: In the corrugated flamelets regime the first two terms balance, while in the thin reaction zones regime there is a balance of the first and the last term.

Using $D_t = a_4 v' \ell$ it follows for the corrugated flamelets regime

$$c_2 b_2 s_L^0 \bar{\sigma}_t = a_4 c_1 v' |\nabla \widetilde{G}|$$

Experimental data (cf. Abdel-Gayed and Bradley, 1981) for fully developed turbulent flames in that regime show that for Re $\to \infty$ and $v'/s_L \to \infty$ the turbulent burning velocity is $s^0_T = b_1 v'$ where $b_1 = 2.0$.

In Peters (2000) it is shown that the turbulent burning velocity s_T is related to the mean flame surface area ratio as

$$(\rho s_T^0) \mid \nabla \tilde{G} \mid = (\rho s_L^0)\, \bar{\sigma}_t .$$

If the turbulent and laminar burning velocities are evaluated at a constant density it follows in that limit that

$$s_L^0 \bar{\sigma}_t = b_1 v' |\nabla \widetilde{G}|$$

Therefore one obtains by comparison with $c_2 b_2 s_L^0 \bar{\sigma}_t = a_4 c_1 v' |\nabla \widetilde{G}|$

$$b_1 b_2 c_2 = a_4 \, c_1$$

which leads to c_2=1.01 using the constants from the table.

constant	equation	suggested value	origin
a_1	$\tilde{\varepsilon} = a_1 v'^3/\ell$	0.37	Bray (1990)
a_2	$\tilde{k} = a_2 v'^2$	1.5	definition
a_3	$\tau = a_3 \ell / v'$	4.05	$\tau = \tilde{k}/\tilde{\varepsilon}$
a_4	$D_t = a_4 v' \ell$	0.78	$D_t = \nu_t/0.7$
b_1	$s_T = b_1 v'$	2.0	experimental data
b_2	$\ell_{F,t} = b_2 \ell$	1.78	$(2a_3a_4/c_s)^{1/2}$
b_3	$s_T^0/s_L^0 = b_3(D_t/D)^{1/2}$	1.0	Damköhler (1940)
c_0	$c_0 = c_{\varepsilon 1} - 1$	0.44	standard value
c_1	Eq. (13.18)	4.63	DNS
c_2	Eq. (13.18)	1.01	$a_4c_1/(b_1b_2)$
c_3	Eq. (13.18)	4.63	$c_1 = c_3$
c_s	Eq. (12.47)	2.0	spectral closure

Similarly, for the thin reaction zones regime we obtain

$$c_3 \, D \bar{\sigma}_t^2 = c_1 \, D_t |\nabla \widetilde{G}|^2$$

This must be compared with

$$\frac{s_T}{s_L} \sim \left(\frac{D_t}{D} \right)^{1/2}$$

written as

$$\frac{s_T^0}{s_L^0} = b_3 \left(\frac{D_t}{D} \right)^{1/2}$$

Damköhler (1940) believed that for the constant of proportionality $b_3 = 1$.

Wenzel (1997) has performed DNS simulations similar to those of Kerstein et al. (1988) and finds $b_3 = 1.07$ which is very close to Damköhler's suggestion.

Then, with

$$\ell_F = D / s_L^0$$

and the relations in the table the equation

$$c_1 \frac{D_t}{\ell^2} - c_2\, b_2\, \frac{s_L^0}{\ell} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - c_3 \frac{D}{\ell^2} \frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} = 0$$

leads to the quadratic equation

$$\frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} + \frac{a_4 b_3^2}{b_1} \frac{\ell}{\ell_F} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - a_4 b_3^2 \frac{v' \ell}{s_L^0 \ell_F} = 0$$

The difference Δs between the turbulent and the laminar burning velocity is

$$\Delta s = s_T^0 - s_L^0 = s_L^0 \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|}$$

Taking only the positive root in the solution of

$$\frac{\bar{\sigma}_t^2}{|\nabla \widetilde{G}|^2} + \frac{a_4 b_3^2}{b_1} \frac{\ell}{\ell_F} \frac{\bar{\sigma}_t}{|\nabla \widetilde{G}|} - a_4 b_3^2 \frac{v' \ell}{s_L^0 \ell_F} = 0$$

this leads to the algebraic expression for Δs

$$\frac{\Delta s}{s_L^0} = -\frac{a_4 b_3^2}{2\, b_1} \frac{\ell}{\ell_F} + \left[\left(\frac{a_4 b_3^2}{2\, b_1} \frac{\ell}{\ell_F} \right)^2 + a_4 b_3^2 \frac{v' \ell}{s_L^0 \ell_F} \right]^{1/2}$$

The modeling constants used in the final equations and are summarized in a table.

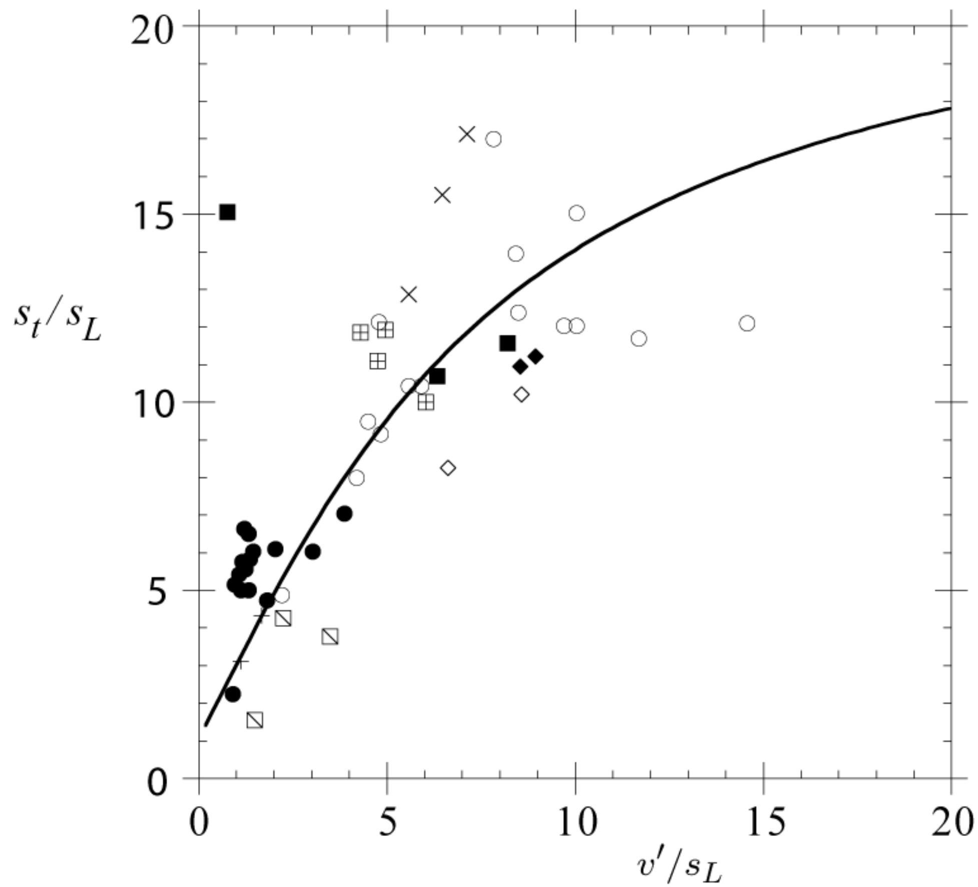
constant	equation	suggested value	origin
a_1	$\tilde{\varepsilon} = a_1 v'^3/\ell$	0.37	Bray (1990)
a_2	$\tilde{k} = a_2 v'^2$	1.5	definition
a_3	$\tau = a_3 \ell / v'$	4.05	$\tau = \tilde{k}/\tilde{\varepsilon}$
a_4	$D_t = a_4 v' \ell$	0.78	$D_t = \nu_t / 0.7$
b_1	$s_T = b_1 v'$	2.0	experimental data
b_2	$\ell_{F,t} = b_2 \ell$	1.78	$(2 a_3 a_4 / c_s)^{1/2}$
b_3	$s_T^0 / s_L^0 = b_3 (D_t/D)^{1/2}$	1.0	Damköhler (1940)
c_0	$c_0 = c_{\varepsilon 1} - 1$	0.44	standard value
c_1	Eq. (13.18)	4.63	DNS
c_2	Eq. (13.18)	1.01	$a_4 c_1 / (b_1 b_2)$
c_3	Eq. (13.18)	4.63	$c_1 = c_3$
c_s	Eq. (12.47)	2.0	spectral closure

Note that b_1 is the only constant that has been adjusted using experimental data from turbulent burning velocity while the constant b_3 was suggested by Damköhler (1940). The constant c_1 was obtained from DNS and all other constants are related to constants in standard turbulence models.

If

$$\frac{\Delta s}{s_L^0} = -\frac{a_4 b_3^2}{2\,b_1}\frac{\ell}{\ell_F} + \left[\left(\frac{a_4 b_3^2}{2\,b_1}\frac{\ell}{\ell_F}\right)^2 + a_4 b_3^2 \frac{v'\ell}{s_L^0 \ell_F}\right]^{1/2}, \quad \mathrm{Re}_t = \frac{v'\ell}{s_L^0 \ell_F}$$

is compared with experimental data as in the burning velocity diagram, the turbulent Reynolds number appears as a parameter.



From the viewpoint of turbulence modeling this seems disturbing, since in free shear flows any turbulent quantity should be independent of the Reynolds number in the large Reynolds number limit.

The apparent Reynolds number dependence turns out to be an artifact, resulting from the normalization of Δs by $s_L{}^0$, which is a molecular quantity whose influence should disappear in the limit of large Reynolds numbers and large values of v'/s_L.

If the burning velocity difference Δs is normalized by v' rather than by $s_L{}^0$, it may be expressed as a function of the turbulent Damköhler number

$$\mathrm{Da}_t = s_L^0 \ell / v' \ell_F$$

instead, and one obtains the form

$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2}$$

This is Reynolds number independent and only a function of a single parameter, the turbulent Damköhler number.

In the limit of large scale turbulence

$$\ell/\ell_F \to \infty \quad \text{or} \quad \mathsf{Da}_t \to \infty$$

it becomes Damköhler number independent.

In the small scale turbulence limit

$$\ell/\ell_F \to 0 \quad \text{or} \quad \mathsf{Da}_t \to 0$$

it is proportional to the square root of the Damköhler number.

A Damköhler number scaling has also been suggested by Gülder (1990) who has proposed

$$\frac{\Delta s}{v'} = 0.62\,\mathsf{Da}_t^{1/4}$$

as an empirical fit to a large number of burning velocity data.

A similar correlation with the same Damköhler number dependence, but a constant of 0.51 instead of 0.62 was proposed by Zimont and Lipatnikov (1995).

Bradley et al. (1992), pointing at flame stretch as a determinant of the turbulent burning velocity, propose to use the product of the Karlovitz stretch factor *K* and the Lewis number as the appropriate scaling parameter

$$\frac{s_T^0}{v'} = 0.88(K\,\mathrm{Le})^{-0.3}$$

where the Karlovitz stretch factor is related to the Damköhler number by

$$K = 0.157\frac{v'}{s_L^0}\mathrm{Da}_t^{-1/2}.$$

This leads to the expression

$$\frac{\Delta s}{v'} = 1.53\left(\frac{s_L^0}{v'}\right)^{0.3}\mathrm{Da}_t^{0.15}\mathrm{Le}^{-0.3} - \frac{s_L^0}{v'}$$

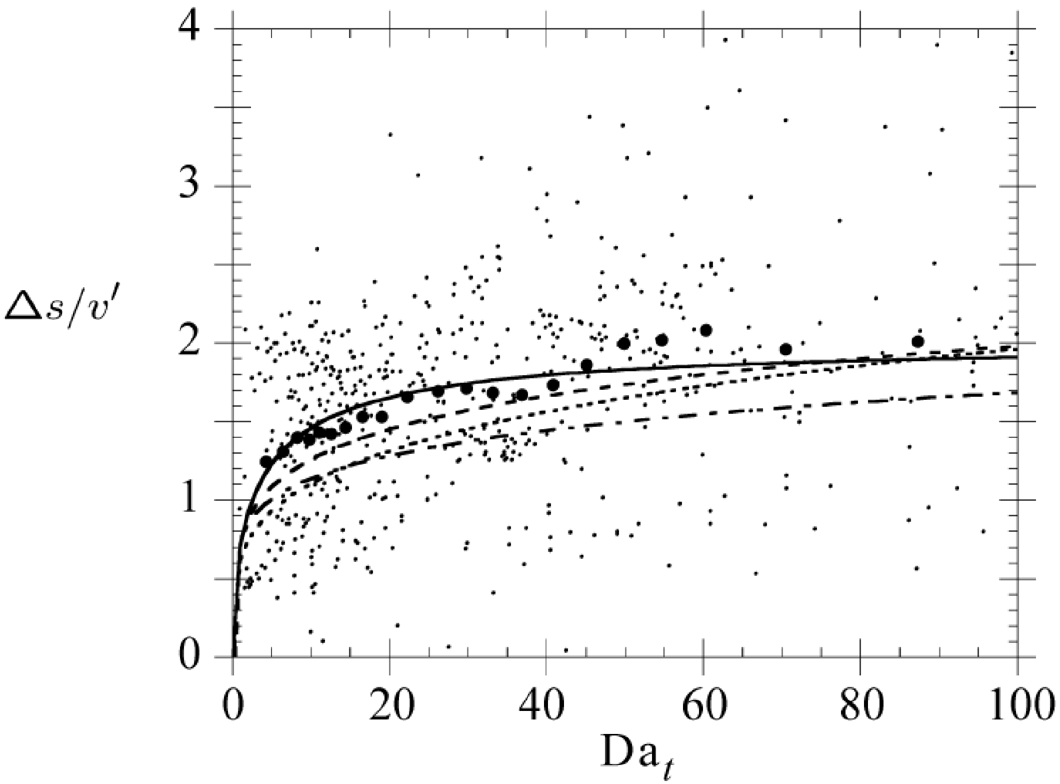
The correlations

$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\,b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\,b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2}$$

$$\frac{\Delta s}{v'} = 0.62\,\mathrm{Da}_t^{1/4}$$

$$\frac{\Delta s}{v'} = 1.53\left(\frac{s_L^0}{v'}\right)^{0.3}\mathrm{Da}_t^{0.15}\mathrm{Le}^{-0.3} - \frac{s_L^0}{v'}$$

are compared among each other and with data from the experimental data collection.



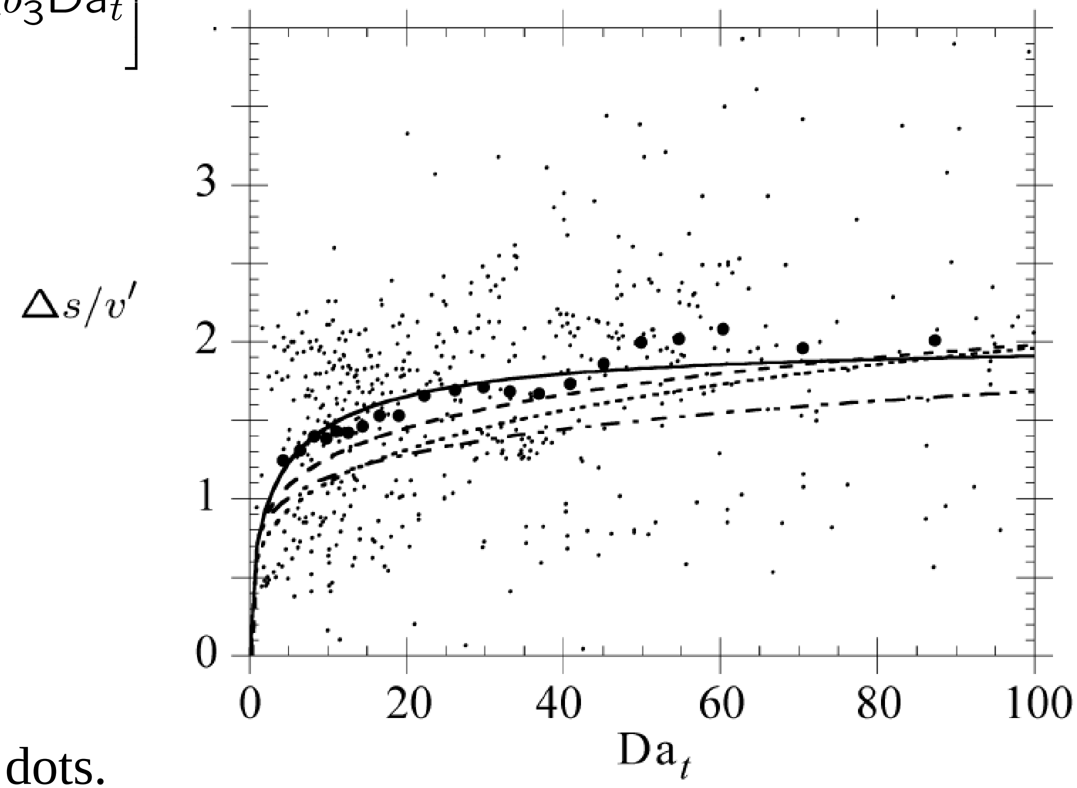
The data points show a large scatter, which is due to the fact that the experimental conditions were not always well defined.

Since unsteady effects have been neglected in deriving

$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2}$$

only data based on steady state experiments were retained from this collection.

These 598 data points and their averages within fixed ranges of the turbulent Damköhler number are shown as small and large dots.



Lewis number effects are often found to influence the turbulent burning velocity (cf. Abdel-Gayed et al. ,1984).

This is supported by two-dimensional numerical simulations by Ashurst et al. (1987) and Haworth and Poinsot (1992), and by three-dimensional simulations by Rutland and Trouvé (1993), all being based on simplified chemistry.

There are additional experimental data on Lewis number effects in turbulent flames at moderate intensities by Lee et al. (1993, 1995).

Appendix

The apparently fractal geometry of the flame surface and the fractal dimension that can be extracted from it, also has led to predictions of the turbulent burning velocity.

Gouldin (1987) has derived a relationship between the flame surface area ratio A_T/A and the ratio of the outer and inner cut-off of the fractal range:

$$\frac{A_T}{A} = \left(\frac{\varepsilon_0}{\varepsilon_i}\right)^{D_f - 2}$$

Here D_f is the fractal dimension.

While there is general agreement that the outer cut-off scale ε_0 should be the integral length scale, there are different suggestions by different authors concerning the inner cut-off scale ε_i.

While Peters (1986) and Kerstein (1988) propose, based on theoretical grounds that ε_i should be the Gibson scale, most experimental studies reviewed by Gülder (1990) and Gülder et al. (1999) favour the Kolmogorov scale η or a multiple thereof.

As far as the fractal dimension is concerned, the reported values in the literature also vary considerably.

Kerstein (1988) has suggested the value D_f=7/3, which, in combination with the Gibson scale as the inner cut-off, is in agreement with Damköhler's result in the corrugated flamelets regime:

$$s_T \sim v'$$

This is easily seen by inserting

$$\frac{A_T}{A} = \left(\frac{\varepsilon_0}{\varepsilon_i}\right)^{D_f - 2}$$

into

$$\frac{s_T}{s_L} = \frac{A_T}{A}$$

using

$$\ell_G = \left(\frac{s_L}{v'}\right)^3$$

On the other hand, if the Kolmogorov scale is used as inner cut-off, one obtains

$$s_T/s_L \sim \mathrm{Re}^{1/4}$$

as Gouldin (1987) has pointed out.

This power law dependence seems to have been observed by Kobayashi et al. (1998) in high pressure flames.

Gülder (1999) shows in his recent review that most of the measured values for the fractal dimension are smaller than D_f=7/3.

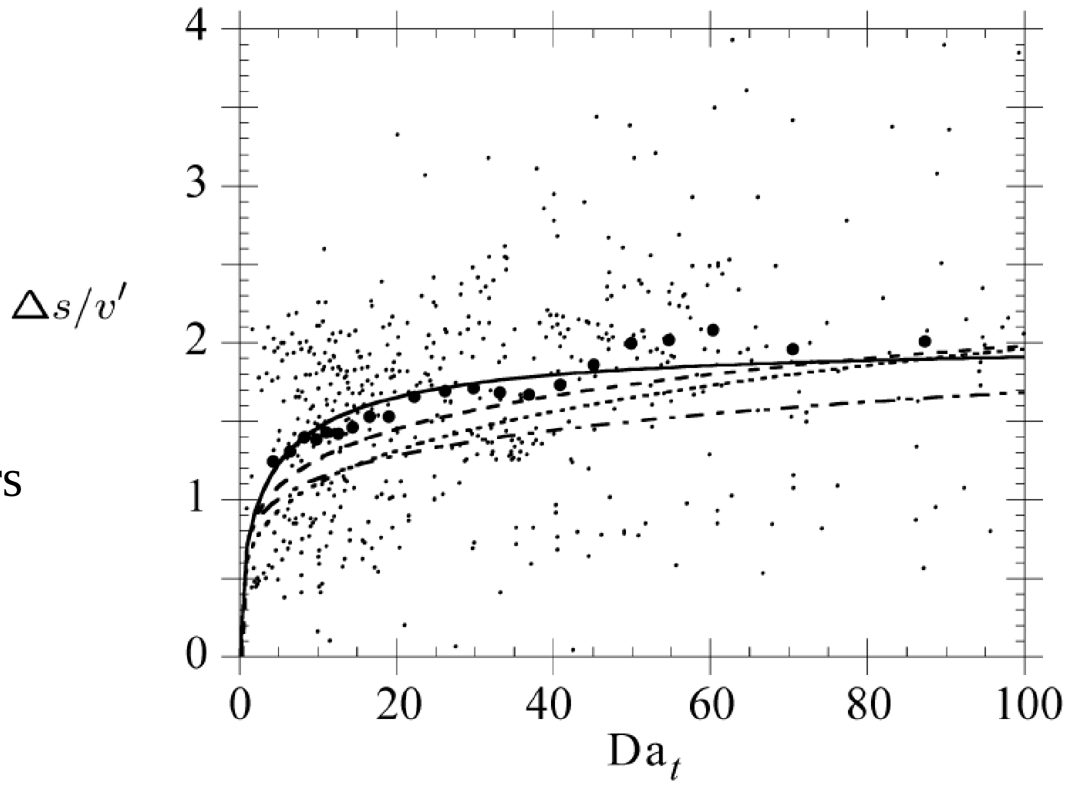
He concludes that the available fractal parameters are not capable of correctly predicting the turbulent burning velocity.

In order to make such a comparison possible, the Lewis number was assumed equal to unity in

$$\frac{\Delta s}{v'} = 1.53 \left(\frac{s_L^0}{v'}\right)^{0.3} \mathrm{Da}_t^{0.15} \mathrm{Le}^{-0.3} - \frac{s_L^0}{v'}$$

and two values of v'/s^0_L were chosen.

As a common feature of all three correlations one may note that $\Delta s/v'$ strongly increases in the range of turbulent Damköhler numbers up to 10, but levels off for larger turbulent Damköhler numbers.



The correlation

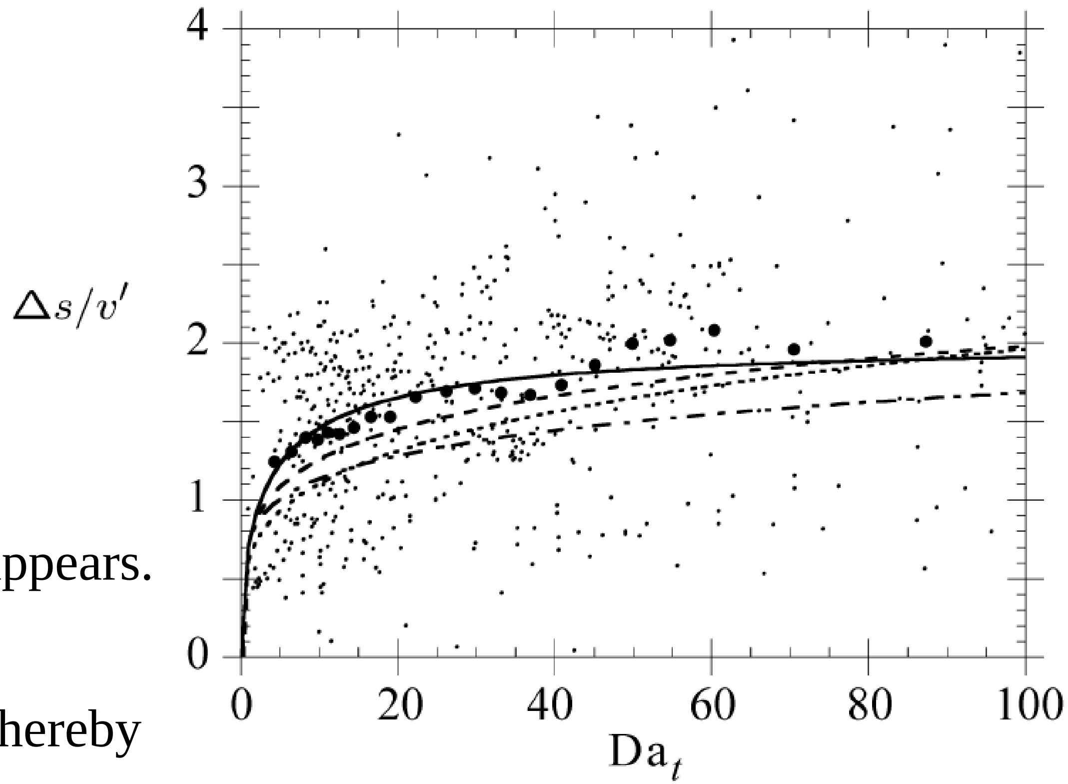
$$\frac{\Delta s}{v'} = -\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t + \left[\left(\frac{a_4 b_3^2}{2\, b_1}\mathrm{Da}_t\right)^2 + a_4 b_3^2 \mathrm{Da}_t\right]^{1/2}$$

is the only one that predicts Damköhler number independence in the large Damköhler number limit.

The model for the turbulent burning velocity derived here is based on

$$\rho\frac{\partial G}{\partial t} + \rho \boldsymbol{v}\cdot\boldsymbol{\nabla} G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

in which the mass diffusivity D rather than the Markstein diffusivity appears.



As a consequence, flame stretch and thereby the Lewis number effects do not enter into the model.