Lecture 15

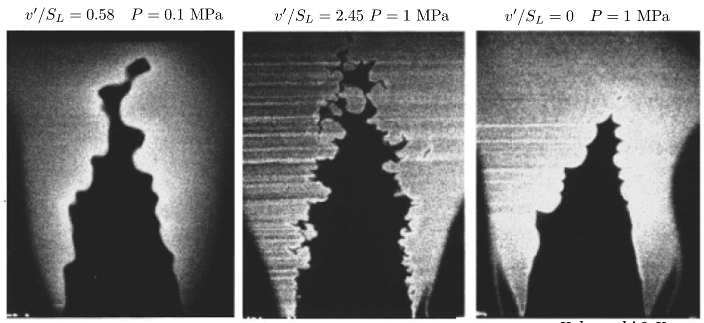
The Turbulent Burning Velocity

The **turbulent burning velocity** is defined as the average rate of propagation of the flame through the turbulent premixed gas mixture.

In the laminar case, solutions of the governing equations of the form $f(x - S_L t)$ implies that the whole structure propagates to the left with a speed S_L - the laminar flame speed.

In the turbulent case it is not clear that a turbulent speed is indeed a well-defined notion. The assumption that it exists could be supported by the observations that turbulent flames propagate a well-defined distance, and if in steady turbulent flows they possess a measurable inclination angle.

But this may only apply to the wrinkled flamelet and reaction sheet regimes; it is less obvious, however, that it applies in the distributed-reaction regime where the basic structure of a flame may no longer exist.



Kobayashi & Kawazoe, 2000

Images of CH_4-air Bunsen-type flames

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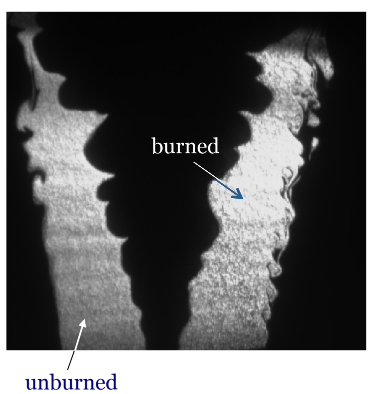

CH_4-air inverted conical flame in a low-intensity turbulent flow

Sattler et al. PCI, 2002

The figure displays the image of a wrinkled methane-air V-flame in a turbulent flow of low intensity. The image is bright at the reactants (as a result of Mie scattering of laser light off the micrometer sized silicone oil droplets that were added to the fresh mixture and consumed in the reaction zone), and dark in the products.

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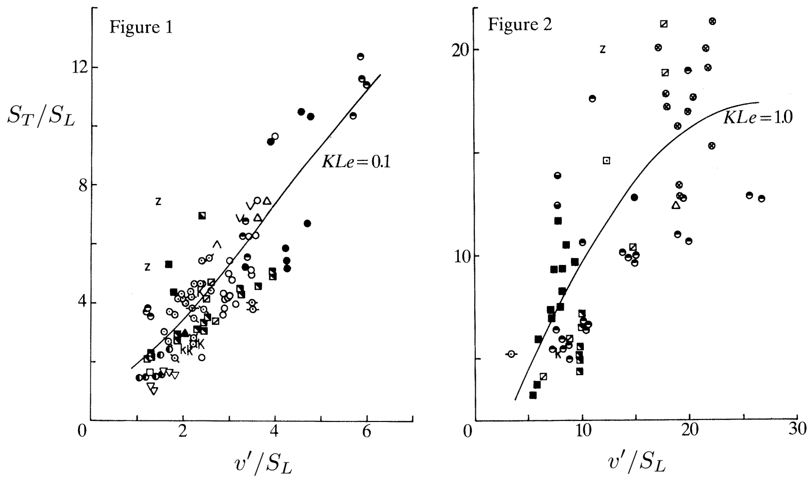

The experimental data collected from various studies exhibit a wide scatter (here K is the Karlovitz number).

Bradley et al., Proc. Royal Soc. 1992

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Damköhler's Conjecture

Z. Elektrochem., 1940.

Damköhler identiifed two distinct limiting regimes:
(i) a *small scale turbulence* regime where small eddies interact with the transport mechanisms within the flame.
(ii) a *large scale turbulence*, where the flame is thin compared to the smallest turbulence scale, turbulence-flame interaction is purely kinematic

These two regimes correspond, in the present terminology to the thin reaction zone and corrugated flamelet regimes, respectively.

The expressions for the turbulent burning velocity S_T proposed by Damköhler, and the modifications due to Shelkin (NACA TM 1110, 1947) are discussed next.

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In the small scale turbulence (reaction sheet) regime, the turbulent eddies modify the transport processes in the preheat zone.
By analogy to the scaling relation for the laminar burning velocity $S_L \sim \sqrt{\mathcal{D}_{\text{th}}/t_f}$ he proposed that $S_T \sim \sqrt{\mathcal{D}_{\text{T}}/t_f}$ where $\mathcal{D}_{\text{T}}$ is the turbulent diffusivity. Hence

$$\frac{S_T}{S_L} = \sqrt{\frac{\mathcal{D}_{\text{T}}}{\mathcal{D}_{\text{th}}}}$$

Since the turbulent diffusivity $\mathcal{D}_{\text{T}} \sim v'\ell$ and $\mathcal{D}_{\text{th}} \sim S_L l_f$, we have

$$\boxed{\frac{S_T}{S_L} = \sqrt{\frac{v'}{S_L}\frac{\ell}{l_f}}}$$

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In the large scale turbulence (corrugated flamelet) regime, Damköhler resorted to a geometrical argument, with analogy to a Bunsen flame.

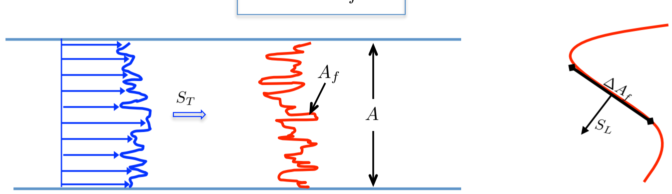
The turbulent speed S_T, in the one-dimensional configuration shown in the figure, is the incoming mean flow velocity. Then

$$\dot{m} = \rho_u A S_T$$

Since all the reactants pass through the wrinkled flame, the mass flow rate can be also calculated from the total contributions of the mass flowing through the segments ΔA_f comprising the wrinkled flame, assuming that each segment propagates normal to itself at the laminar flame speed S_L. Thus

$$\dot{m} = \rho_u A_f S_L$$

$$\boxed{A S_T = A_f S_L}$$



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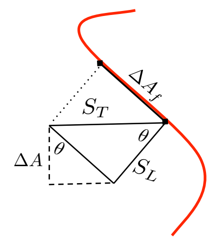
The interaction between the wrinkled flame front and the turbulent flow field is assumed purely kinematic. Using the geometrical analogy with a Bunsen flame, the velocity component normal to the surface ΔA_f is the laminar flame speed S_L, while that tangential to the surface is due to the turbulent eddy and is proportional to the turbulent intensity v'. Then

$$\frac{\Delta A_f}{\Delta A} = \frac{1}{\cos\theta} = \sqrt{1+\tan^2\theta}$$

$$\tan\theta = \frac{v'}{S_L}$$

$$S_T = S_L\sqrt{1+\left(\frac{v'}{S_L}\right)^2}$$



$$S_T \sim S_L\left[1+\frac{1}{2}\left(\frac{v'}{S_L}\right)^2\right] \qquad \text{for } v' \ll S_L$$

$$S_T \sim v' \qquad \text{for } v' \gg S_L$$

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More generally, let $F(\mathbf{x},t)=0$ represents the flame sheet

$$\frac{A_f}{A} = \overline{\left(\frac{|\nabla F|}{|\nabla F\cdot\mathbf{e}|}\right)}$$

where $\mathbf{e}$ is a unit vector in the direction of propagation and an suitable average is taken.

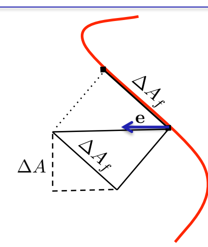
If the flame surface does not fold back on itself[1], it may be represented explicitly in the form

$$x = f(y,z,t)$$

with x in the direction of propagation; along $\mathbf{e}$

$$\Rightarrow \qquad \frac{A_f}{A} = \overline{\sqrt{1+|\nabla f|^2}}$$

$$S_T = S_L\,\overline{\sqrt{1+|\nabla f|^2}}$$



the area of a surface element on the flame sheet $\Delta A_f = |\nabla F|$

the projection in the direction of propagation is $\Delta A = |\nabla F\cdot\mathbf{e}|$

[1] For more general surfaces, see the discussion in Williams (1985) and Peters (2000)

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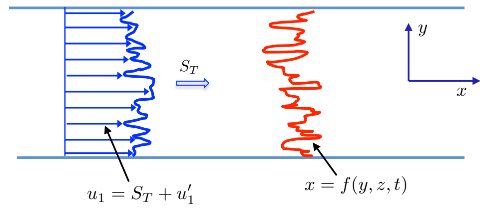
This expression, for the turbulent flame speed, can be also obtained from the hydrodynamic theory, which applies to laminar as well as turbulent flow.

We consider the flame to be in statistical steady state within an isotropic homogeneous turbulent incident field

$$\mathbf{v} = (S_T + u',\, u_2',\, u_3')$$

$$\overline{u'} = 0$$

choose $\overline{f} = 0$



By definition, the flame speed is given by $S_f = \mathbf{v} \cdot \mathbf{n} - V_f$ where velocities are evaluated just ahead of the flame. Then

$$S_T + u_1' - u_2' f_y - u_3' f_z - f_t = S_f \sqrt{1 + f_y^2 + f_z^2)}$$

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$$S_T + u_1' - u_2' f_y - u_3' f_z - f_t = S_f \sqrt{1 + f_y^2 + f_z^2)}$$

upon averaging along the transverse directions y, z and in time,

$\overline{f_t} = 0$: the flame is in statistical steady state

$\overline{u_2' f_y} = \overline{u_3' f_z} = 0$: f_y and $-f_y$ are statistically identical (similarly for f_z)

$$\boxed{S_T = \overline{S_f \sqrt{1 + f_y^2 + f_z^2}}}$$

This equation highlights two main contributions to the turbulent propagation speed, namely a contribution due to flame area increase (as in Damköhler conjecture) and one due to S_f, the displacement speed of the front relative to the incoming flow.

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1. Effect of mean area increase on the turbulent burning velocity

Neglecting the effect of flame stretch, $S_f = S_L$ and

$$S_T = S_L \sqrt{1 + f_y^2 + f_z^2}$$

as in Damköhler's proposition.

It remains to relate the average area ratio to the properties of the incoming turbulence.

- Approximate "asymptotic" relations
- Numerical calculations employing the hydrodynamic theory (two-dimensional "turbulence")

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The leading order solution of the flame structure (in the asymptotic hydrodynamic model), for weak transverse gradients ($f_y \ll 1$, $f_z \ll 1$), shows that $f_t = u_1'/S_L$ i.e., the flame is advected upstream by the velocity perturbations.

$$S_T = S_L \sqrt{1 + \left[\frac{\partial}{\partial y}\int u_1' dt\right]^2 + \left[\frac{\partial}{\partial z}\int u_1' dt\right]^2}$$

If Taylor hypothesis[1] is invoked, and isotropy assumed, the statistics of the transverse gradients could be related to the statistics of the non dimensional longitudinal fluctuation u_1'/S_T, so that

$$S_T = S_L \sqrt{1 + 2\left(\frac{u_1'}{S_T}\right)^2}$$

which may be evaluated if the *pdf* of u_1'/S_T is known, or with further simplifications.

[1] Taylor hypothesis provides a relation between temporal and spatial correlations, in the limit of weak turbulence and when there is a predominantly mean flow in one direction.

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Some additional details of slide 14 are given in the next three slides

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The leading order solution of the flame structure (in the asymptotic hydrodynamic model), for weak transverse gradients ($f_y \ll 1$, $f_z \ll 1$), shows that $f_t = u_1'/S_L$ i.e., the flame is advected upstream by the velocity perturbations.

$$S_T = S_L \sqrt{1 + \left[\frac{\partial}{\partial y}\int u_1' dt\right]^2 + \left[\frac{\partial}{\partial z}\int u_1' dt\right]^2}$$

If the streamwise Eulerian displacement of the fluid elements by turbulence is $a = \int u_1' dt$, we obtain

$$S_T = S_L \sqrt{1 + a_y^2 + a_z^2)}$$

note that within the current approximation a determines the extent of longitudinal motion of the flame sheet, and the gradients a_y and a_z the increase in area of the flame sheet.

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If Taylor hypothesis[1] is invoked,

$$\frac{\partial}{\partial y}\int u_1' dt = \frac{1}{S_T}\frac{\partial}{\partial y}\int u_1' dx = \frac{1}{S_T}\int \frac{\partial u_1'}{\partial y} dx$$

and if the turbulence is isotropic, then

$$\frac{1}{S_T}\int \frac{\partial u_1'}{\partial y} dx = \frac{1}{S_T}\int \frac{\partial u_1'}{\partial x} dx = \frac{u_1'}{S_T}$$

$$S_T = S_L \sqrt{1 + 2\overline{\left(\frac{u_1'}{S_T}\right)^2}}$$

[1] Taylor hypothesis provides a relation between temporal and spatial correlations, in the limit of weak turbulence and when there is a predominantly mean flow in one direction. Specifically, If the mean flow is in the x-direction, say, and $u'/\bar{u} \ll 1$, then

$$\frac{\partial}{\partial t} = \bar{u}\frac{\partial}{\partial x}$$

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Additional simplifications (Williams, 1985) yields an expression of the form

$$S_T = S_L \sqrt{1 + \frac{2}{3}C\left(\frac{v'}{S_T}\right)^2}$$

where C is a correction factor, and $v'^2 = 3\,\overline{u_1'^{\,2}}$ with v' the turbulent intensity (assuming isotropic turbulence).

Squaring the LHS and solving the quadratic equation for S_T yields

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Additional simplifications (such as bringing the average inside the square root, invoking the condition of isotropic turbulence, etc..) suggest expressions the form

$$\frac{S_T}{S_L} = 1 + C\left(\frac{v'}{S_L}\right)^n$$

and for low turbulent intensity

$$\frac{S_T}{S_L} = 1 + C\left(\frac{v'}{S_L}\right)^2$$

A similar scaling was recently proposed based on numerical calculations employing the hydrodynamic theory (the flame treated as a sheet propagating with a speed S_F that depends on the local stretch rate), where C exhibits a dependence on thermal expansion and turbulent scale.

Creta & Matalon, JFM 2011

Additional results based on a simpler model, i.e., the Michelson-Sivashinsky equation will be also presented illustrating in particular the effect of hydrodynamic instability on the turbulent flame speed.

Creta et al., CTM 2011

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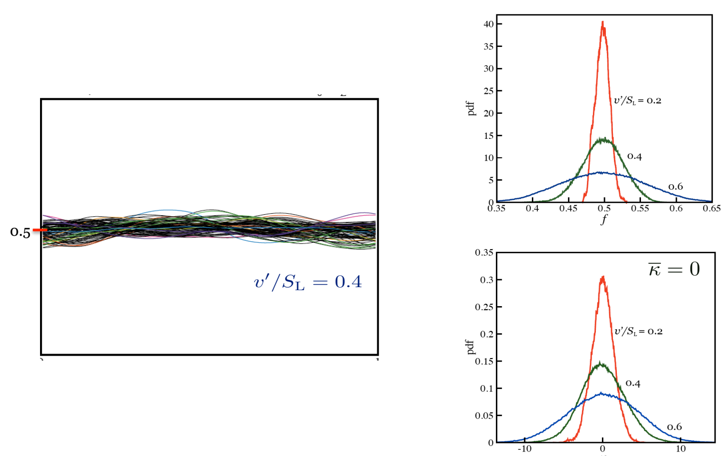
Statistically stationary flame profiles



- The variance of both distributions increases with turbulence intensity, indicating thicker flame brushes and higher local curvatures.
- The mean curvature is zero; the planar (on the average) flames are likely to be convex as they are to be concave.

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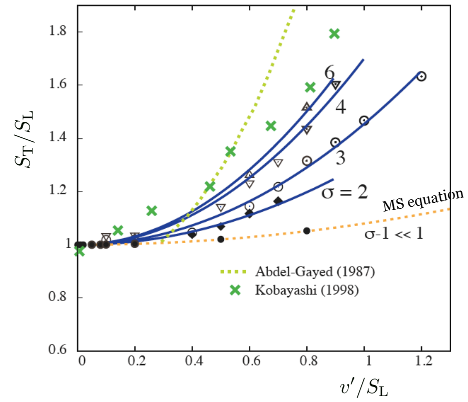
The turbulent propagation speed as a function of turbulence intensity, parametrized for different values of the expansion ratio σ. The solid lines are quadratic fits of the kind

$$S_T/S_L = 1 + c\,(u_0'/S_L)^2$$

with $c = c(\sigma, \ell)$ a constant that depends in general on the expansion ratio σ and, as we shall see, on turbulence scale ℓ.



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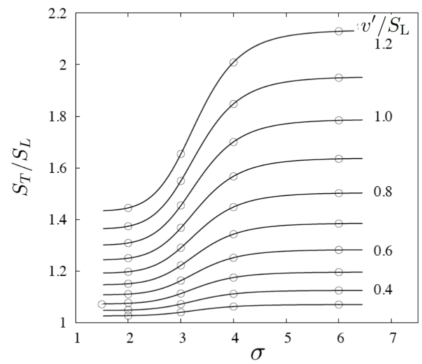
Assuming the quadratic law, the figure displaying the turbulent propagation speed as a function of σ, visualizes the function $c = c(\sigma)$, for a given ℓ, which effectively collapse on a unique curve.

The effect of an increase of thermal expansion enhances the turbulent flame speed, but plateau towards a certain value when σ increases.



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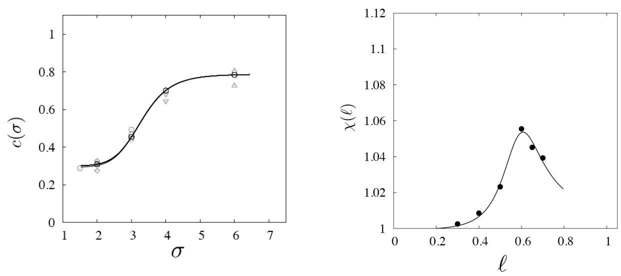
The dependence of the turbulent propagation speed on the integral; scale ℓ shows that a particular intermediate scale exists at which S_T experiences a maximum. For a given turbulence intensity there exists, therefore, a preferred eddy size that most effectively perturbs the flame front.

The proposed expression for the turbulent propagation speed

$$\frac{S_{\mathrm{T}}}{S_{\mathrm{L}}} \sim 1 + a\, c(\sigma)\, \chi(\ell) \left(\frac{v'}{S_{\mathrm{L}}}\right)^2$$

highlights the quadratic dependence from turbulence intensity being modulated by two coefficients depending respectively on expansion ratio and on integral scale, and a is a proportionality constant.



2. Effect of the mean flame stretching on the turbulent burning velocity

Beyond the mere geometric corrugation of the flame's surface due to turbulence influencing the burning rate, an important role in turbulent propagation is played by the mean stretching of the flame which can influence the local flame speed.

So far, effect of stretch was neglected by setting $\mathbb{K} = 0$ and $S_f = S_L$. With

$$S_f = S_L - \mathcal{L}\mathbb{K}$$

$$S_T = \overline{S_f \sqrt{1 + f_y^2 + f_z^2}}$$

$$S_T = S_L \underbrace{\overline{(1 + f_x^2)^{1/2}}}_{\text{area increase}} - \mathcal{L} \underbrace{\overline{\mathbb{K}(1 + f_x^2)^{1/2}}}_{\text{stretching}}.$$

hydrodynamic strain

where, we recall that the flame stretch rate consists of the effects of curvature and strain; i.e., $\mathbb{K} = S_L\kappa + K_S$.

Having established that the mean curvature (for a planar flame brush) is zero, the mean flame speed

$$\overline{S_f} = S_L - \mathcal{L}S_L\overline{\kappa} - \mathcal{L}\overline{K_S} = S_L - \mathcal{L}\overline{K_S}.$$

The overall statistical properties of hydrodynamic strain during the flame propagation indicate that K_S has a positive mean ($\overline{K_S} > 0$), which reveals a net expanding effect of hydrodynamic strain during turbulent propagation.

The mean hydrodynamic strain rate was also found to increase with turbulence intensity.

Since the mean flame speed $\overline{S_f}$ is linearly correlated to the mean strain rate, with the correlation having a negative slope for positive Markstein lengths ($\mathcal{L} > 0$), the mean flame speed will decrease with turbulent intensity in a similar way as the mean strain rate increases.

The decrease of $\overline{S_f}$ with turbulent result in a decrease in turbulent propagation speed S_T/S_L with respect to a flame for which stretch effects are neglected.

As is clearly visible in the figure, the turbulent propagation speed is observed to drop even below the laminar speed, at least for moderate intensities. At greater intensities, the increase in flame surface area dominates over the decrease in mean flame speed $\overline{S_f}$, resulting in the turbulent propagation speed increasing above the laminar speed.

Rescaling the turbulent speed with the mean flame speed, however, is seen to eliminate the effect due to the decrease of $\overline{S_f}$ and recovers the quadratic scaling which represents the geometric effect due to area increase alone.

$$S_T = \left[1 + a\,\chi(\ell)\,c(\sigma)\,(u_0'/S_L)^2\right][S_L - \mathcal{L}\overline{K_S}],$$