Lecture 12

The Level Set Approach for Turbulent Premixed Combustion

- A model for premixed turbulent combustion, based on the non-reacting scalar G rather than on progress variable, has been developed in recent years

- It avoids complications associated with counter-gradient diffusion and, since G is non-reacting, there is no need for a source term closure

- An equation for G can be derived by considering an iso-scalar surface

$$G(\boldsymbol{x}, t) = G_0$$

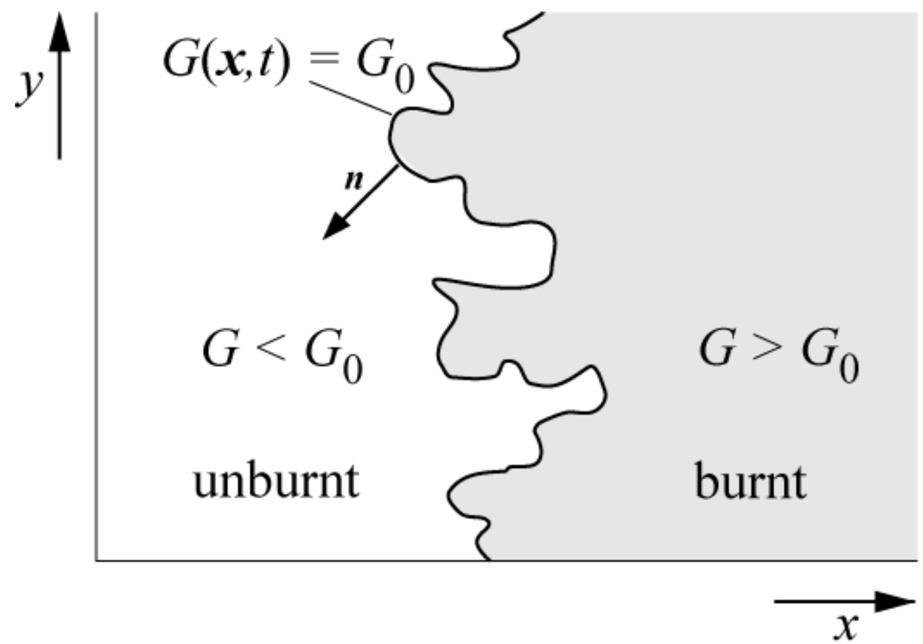
- This surface divides the flow field into two regions where

  $G > G_0$ is the region of burnt gas

  and

  $G < G_0$ is that of the unburnt mixture

- The choice of G_0 is arbitrary and typically taken as $G_0 = 0$



Recapitulation

- Flame front normal vector in direction of the unburnt gas

$$\boldsymbol{n} = -\frac{\boldsymbol{\nabla} G}{|\boldsymbol{\nabla} G|}$$

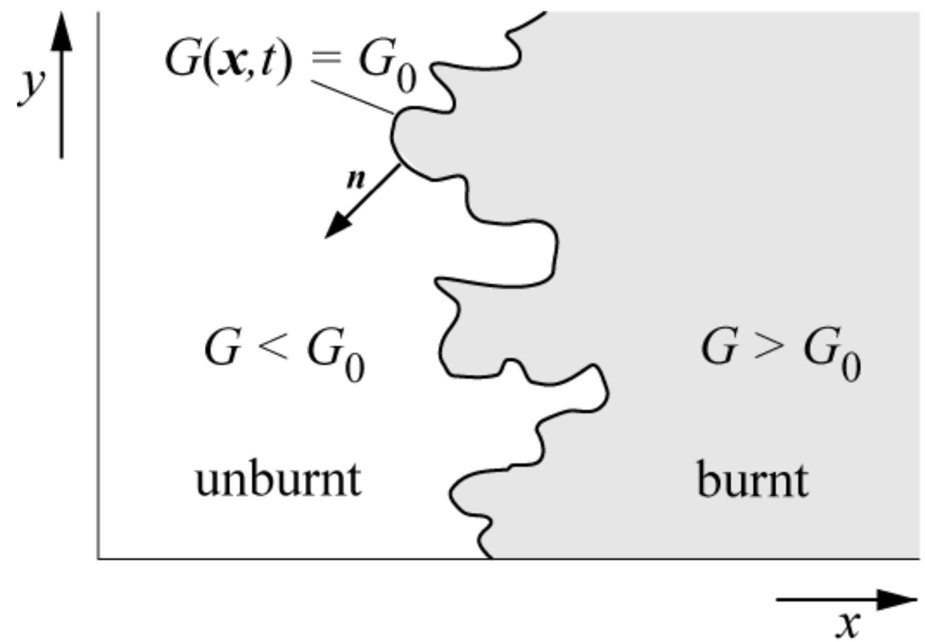
- Flame displacement speed dx_f/dt

$$\frac{\mathrm{d}\boldsymbol{x}_f}{\mathrm{d}t} = \boldsymbol{v}_f + \boldsymbol{n}\, s_L$$

- A field equation can now be derived by differentiating

$$G(\boldsymbol{x}, t) = G_0.$$

with respect to time



- This leads to

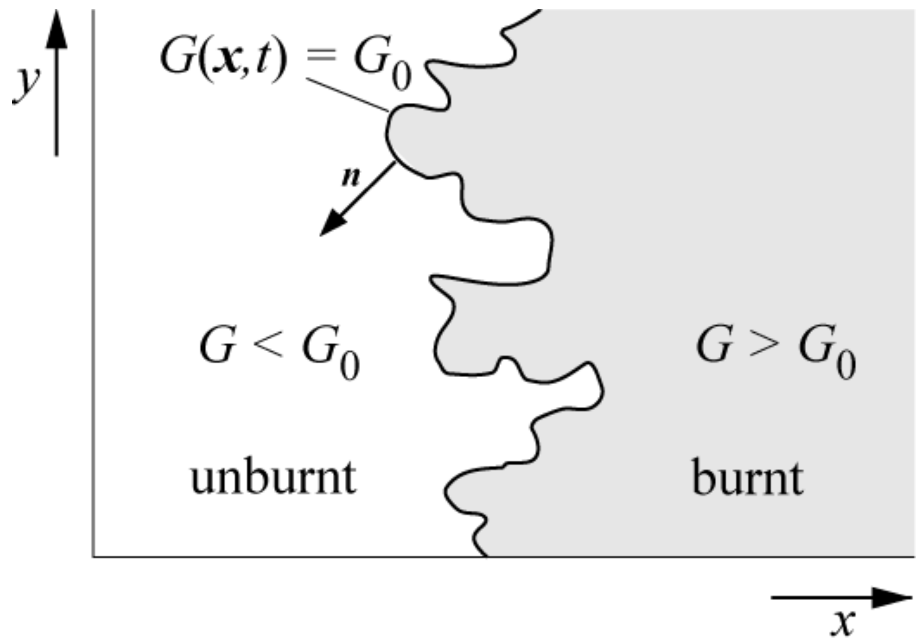
$$\frac{DG}{Dt} = 0 \quad \Rightarrow \quad \frac{\partial G}{\partial t} + \boldsymbol{\nabla} G \cdot \frac{\mathrm{d}\boldsymbol{x}_f}{\mathrm{d}t} = 0$$

- Introducing

$$\frac{\mathrm{d}\boldsymbol{x}_f}{\mathrm{d}t} = \boldsymbol{v}_f + \boldsymbol{n}\, s_L \quad \text{and} \quad \boldsymbol{\nabla} G = -\boldsymbol{n}|\boldsymbol{\nabla} G|$$

one obtains the field equation

$$\frac{\partial G}{\partial t} + \boldsymbol{v}_f \cdot \boldsymbol{\nabla} G = s_L |\boldsymbol{\nabla} G|$$



- This equation was introduced by Williams (1985)

- It is known as the *G*-equation

- *G*-equation is applicable to thin flame structures propagating with a well-defined burning velocity

- It therefore is well-suited for the description of premixed turbulent combustion in the corrugated flamelets regime, where it is assumed that the laminar flame thickness is smaller than the smallest turbulent length scale, the Kolmogorov scale

- Therefore, the entire flame structure is embedded within a locally quasi-laminar flow field and the laminar burning velocity remains well-defined

- The *G*-equation

$$\frac{\partial G}{\partial t} + \boldsymbol{v}_f \cdot \nabla G = s_L |\nabla G|$$

has no diffusion term

- *G* is a scalar quantity which is defined at the flame surface only, while the surrounding *G*-field is not yet uniquely defined

- This originates from the fact that the kinematic balance

$$\frac{\mathrm{d}\boldsymbol{x}_f}{\mathrm{d}t} = \boldsymbol{v}_f + \boldsymbol{n}\, s_L$$

is valid only for the flame surface and definition of remaining *G*-field is (within some constraints) arbitrary

- G-field typically defined to be the closest distance from the flame

- The burning velocity s_L appearing in

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L |\boldsymbol{\nabla} G|$$

may be modified to account for the effect of flame stretch

- Performing two-scale asymptotic analyses of corrugated premixed flames, Pelce and Clavin (1982), Matalon and Matkowsky (1982) derived first order correction terms for small curvature and strain

- Expression for the modified burning velocity becomes

$$s_L = s_L^0 - s_L^0 \mathcal{L} \, \kappa - \mathcal{L} \, S$$

- The expression for the modified burning velocity becomes

$$s_L = s_L^0 - s_L^0 \mathcal{L}\,\kappa - \mathcal{L}\,S$$

- Here s_L^0 is the burning velocity of the unstretched flame, κ is the curvature and S is the strain rate

- Flame curvature is defined in terms of the *G*-field as

$$\kappa = \boldsymbol{\nabla}\cdot\boldsymbol{n} = \boldsymbol{\nabla}\cdot\left(-\frac{\boldsymbol{\nabla}G}{|\boldsymbol{\nabla}G|}\right) = -\frac{\boldsymbol{\nabla}^2 G - \boldsymbol{n}\cdot\boldsymbol{\nabla}(\boldsymbol{n}\cdot\boldsymbol{\nabla}G)}{|\boldsymbol{\nabla}G|}$$

  where

$$\boldsymbol{\nabla}(|\boldsymbol{\nabla}G|) = -\boldsymbol{\nabla}(\boldsymbol{n}\cdot\boldsymbol{\nabla}G)$$

  has been used

- Flame curvature is positive if flame is convex with respect to unburnt mixture

- The strain rate imposed on the flame by velocity gradients is defined as

$$S = -\boldsymbol{n} \cdot \boldsymbol{\nabla} \boldsymbol{v} \cdot \boldsymbol{n}.$$

- The Markstein length $\mathcal{L}$ is of the same order of magnitude and proportional to the laminar flame thickness

- The ratio $\mathcal{L}/\ell_F$ is called the Markstein number

- For the case of a one-step reaction with a large activation energy, constant transport properties and a constant heat capacity, the Markstein length with respect to the unburnt mixture reads, for example (Clavin &Williams, 1982 and Matalon & Matkowsky, 1982)

$$\frac{\mathcal{L}_u}{\ell_F} = \frac{1}{\gamma}\ln\frac{1}{1-\gamma} + \frac{\mathrm{Ze}(\mathrm{Le}-1)}{2}\frac{(1-\gamma)}{\gamma}\int_0^{\gamma/(1-\gamma)}\frac{\ln(1+x)}{x}\mathrm{d}x$$

- Here

$$\mathrm{Ze} = E(T_b - T_u)/\mathcal{R}T_b^2$$

is the Zeldovich number, where *E* is the activation energy, and Le is the Lewis number of the deficient reactant

- The Lewis number is approximately unity for methane flames and larger than unity for fuel-rich hydrogen and all fuel-lean hydrocarbon flames other than methane

- Therefore, since the first term on the r.h.s. of

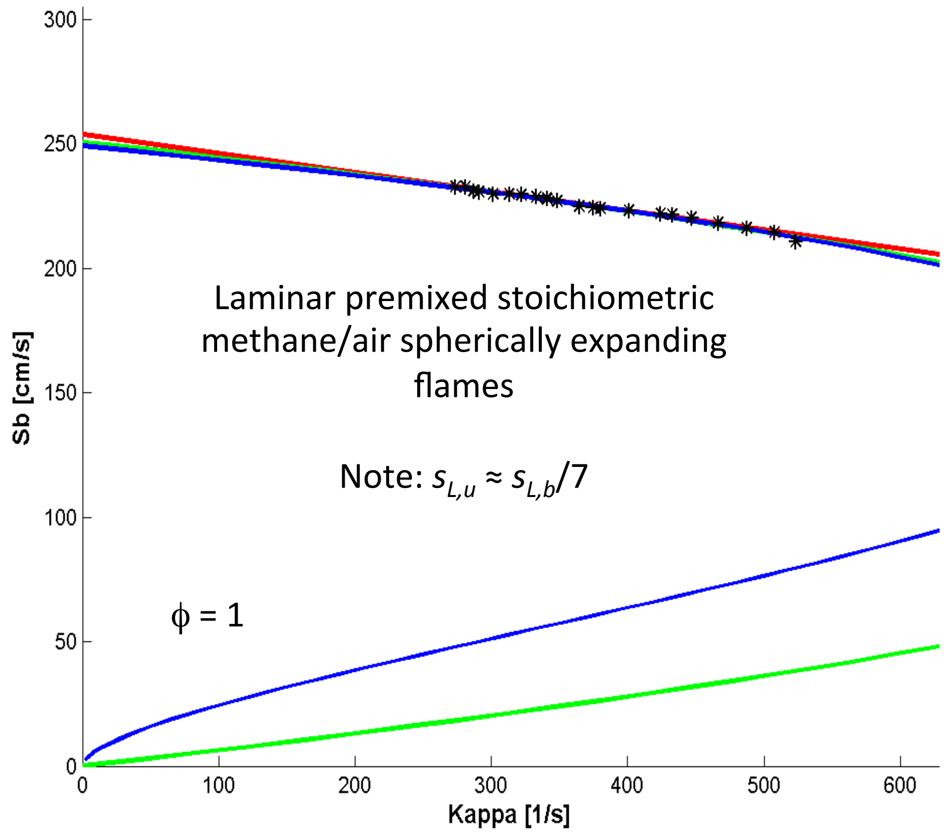
$$\frac{\mathcal{L}_u}{\ell_F} = \frac{1}{\gamma}\ln\frac{1}{1-\gamma} + \frac{\mathrm{Ze}(\mathrm{Le}-1)}{2}\frac{(1-\gamma)}{\gamma}\int\limits_0^{\gamma/(1-\gamma)}\frac{\ln(1+x)}{x}\mathrm{d}x$$

  is always positive, the Markstein length is positive for most practical applications of premixed hydrocarbon combustion, occurring typically under stoichiometric or fuel-lean conditions

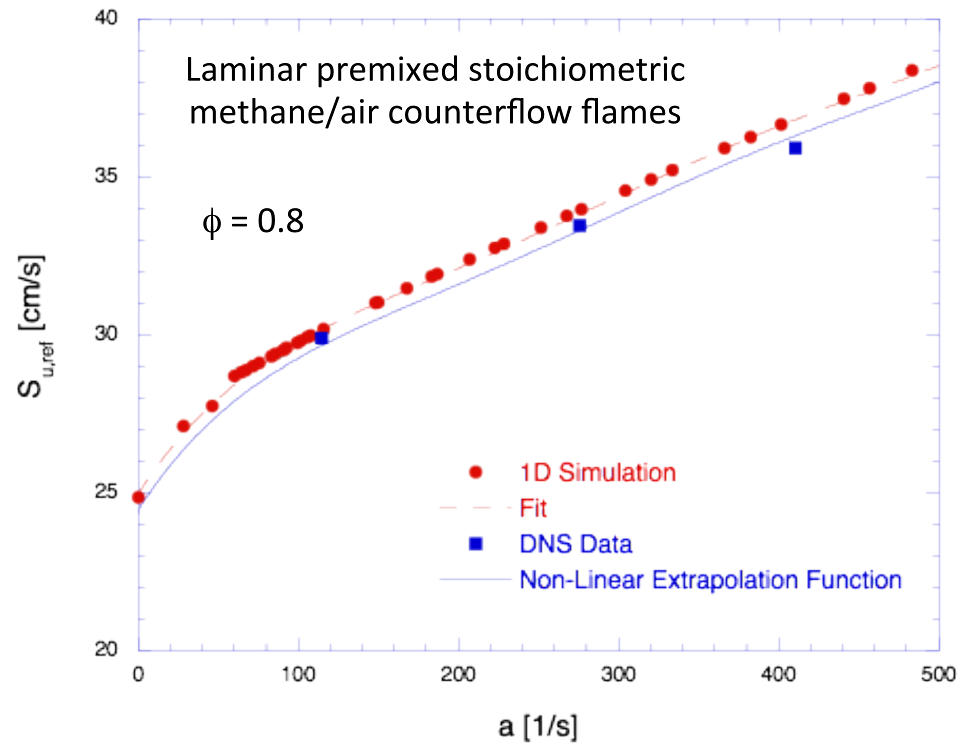
- Whenever the Markstein length is negative, as in lean hydrogen-air mixtures, diffusional-thermal instabilities tend to increase the flame surface area
- This is believed to be an important factor in gas cloud explosions of hydrogen-air mixtures
- Although turbulence tends to dominate such local effects the combustion of diffusional-thermal instabilities and instabilities induced by gas expansion could lead to strong flame accelerations

- Effects of curvature and strain on laminar burning velocity

Curvature Effect on Laminar Burning Velocity from Experiments and Theory



Strain Effect on Laminar Burning Velocity from Numerical Simulations



- Introducing

$$s_L = s_L^0 - s_L^0 \mathcal{L}\,\kappa - \mathcal{L}\,S$$

into the *G*-equation, it may be written as

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L^0 |\boldsymbol{\nabla} G| - \mathcal{D}_{\mathcal{L}} \kappa |\boldsymbol{\nabla} G| - \mathcal{L} S |\boldsymbol{\nabla} G|$$

- Here

$$\mathcal{D}_{\mathcal{L}} = s_L^0 \mathcal{L}$$

is defined as the Markstein diffusivity

- The curvature term adds a second order derivative to the *G*-equation

- This avoids the formation of cusps that would result from

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L^0 |\boldsymbol{\nabla} G|$$

  for a constant value of s_L^0

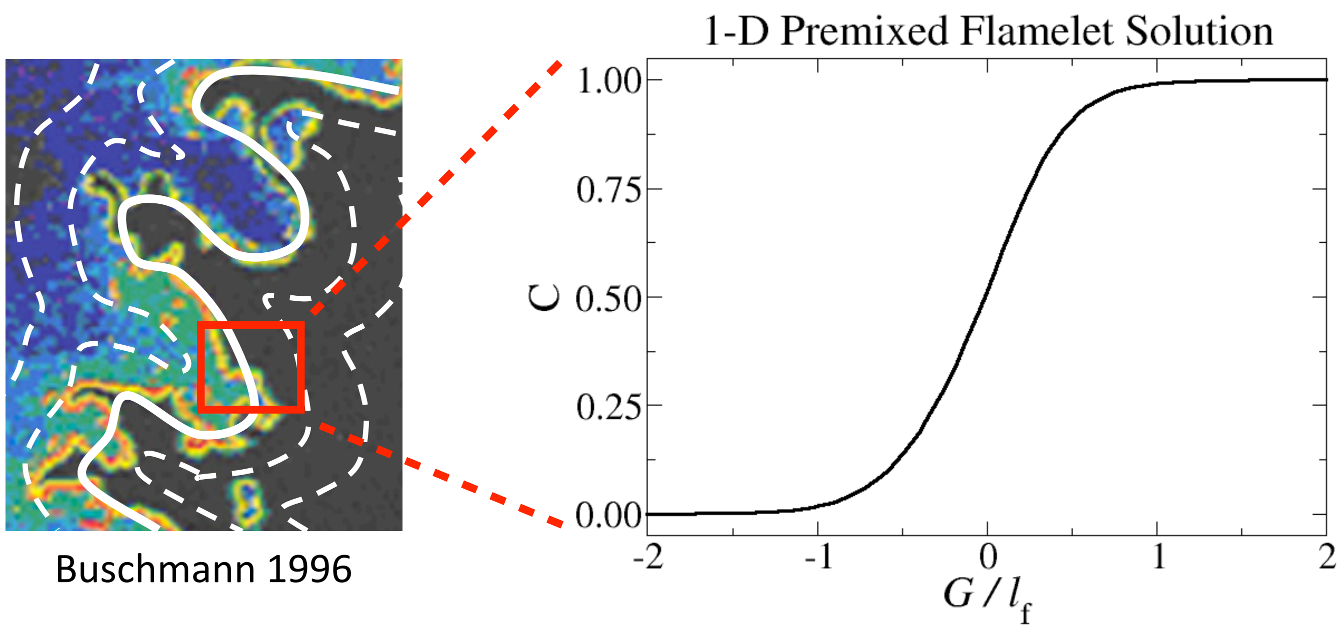
- While the solution of the *G*-equation with a constant s_L^0 is solely determined by specifying the initial conditions, the parabolic character of

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L^0 |\boldsymbol{\nabla} G| - \mathcal{D}_{\mathcal{L}} \kappa |\boldsymbol{\nabla} G| - \mathcal{L} S |\boldsymbol{\nabla} G|$$

  requires that the boundary conditions for each iso-surface *G* must be specified

- The *G*-equation model in the corrugated flamelets regime
  - Flame thin compared to small turbulent scales
  - Level set describes interface between unburned and burned
  - Flame structure embedded in the interface, but variations on much smaller scale

    → Multi-scale model
  - *G*-field defined as distance function



Buschmann 1996

The Level Set Approach for the Thin Reaction Zones Regime

- The equation

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \nabla G = s_L^0 |\nabla G| - \mathcal{D}_{\mathcal{L}} \kappa |\nabla G| - \mathcal{L} S |\nabla G|$$

is suitable for thin flame structures in the corrugated flamelets regime,

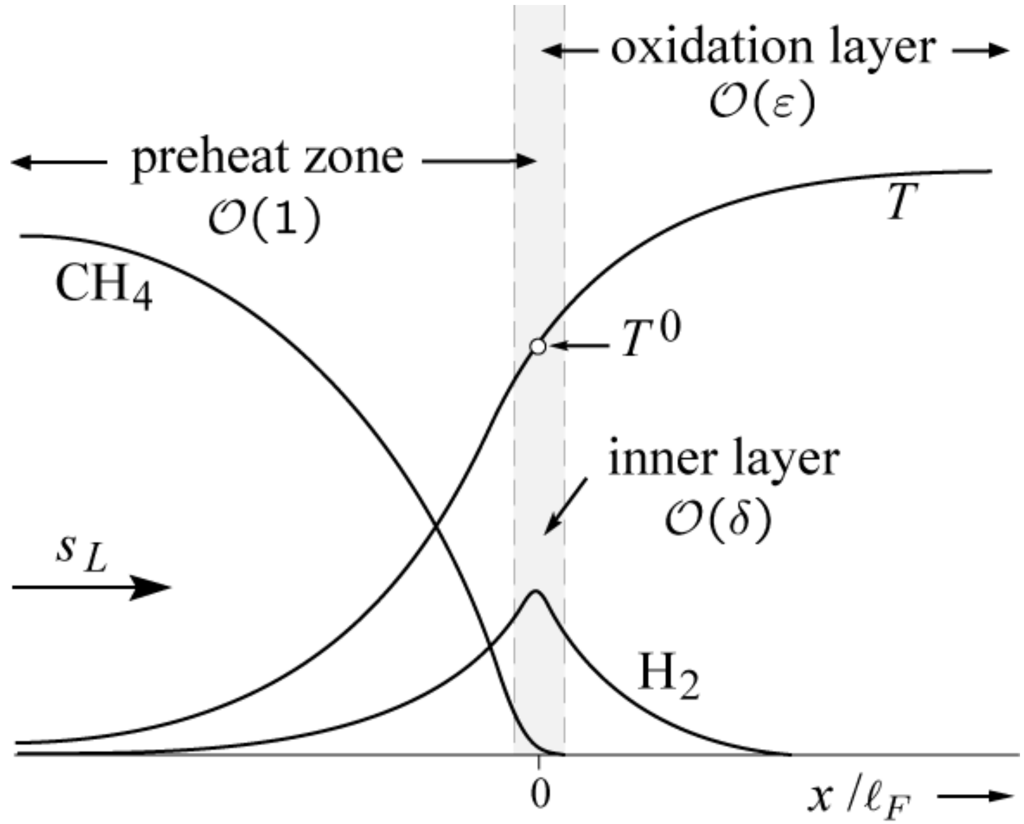
- Entire flame structure quasi-steady
- Laminar burning velocity well defined

- However, not suitable for thin reaction zones regime

➡ Derivation of level set equation valid in the thin reaction zones regime

- Since the inner layer shown previously is responsible for maintaining the reaction process alive, we define the thin reaction zone as the inner layer

- The location of the inner layer will be determined by the iso-scalar surface of the temperature setting $T(\boldsymbol{x},t)=T^0$, where T^0 is the inner layer temperature



- The temperature equation reads

$$\rho\frac{\partial T}{\partial t} + \rho \boldsymbol{v}\cdot\boldsymbol{\nabla} T = \boldsymbol{\nabla}\cdot(\rho D \boldsymbol{\nabla} T) + \omega_T$$

where D is the thermal diffusivity ω_T the chemical source term

- Similar to

$$\frac{\partial G}{\partial t} + \boldsymbol{\nabla} G\cdot\frac{\mathrm{d}\boldsymbol{x}_f}{\mathrm{d}t} = 0$$

for the scalar *G,* the iso-temperature surface $T(\boldsymbol{x},t)=T^0$ satisfies the condition

$$\left.\frac{\partial T}{\partial t} + \boldsymbol{\nabla} T\cdot\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}\right|_{T=T^0} = 0$$

- Gibson (1968) has derived an expression for the displacement speed s_d of an iso-surface of non-reacting diffusive scalars

- Extending this result to the reactive scalar T this leads to

$$\left.\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}\right|_{T=T^0} = \boldsymbol{v}_0 + \boldsymbol{n}\, s_d$$

where the displacement speed s_d is given by

$$s_d = \left[\frac{\boldsymbol{\nabla}\cdot(\rho D \boldsymbol{\nabla} T) + \omega_T}{\rho|\boldsymbol{\nabla} T|}\right]_0$$

- Here the index 0 defines conditions immediately ahead of the thin reaction zone

- The normal vector on the iso-temperature surface is defined as

$$\boldsymbol{n} = -\frac{\boldsymbol{\nabla} T}{|\boldsymbol{\nabla} T|}\bigg|_{T=T^0}$$

- We now want to formulate a *G*-equation describing the location of the thin reaction zones such that the iso-surface $\mathbf{T}(\boldsymbol{x},t)=T^0$ coincides with the iso-surface defined by $G(\boldsymbol{x},t)=G^0$

- Then, the normal vector defined by $\boldsymbol{n} = -\frac{\boldsymbol{\nabla} T}{|\boldsymbol{\nabla} T|}\Big|_{T=T^0}$

  is equal to that defined by

$$\boldsymbol{n} = -\frac{\boldsymbol{\nabla} G}{|\boldsymbol{\nabla} G|}$$

  and also points towards the unburnt mixture

- Using $\boldsymbol{n} = -\frac{\boldsymbol{\nabla} G}{|\boldsymbol{\nabla} G|}$ and $\frac{\partial G}{\partial t} + \boldsymbol{\nabla} G \cdot \frac{\mathrm{d}\boldsymbol{x}_f}{\mathrm{d}t} = 0$

  together with

$$s_d = \left[\frac{\boldsymbol{\nabla} \cdot (\rho D \boldsymbol{\nabla} T) + \omega_T}{\rho |\boldsymbol{\nabla} T|}\right]_0$$

  leads the *G*-equation

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = \left[\frac{\boldsymbol{\nabla} \cdot (\rho D \boldsymbol{\nabla} T) + \omega_T}{\rho |\boldsymbol{\nabla} T|}\right]_0 |\boldsymbol{\nabla} G|$$

- Peters et al. (1998) show that the diffusive term appearing in the brackets in this equation may be split into one term accounting for curvature and another for diffusion normal to the iso-surface

$$\nabla\cdot(\rho D\nabla T) = -\rho D|\nabla T|\nabla\cdot\boldsymbol{n} + \boldsymbol{n}\cdot\nabla(\rho D\boldsymbol{n}\cdot\nabla T)$$

- This is consistent with the definition of the curvature

$$\kappa = \nabla\cdot\boldsymbol{n} = -\frac{\nabla^2 G - \boldsymbol{n}\cdot\nabla(\boldsymbol{n}\cdot\nabla G)}{|\nabla G|}$$

if the iso-surface $G(\boldsymbol{x},t)=G^0$ is replaced by the iso-surface $T(\boldsymbol{x},t)=T^0$ and if ρD is assumed constant

- Introducing

$$\nabla\cdot(\rho D\nabla T) = -\rho D|\nabla T|\nabla\cdot\boldsymbol{n} + \boldsymbol{n}\cdot\nabla(\rho D\boldsymbol{n}\cdot\nabla T)$$

into

$$\frac{\partial G}{\partial t} + \boldsymbol{v}\cdot\nabla G = \left[\frac{\nabla\cdot(\rho D\nabla T) + \omega_T}{\rho|\nabla T|}\right]_0 |\nabla G|$$

one obtains

$$\frac{\partial G}{\partial t} + \boldsymbol{v}\cdot\nabla G = (s_n + s_r)|\nabla G| - D\,\kappa|\nabla G|$$

Here

$$\kappa = \nabla\cdot\boldsymbol{n} = -\frac{\nabla^2 G - \boldsymbol{n}\cdot\nabla(\boldsymbol{n}\cdot\nabla G)}{|\nabla G|}$$

is to be expressed by in terms of the *G*-field

- The quantities s_n and s_r are contributions due to normal diffusion and reaction to the displacement speed of the thin reaction zone and are defined as

$$s_n = \frac{\boldsymbol{n}\cdot\boldsymbol{\nabla}(\rho D\boldsymbol{n}\cdot\boldsymbol{\nabla}T)}{\rho|\boldsymbol{\nabla}T|}\,,$$

$$s_r = \frac{\omega_T}{\rho|\boldsymbol{\nabla}T|}$$

- In a steady unstretched planar laminar flame we would have

$$s_n + s_r = s_L^0$$

- In the thin reaction zones regime, however, the unsteady mixing and diffusion of chemical species and the temperature in the regions ahead of the thin reaction zone will influence the local displacement speed

- Then the sum of

$$s_n + s_r = s_{L,s} \neq s_L^0$$

  is a fluctuating quantity that couples the *G*-equation to the solution of the balance equations of the reactive scalars

- There is reason to expect, however, that $s_{L,s}$ is of the same order of magnitude as the laminar burning velocity

- The evaluation of DNS-data by Peters et al. (1998) confirms this estimate

- In that paper it was also found that the mean values of s_n and s_r slightly depend on curvature

- This leads to a modification of the diffusion coefficient which partly takes Markstein effects into account

- We will ignore these modifications here and consider the following level set equation for flame structures of finite thickness

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \nabla G = s_{L,s} |\nabla G| - D\kappa |\nabla G|$$

- This equation is defined at the thin reaction zone
  v, $s_{L,s}$, and D are values at that position

- The G-equation in the thin reaction zones regime

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \nabla G = s_{L,s} |\nabla G| - D\kappa |\nabla G|$$

is very similar to that for the corrugated flamelets regime

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \nabla G = s_L^0 |\nabla G| - \mathcal{D}_{\mathcal{L}} \kappa |\nabla G| - \mathcal{L} S |\nabla G|$$

- Important is the difference between $\mathcal{D}_{\mathcal{L}}$ and *D* and the disappearance of the strain term

- The latter is implicitly contained in the burning velocity $s_{L,s}$

- In an analytical study of the response of one-dimensional constant density flames to time-dependent strain and curvature, Joulin (1994) has shown that in the limit of high frequency perturbations, the effect of strain disappears entirely and Lewis-number effects also disappear in the curvature term such that

$$\mathcal{D}_{\mathcal{L}} \approx D$$

- This analysis was based on one-step large activation energy asymptotics with the assumption of a single thin reaction zone

- It suggests that

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_{L,s}|\boldsymbol{\nabla} G| - D\kappa|\boldsymbol{\nabla} G|$$

could also have been derived from

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L^0|\boldsymbol{\nabla} G| - \mathcal{D}_{\mathcal{L}}\kappa|\boldsymbol{\nabla} G| - \mathcal{L}S|\boldsymbol{\nabla} G|$$

for the limit of high frequency perturbations of the flame structure

- This strongly supports it as level set equation for flame structures of finite thickness and shows that unsteadiness of that structure is an important feature in the thin reaction zones regime

- The important difference between the level set formulation

$$\frac{\partial G}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} G = s_{L,s}|\boldsymbol{\nabla} G| - D\kappa|\boldsymbol{\nabla} G|$$

and the equation for the reactive scalar

$$\rho\frac{\partial T}{\partial t} + \rho\boldsymbol{v}\cdot\boldsymbol{\nabla} T = \boldsymbol{\nabla}\cdot(\rho D\boldsymbol{\nabla} T) + \omega_T$$

is the appearance of a burning velocity which replaces normal diffusion and reaction at the flame surface

- It should be noted again that the level set equations

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L^0 |\boldsymbol{\nabla} G| - \mathcal{D}_{\mathcal{L}} \kappa |\boldsymbol{\nabla} G| - \mathcal{L} S |\boldsymbol{\nabla} G|$$

and

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_{L,s} |\boldsymbol{\nabla} G| - D \kappa |\boldsymbol{\nabla} G|$$

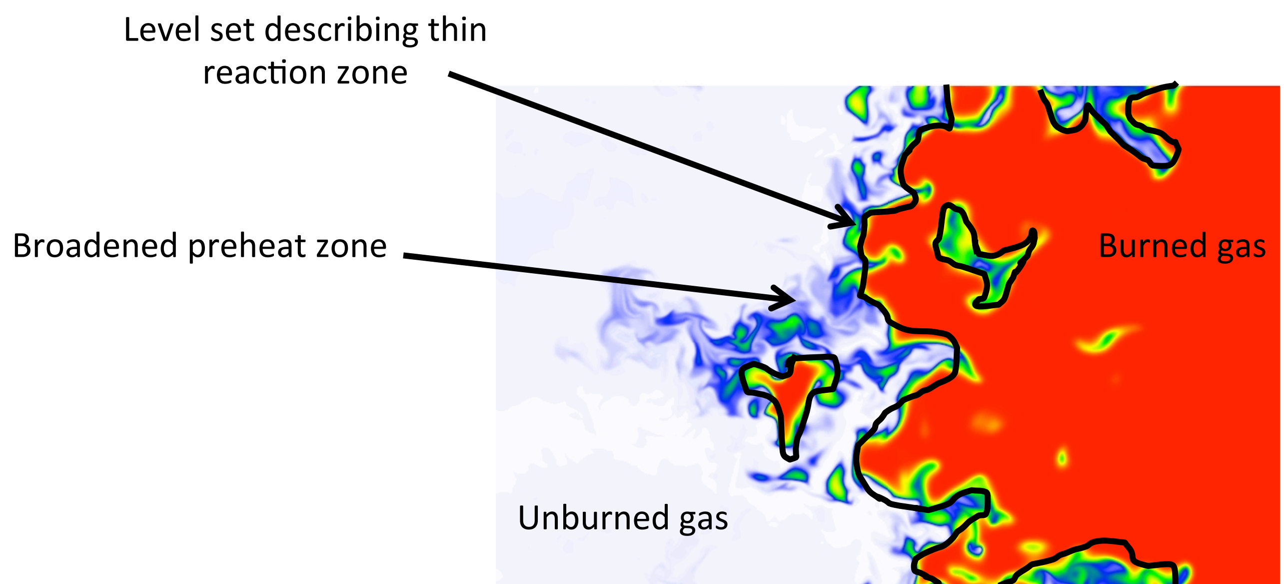
are only defined at the flame surface,

while

$$\rho \frac{\partial T}{\partial t} + \rho \boldsymbol{v} \cdot \boldsymbol{\nabla} T = \boldsymbol{\nabla} \cdot (\rho D \boldsymbol{\nabla} T) + \omega_T$$

is valid in the entire field

- The *G*-equation model in the thin reaction zones regime
  - Flame broadened by turbulence, but
  - Reaction zone still thin compared to turbulent scales and appears as interface
  - Level set describes location of reaction zone



DNS of Premixed Combustion, Knudsen 2010

A Common Level Set Equation for Both Regimes

- The *G*-equation applies to different regimes in premixed turbulent combustion:
  - Corrugated flamelets regime

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_L^0 |\boldsymbol{\nabla} G| - \mathcal{D}_{\mathcal{L}} \kappa |\boldsymbol{\nabla} G| - \mathcal{L} S |\boldsymbol{\nabla} G|$$

  - Thin reaction zones regime

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = s_{L,s} |\boldsymbol{\nabla} G| - D \kappa |\boldsymbol{\nabla} G|$$

- In order to show this, we will analyze the order of magnitude of the different terms in the second equation

- This can be done by normalizing the independent variables and the curvature in this equation with respect to Kolmogorov length, time and velocity scales

$$t^* = t/t_\eta\,, \quad \boldsymbol{x}^* = \boldsymbol{x}/\eta\,, \quad \boldsymbol{v}^* = \boldsymbol{v}/v_\eta\,,$$

$$\kappa^* = \eta\kappa\,, \quad \boldsymbol{\nabla}^* = \eta\boldsymbol{\nabla}.$$

- Using $\eta^2/t_\eta = \nu$ one obtains

$$\frac{\partial G}{\partial t^*} + \boldsymbol{v}^* \cdot \boldsymbol{\nabla}^* G = \frac{s_{L,s}}{v_\eta}|\boldsymbol{\nabla}^* G| - \frac{D}{\nu}\kappa^*|\boldsymbol{\nabla}^* G|,$$

- In flames, D/ν is of order unity

- Since Kolmogorov eddies can perturb the flow as well as the *G*-field, all derivatives, the curvature and the velocity v^* are typically of order unity

- However, since $s_{L,s}$ is of the same order of magnitude as s_L, the definition

$$\text{Ka} = \frac{t_F}{t_\eta} = \frac{\ell_F^2}{\eta^2} = \frac{v_\eta^2}{s_L^2}$$

shows that the ratio $s_{L,s}/v_\eta$ is proportional to $\text{Ka}^{-1/2}$.

- Since Ka > 1 in the thin reaction zones regime, it follows for this regime that

$$s_{L,s} < v_\eta$$

- In the thin reaction zones regime, the propagation term in the equation

$$\frac{\partial G}{\partial t^*} + \boldsymbol{v}^* \cdot \boldsymbol{\nabla}^* G = \frac{s_{L,s}}{v_\eta} |\boldsymbol{\nabla}^* G| - \frac{D}{\nu} \kappa^* |\boldsymbol{\nabla}^* G|,$$

is therefore small and the curvature term will be dominant

- In the corrugated flamelets regime, $s_{L,s}/v_\eta$ becomes large and propagation dominates

- We want to base the following analysis on an equation which contains only the leading order terms in both regimes

- Therefore we take the propagation term with a constant laminar burning velocity s_L^0 from the corrugated flamelets regime and the curvature term multiplied with the diffusivity D from the thin reaction zones regime

- The strain term $\mathcal{L}\, S$ will be neglected in both regimes

- The leading order equation valid in both regimes then reads

$$\rho\frac{\partial G}{\partial t} + \rho\boldsymbol{v}\cdot\boldsymbol{\nabla}G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

- For consistency with other field equations that will be used as a starting point for turbulence modeling, we have multiplied all terms in this equation with ρ. This will allow to apply Favre averaging to all equations

- Furthermore, we have set $\rho\, s_L^0 = \rho_u\, s_{L,u}$ constant and denoted this by paranthesis

Modeling Premixed Turbulent Combustion Based on the Level Set Approach

- If the G-equation is to be used as a basis for turbulence modeling, it is convenient to ignore at first its non-uniqueness outside the surface $G(\boldsymbol{x}, t) = G_0$.

- Then the G-equation would have similar properties as other field equations used in fluid dynamics and scalar mixing

- This would allow to define, at point $\boldsymbol{x}$ and time t in the flow field, a probability density function $P(G;\boldsymbol{x},t)$ for the scalar G

- Then, the probability density of finding the flame surface $G(\boldsymbol{x},t)=G_0$ at $\boldsymbol{x}$ and t is given by

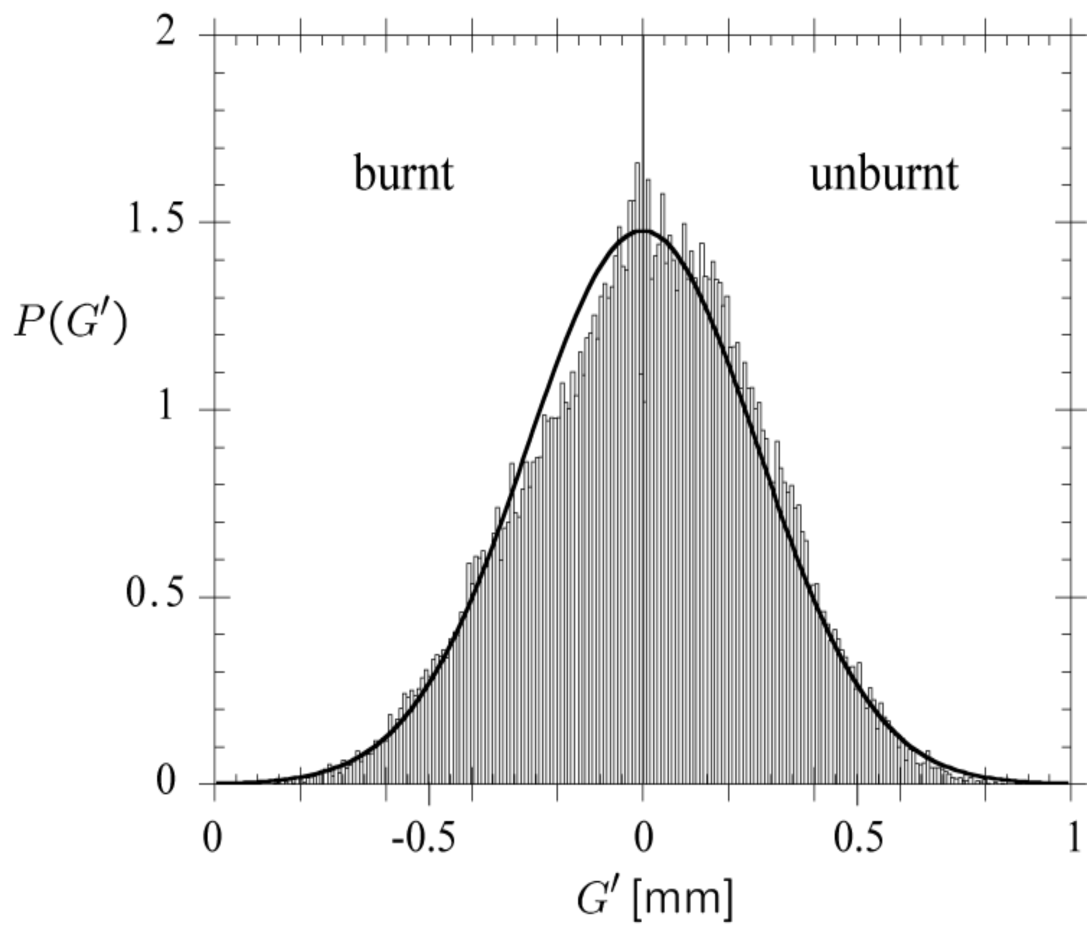
$$P(G_0,\boldsymbol{x},t) = \int_{-\infty}^{+\infty} \delta(G-G_0)P(G;\boldsymbol{x},t)\mathrm{d}G = P(\boldsymbol{x},t).$$

- This quantity can be measured, for instance, by counting the number of flame crossings in a small volume ΔV located at $\boldsymbol{x}$ over a small time difference Δt

- Experimental data for the pdf (Wirth et al., 1992, 1993) from a transparent spark-ignition engine

- Smoke particles, which burnt out immediately in the flame front, were added to the unburnt mixture

- Thereby, the front could be visualized by a laser sheet as the borderline of the region where Mie scattering of particles could be detected



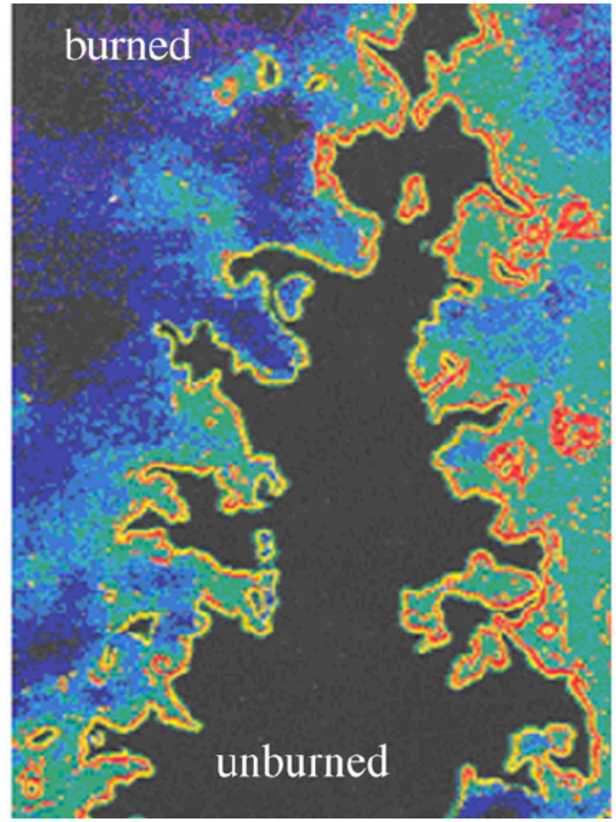
- The pdf represents the pdf of fluctuations around the mean flame contour of several instantaneous images

- By comparing the measured pdf with a Gaussian distribution, it is seen to be slightly skewed to the unburnt gas side

- This is due to the non-symmetric influence of the laminar burning velocity on the shape of the flame front

  ⇒ There are rounded leading edges towards the unburnt mixture, but sharp and narrow troughs towards the burnt gas

- This non-symmetry is also found in other experimental pdfs



Buschmann 1996

- Without loss of generality, we now want to consider a one-dimensional steady turbulent flame propagating in *x*-direction

- We will analyze its structure by introducing the flame-normal coordinate *x*, such that all turbulent quantities are a function of this coordinate only

- Then, the pdf of finding the flame surface at a particular location *x* within the flame brush simplifies to $P(G_0; x)$, which we write as $P(x)$

- We normalize $P(x)$ by

$$\int_{-\infty}^{+\infty} P(x)\mathrm{d}\boldsymbol{x} = 1$$

and define the mean flame position x_f as

$$x_f = \int_{-\infty}^{+\infty} xP(x)\mathrm{d}x$$

- The turbulent flame brush thickness $\ell_{F,t}$ can also be defined using *P*(*x*)

- With the definition of the variance

$$\overline{(x - x_f)^2} = \int\limits_{-\infty}^{+\infty} (x - x_f) P(x) \, \mathrm{d}x$$

a plausible definition is

$$\ell_{F,t} = \left(\overline{(x - x_f)^2}\right)^{1/2}$$

- We note that from *P*(*x*) two important properties of a premixed turbulent flame, namely the mean flame position and the flame brush thickness can be calculated

- Peters (1992) considered turbulent modeling of the *G*-equation in the corrugated flamelets regime and derived Reynolds-averaged equations for the mean and the variance of *G*

- The main sink term in the variance equation was defined as

$$\overline{\omega} = -2\, s_L^0 \overline{G'\sigma'}$$

- This quantity was called kinematic restoration in order to emphasize the effect of local laminar flame propagation in restoring the *G*-field and thereby the flame surface

- Corrugations produced by turbulence, which would exponentially increase the flame surface area with time of a non-diffusive iso-scalar surface are restored by this kinematic effect

- From that analysis resulted a closure assumption which relates the main sink term to the variance of *G* and the integral time scale

$$\overline{\omega} = c_\omega \, \frac{\varepsilon}{k} \, \overline{G'^2}$$

where c_ω=1.62 is a constant of order unity

- This expression shows that kinematic restoration plays a similar role in reducing fluctuations of the flame front as scalar dissipation does in reducing fluctuations of diffusive scalars

- It was also shown by Peters (1992) that kinematic restoration is active at the Gibson scale, since the cut-off of the inertial range in the scalar spectrum function occurs at that scale

Equations for the Mean and the Variance of *G*

- In order to obtain a formulation that is consistent with the well-established use of Favre averages in turbulent combustion, we split *G* and the velocity vector v into Favre means and fluctuations

$$G = \widetilde{G} + G'' \,, \quad \boldsymbol{v} = \tilde{\boldsymbol{v}} + \boldsymbol{v}''$$

- Here, the Favre means are at first viewed as unconditional averages

- At the end, however, only the respective conditional averages are of interest

- The turbulent burning velocity s_T^0 is obtained by averaging over

$$\frac{\partial G}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} G = s_L^0\sigma - \mathcal{D}_{\mathcal{L}}\kappa|\boldsymbol{\nabla} G| - \mathcal{L}S|\boldsymbol{\nabla} G|$$

- By setting $\bar{v} = s_T^0$ one finds for the stationary unstretched flame front

$$s_T^0|\boldsymbol{\nabla} G| = \overline{s_L^0\sigma}$$

- Using a number of additional closure assumptions described in Peters (2000), one finally obtains the following equations for the Favre mean and variance of *G*:

$$\bar{\rho}\frac{\partial \widetilde{G}}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\widetilde{G} = (\bar{\rho}s_T^0)|\boldsymbol{\nabla}\widetilde{G}| - \bar{\rho}D_t\tilde{\kappa}|\boldsymbol{\nabla}\widetilde{G}|$$

$$\bar{\rho}\frac{\partial \widetilde{G''^2}}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\boldsymbol{\nabla}\widetilde{G''^2} = \boldsymbol{\nabla}_{||}(\bar{\rho}D_t\boldsymbol{\nabla}_{||}\widetilde{G''^2}) + 2\bar{\rho}D_t(\boldsymbol{\nabla}\widetilde{G})^2 - c_s\bar{\rho}\frac{\tilde{\varepsilon}}{\tilde{k}}\widetilde{G''^2}$$

- It is easily seen that

$$\bar{\rho}\frac{\partial \widetilde{G}}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\widetilde{G} = (\bar{\rho}s_T^0)|\boldsymbol{\nabla}\widetilde{G}| - \bar{\rho}D_t\tilde{\kappa}|\boldsymbol{\nabla}\widetilde{G}|$$

has the same form as

$$\rho\frac{\partial G}{\partial t} + \rho\boldsymbol{v}\cdot\boldsymbol{\nabla}G = (\rho s_L^0)\sigma - (\rho D)\kappa\sigma$$

and therefore shares its mathematical properties

- It also is valid at $\widetilde{G}(\boldsymbol{x},t) = G_0$ only

- The solution outside of that surface depends on the ansatz for the Favre mean of *G* that is introduced

- In order to solve

$$\bar{\rho}\frac{\partial \widetilde{G}}{\partial t} + \bar{\rho}\tilde{\boldsymbol{v}}\cdot\nabla\widetilde{G} = (\bar{\rho}s_T^0)|\nabla\widetilde{G}| - \bar{\rho}D_t\tilde{\kappa}|\nabla\widetilde{G}|$$

  a model for the turbulent burning velocity s^0_T must be provided.

- A first step would be to use empirical correlations from the literature

- Alternatively, a modeled balance equation for the mean gradient $\bar{\sigma}$ will be derived next

- According to Kerstein (1988), this quantity represents the flame surface area ratio, which is proportional to the turbulent burning velocity