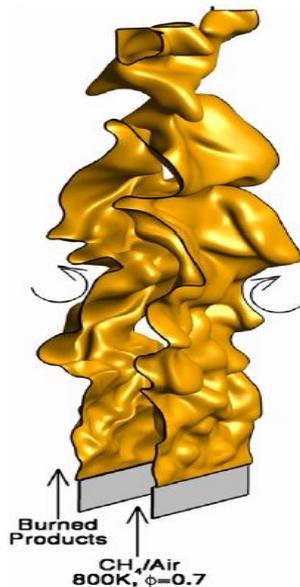


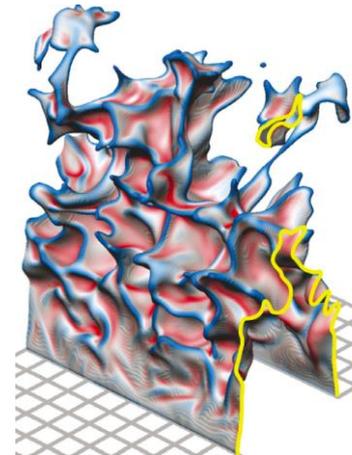
Turbulent Combustion

Experiments and Fundamental Models

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Jackie Chen
T. Lu, C. K. Law
Premixed DNS
PROCI 31, 1291



Bell, Day,
Driscoll
“corrugated”
premixed DNS
PROCI 31, 1299

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Outline for the week

Mon: **Physical concepts** faster mixing, faster propagation, optimize liftoff, flame surface density, reaction rate, PDF

Tues: **Kilohertz PLIF, PIV measurements of flame structure** - to assess models

Wed: **Non-Premixed and Premixed flames** - measurements, models
gas turbine example

Thurs: **Partially premixed flames** - and some examples

Fri: **Future challenges:** Combustion Instabilities (Growl) , Extinction



Outline for Monday = Physical concepts

Motivation - premixed is the way of the future – low NO_x, CO, soot

What problems are important ?

Background what does turbulence do ?
what do we need to model and to measure ? turbulent
burning velocity, flame surface density, reaction rate

Turbulent reaction rate

Probability density function helps to model turbulent reaction rate

Fuel air mixing and Flame stability



Good references

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C. K. Law and C. J. Sung, Structure, aerodynamics, and geometry of premixed flamelets, PECS, 26, 4-6, 2000, 459-505RS

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Pitsch H, Large-eddy simulation of turbulent combustion, Ann Rev Fluid Mech 38, 2006

Duclos JM, Veynante D, Poinso T. A comparison of flamelet models for premixed turbulent combustion. Combust Flame 1993; 95: 1 - 16.

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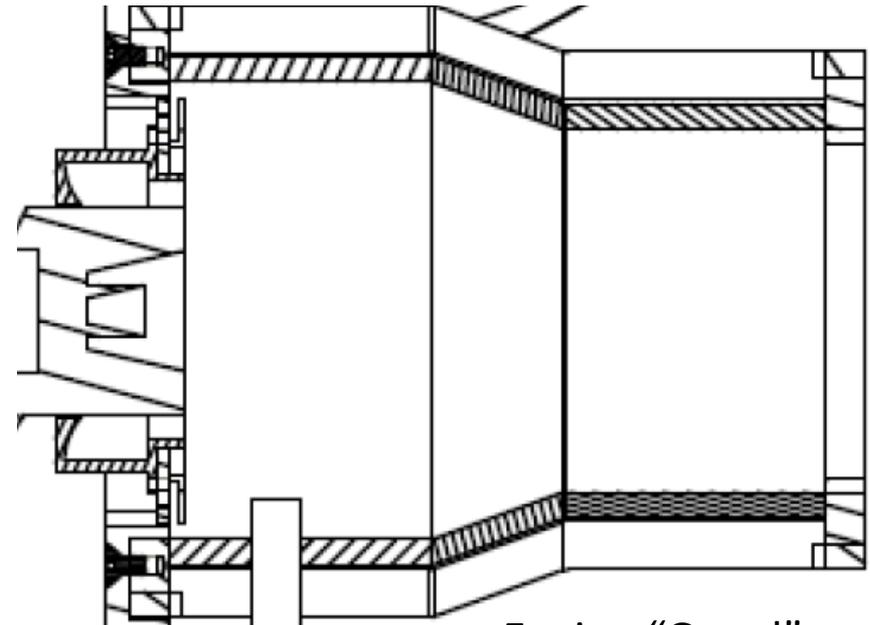
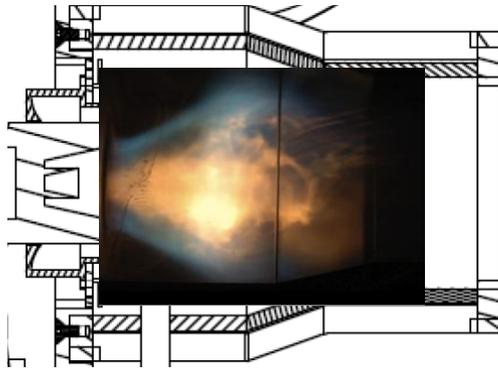
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Kalghatgi, G.. Blowout stability limits of gaseous jet diffusion flames in still air, Combust Sci Technol 26, 233, 1981.



Motivation: Premixed is the way of the future - low NOx, CO, soot

GE-TAPS in Michigan High Pressure GT Combustor



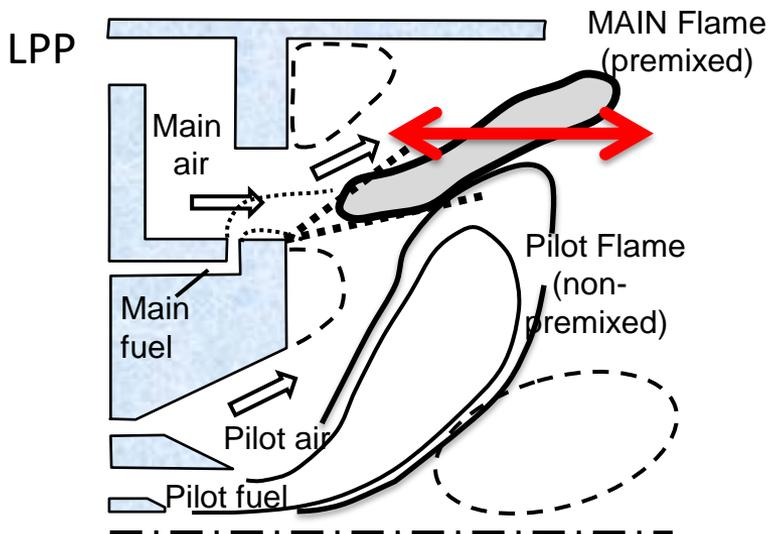
Engine "Growl"

premixed flame difficult to anchor

Temme, Driscoll, Combust. Flame 161, 958

Tim Lieuwen – equivalence ratio oscillations

PROCI 27, 1809



Motivation - challenges

Most practical problems are “Partially-Premixed”

Partially-Premixed = ER varies in space from 0 to ∞ , a point sometimes sees premixed, sometimes non-premixed flamelets

Stratified Premixed = ER varies in space, reactants are within flam. Limits

see: Masri, PROCI 35, 1115, Driscoll Comb Flame 162, 2808

No dependable model of flame blowout, combustion instabilities or turbulent burning velocity at large turbulence level

- Need “robust” LES submodel: **flamelets**: Bray / Flame surface density progress variable (Moin, Pitsch, Ihme)
thickened: Poinot (TFM)
distributed: Menon(LEM), Pope (PDF)
pyrolysis chemistry (?)
- No measurements of boundaries of regimes - when are flamelet models appropriate ?
- Premixed turbulent combustion is **more difficult** than non-premixed turbulent combustion, not just a mixing problem, also wave propagation



Motivation: What problems do we want to solve ?

1. Premixed: Engines: IC engine & HCCI, industrial burners, premixed GT
2. Non-premixed: jet, jet in cross flow, jet in co-flow, jet in swirl flow
3. Partially-premixed: gas turbine, afterburner, base of lifted jet
4. Canonical experiments for model assessment:
 - non-premixed: piloted jet flame (Sandia flame D)
 - premixed: Bunsen (high-Re), premixed jet, low-swirl, spherical

Motivation– kilohertz imaging

Cam Carter (AFRL)
Tonghun Lee (UIUC)
10 kilohertz CH
Reaction layer
Applied Optics B 116: 515

Steinberg, Driscoll
Michigan kHz PIV
eddies passing
through flame
Comb Flame 156, 2285



Best current models ?

<i>PREMIXED LES</i>			React Rate	authors
1	FSD	Flame surface density, also called F-TACLES = tabulated chemistry LES	FSD eqn w flamelet state relns	Bray, Vervisch Veynante, Fureby Ihme,
2	FPV	Flamelet progress variable	flamelet state relns	Moin, Ihme Pitsch, Kempf
3	TFM	Thickened flamelet model	flamelet	Poinsot
4	CMC	Conditional Moment Closure	flamelet	Swami, Huh
5	G Eqn	G equation	G-Eqn	Pitsch, Bai
6	LEM	Linear eddy model	LEM	Menon
<i>NON PREMIXED LES</i>				
1	SSLF	Steady strained laminar flamelet Z eqn with scalar dissipation rate	strained flamelet	Peters, Pitsch
2	FPV	Flamelet progress variable Z and c eqns	flamelet state relns	Moin
3	CMC	Conditional Moment Closure	flamelet	Bilger, Masri
4	PDF	PDF transport method	parcels	Pope

References – Premixed models

■ **FSD = Flame surface density LES models F-TACLES**

Mercier, Veynante, PROCI 35, 1259, Hawkes, Cant, Comb Flame 126, 1617
See, Ihme, PROCI 35, 125, Duwig, Flow,Turb Comb 79, 433

FPV = Flamelet progress variable

Pierce, Moin, J. Fluid Mech 504-73, Chen, Ihme Comb Flame 160, 2896
Lamouroux, Ihme, Comb Flame 161, 2120

TFM = Thickened flamelet model

Selle, Poinot, Comb Flame 137, 489, Esclapez, Cuenot, PROCI 35, 3133
De, Acharya, Comb Sci Technol 181, 1231

CMC = Conditional Moment Closure

Amzin, Swaminathan, Comb Sci Tech 184, 1743, Amzin Cant, Comb Sci Tech 187, 1705

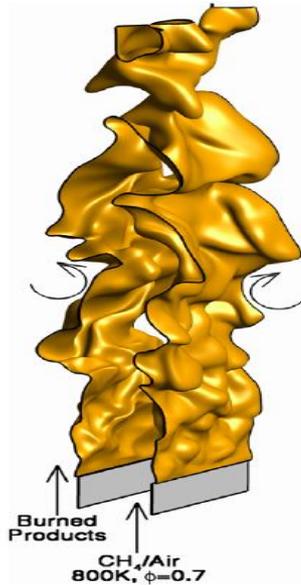
G-Equation

Knudsen, Pitsch, Comb Flame 159, 242, Nogenmyr, Comb Flame 156, 25

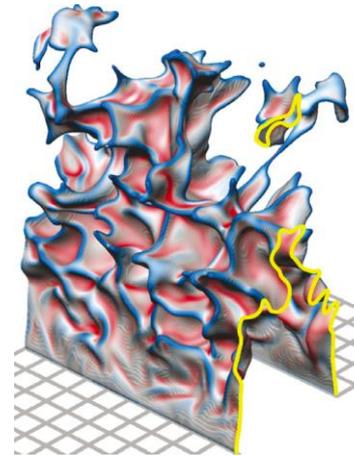
LEM = Linear Eddy Model

Srinivasan, Menon, FlowTurb Comb 94, 237, Sankaran, Menon, PROCI 30, 575

Motivation: DNS examples (at low Reynolds numbers)



R. Sankaran,
E. Hawkes,
Jackie Chen) &
T. Lu, C. K. Law
Premixed DNS
PROCI 31, 1291



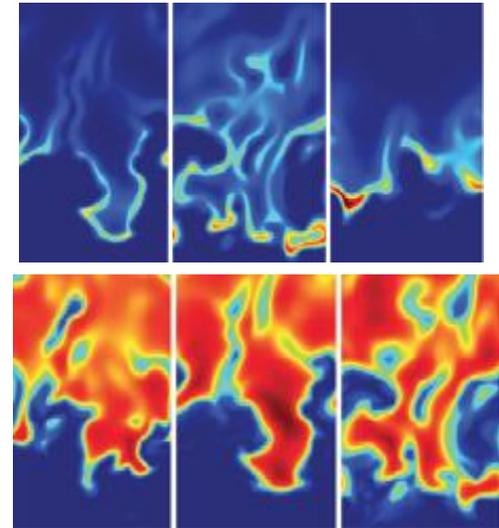
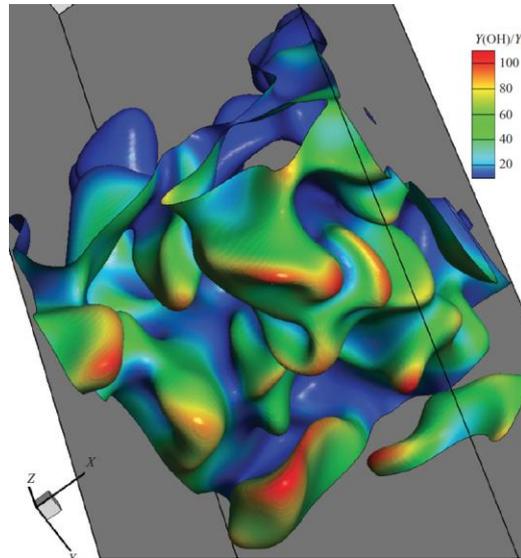
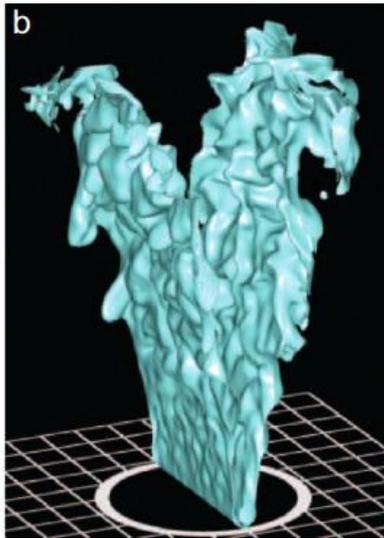
Bell, Day,
Driscoll
"corrugated"
premixed DNS
PROCI 31, 1299

DNS

flame in rectangular duct - L. Vervisch, A. Poludnenko,
Bunsen, V-flame, jet in cross-flow J.H. Chen
IC engine, Gas turbine, industrial burner:

RANS = KIVA (FSD/coherent flamelet), Fluent (empirical)

DNS - 3-D, complex chemistry



Bell, Proc Natl Acad
Sciences 102, 29
10006–10011

Aspden J. Fluid Mech 680, 287
Flame in a duct – periodic boundary conditions
Sees broken reactions
Claims to see distributed reactions

Background – what does turbulence do ?

Faster mixing - large turbulent diffusivity, shorter flames, smaller combustors

Faster propagation - large turbulent burning velocity avoids blowout

Optimize liftoff height - keep flame away from walls but do not blowout

Reduce Pollutants - less NO_x if velocities are large (for short residence times)
and fuel-air mixing is fast and lean

Avoid growl - combustion oscillations in gas turbines, rockets, due to poor flame anchoring

Challenge: to model the turbulent reaction rate for
non-premixed, premixed and partially-premixed combustion
what are the best current models, and how good are they ?



Background

turbulent kinetic energy

$$k = \left(\frac{1}{2}\right) (\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) = \left(\frac{3}{2}\right) \overline{u'^2} \text{ if isotropic}$$

Favre (density-weighted) average

$$\tilde{u} = \frac{\overline{\rho u}}{\bar{\rho}} = \frac{\overline{(\bar{\rho} + \rho')(\bar{u} + u')}}{\bar{\rho}} = \bar{\rho} \frac{\bar{u}}{\bar{\rho}} + \frac{\overline{\rho' u'}}{\bar{\rho}} \quad \text{so:}$$

$$\tilde{u} = \bar{u} + \frac{\overline{\rho' u'}}{\bar{\rho}}$$

replacing mean quantities in Navier Stokes Equations with their Favre averages removes all the density fluctuations - this removes one unknown (ρ') but it adds one new unknown: turbulent mass flux: $\overline{\rho' u'}$ which we determine using:

$$\overline{\rho' u'} = -D_T \frac{\partial \bar{\rho}}{\partial x}$$

Prandtl's gradient diffusion assumption



Gradient diffusion assumption - relates Favre avg to time average

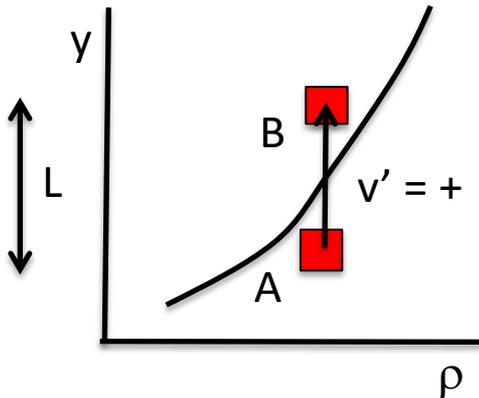
$$\tilde{u} = \bar{u} + \frac{\overline{\rho' u'}}{\bar{\rho}}$$

$$\overline{\rho' u'} = -D_T \frac{\partial \bar{\rho}}{\partial x}$$

Favre average gets rid of one unknown (ρ') but introduces another one: $\overline{\rho' u'}$

$$\overline{\rho' v'} = -D_T \frac{\partial \bar{\rho}}{\partial y}$$

Gradient
Diffusion
assumption



Suppose $\frac{\partial \bar{\rho}}{\partial y}$ and v' are positive

Small element will carry a low density from A To the higher density region at B. Therefore fluid at B will see a low density fluctuation, thus ρ' is negative at B

So: $\rho' = -L \frac{\partial \bar{\rho}}{\partial y}$ where $L =$ integral scale

Thus $\overline{\rho' v'} = -L \overline{v' \frac{\partial \bar{\rho}}{\partial y}}$ taking time average \rightarrow eqn above

We also proved that

$$D_T = v' L$$



Mixing: Turbulent viscosity = diffusivity is large

Since molecular viscosity $\mu / \rho = (\text{speed of sound}) (\text{mean free path})$

Prandtl suggested: $\mu_T / \rho = (u' L) \quad L = \text{integral scale}$

Dissipation rate of TKE $\varepsilon = u'^3 / L \quad \text{see turbulence text by Tennekes}$

Since $u' = k^{1/2}$ then $\varepsilon = k^{3/2} / L$ or $L = k^{3/2} / \varepsilon$

Since we said above $\mu_T = \rho (u') (L)$, then:
$$\mu_T = c_\mu \bar{\rho} \frac{k^2}{\varepsilon}$$

How do we compute k and ε ? Prandtl suggested the k - equation

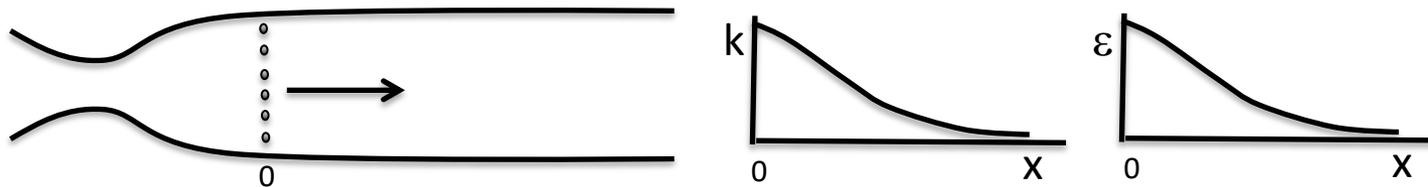


How to compute turbulence level k and dissipation rate ε ?

$$\bar{\rho} \tilde{u} \frac{\partial k}{\partial x} + \bar{\rho} \tilde{v} \frac{\partial k}{\partial y} = \frac{\partial}{\partial y} \left(\mu_T \frac{\partial k}{\partial y} \right) + \mu_T \left(\frac{\partial \tilde{u}}{\partial y} \right)^2 - \bar{\rho} \varepsilon \quad \text{k-epsilon equations}$$

$$\bar{\rho} \tilde{u} \frac{\partial \varepsilon}{\partial x} + \bar{\rho} \tilde{v} \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y} \left(\mu_T \frac{\partial \varepsilon}{\partial y} \right) + C_\varepsilon C_D \bar{\rho} k \left(\frac{\partial \tilde{u}}{\partial y} \right)^2 - \frac{C_\varepsilon \bar{\rho} (\varepsilon)^2}{C_D k}$$

convection = diffusion + creation - destruction



Example: Grid in a Wind tunnel $\bar{\rho}, \tilde{u}$ are constant, $\frac{\partial}{\partial y} = 0$ at $x > 0$ behind grid

Given: at $x = 0$ (grid) $k = k_0 = 10 \text{ m}^2/\text{s}^2$ and $\varepsilon = \varepsilon_0 = 1 \text{ (m/s)}^3/\text{m}$ then integrate:

$$\tilde{u} \frac{\partial k}{\partial x} = -\varepsilon \quad \text{and} \quad \tilde{u} \frac{\partial \varepsilon}{\partial x} = -\frac{C_\varepsilon (\varepsilon)^2}{C_D k}$$

Solve on
MATLAB



Non – premixed turbulent flames

Mixture fraction (Z) = mass fraction of H-atoms

$$Z = (Y_H - Y_{H,2}) / (Y_{H,1} - Y_{H,2})$$

Y_H = mass fraction of H atoms contained in all molecules at a point

$Y_{H,1}$ = mass fraction of H atoms entering in stream 1

$Y_{H,2}$ = mass fraction of H atoms entering in stream 2 = 0 for our examples

Ex. Suppose a jet of methane CH₄ is surrounded by a large stream of pure O₂.

Stream 1 = methane, $Y_{H,1} = 4/16 = 0.25$, Stream 2 = O₂, so $Y_{H,2} = 0.0$

At some point P downstream suppose we have 2 moles H₂O and 1 mole CO₂.

At that point,

$$Y_H = \{ 4 \text{ g} / [44 \text{ g} + 2(18 \text{ g})] \} = 0.05$$

$$Z = (Y_H - Y_{H,2}) / (Y_{H,1} - Y_{H,2}) = (0.05 - 0) / [0.25 - 0] = 0.20$$



Conservation equations for, Z, mass fractions of H₂ and O₂

Now consider a planar 2-D jet of H₂ surrounded by a stream of O₂

$$\begin{aligned}\bar{\rho} \tilde{u} \frac{d\tilde{Y}_{H_2}}{dx} + \bar{\rho} \tilde{v} \frac{d\tilde{Y}_{H_2}}{dy} &= \bar{\rho} D_T \frac{d^2 \tilde{Y}_{H_2}}{dy^2} + \overline{\dot{\omega}_{H_2}} \\ \bar{\rho} \tilde{u} \frac{d\tilde{Y}_{O_2}}{dx} + \bar{\rho} \tilde{v} \frac{d\tilde{Y}_{O_2}}{dy} &= \bar{\rho} D_T \frac{d^2 \tilde{Y}_{O_2}}{dy^2} + \overline{\dot{\omega}_{O_2}} \\ \bar{\rho} \tilde{u} \frac{d\tilde{Y}_{H_2O}}{dx} + \bar{\rho} \tilde{v} \frac{d\tilde{Y}_{H_2O}}{dy} &= \bar{\rho} D_T \frac{d^2 \tilde{Y}_{H_2O}}{dy^2} + \overline{\dot{\omega}_{H_2O}}\end{aligned}$$

The rate at which H₂O is formed is related to rate H₂ is consumed:

$$H_2 + \frac{1}{2} O_2 \rightarrow H_2O \quad \overline{\dot{\omega}_{H_2O}} = \left(-\frac{18}{2}\right) \overline{\dot{\omega}_{H_2}} \quad \overline{\dot{\omega}_{O_2}} = \left(\frac{16}{2}\right) \overline{\dot{\omega}_{H_2}}$$

Inserting these into the above and put in terms of Z to get:

$$\bar{\rho} \tilde{u} \frac{d\bar{Z}}{dx} + \bar{\rho} \tilde{v} \frac{d\bar{Z}}{dy} = \bar{\rho} D_T \frac{d^2 \bar{Z}}{dy^2}$$

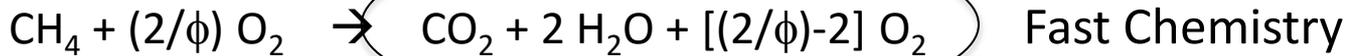
Mixture fraction
Conservation eqn
has no source term



Flamelet idea: turbulent flame is filled with wrinkled, strained flamelets

State relation - relates mass fraction of O_2 at any point to mixture fraction Z

For our methane mixing into O_2 problem, for lean combustion at any equivalence ratio ϕ :



The gas mixture at any point that resulted from lean combustion must be the mixture in the oval. Assuming fast chemistry. How to relate ϕ to Z ?

For the mixture in the oval, $Z = 4 / [44 + 2(18) + (2/\phi - 2)32] / Y_{H,1}$, where $Y_{H,1} = 0.25$

Solve this for ϕ to get : $\phi = 4Z / (1-Z)$

Now what is Y_{O_2} for the mixture in the oval ? It is

$$Y_{O_2} = [(2/\phi)-2] 32 / \{44 + 36 + [(2/\phi)-2] 32\} = (4 - 4\phi) / (4 + \phi)$$

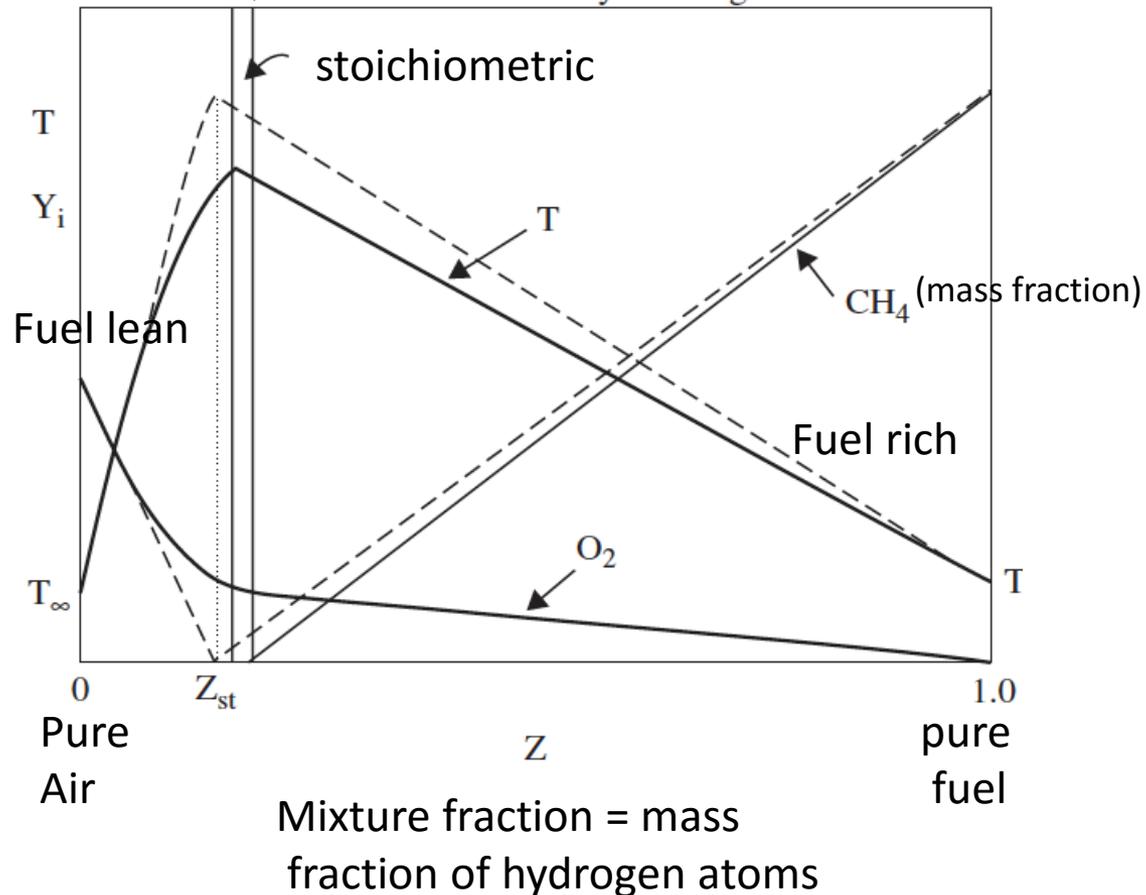
Insert into this the above formula for ϕ to get:

$$Y_{O_2} = (1 - 5 Z)$$

(state relation for lean combustion,
and Z only varies from 0 to 0.25)



State relations for unstrained non-premixed flamelets



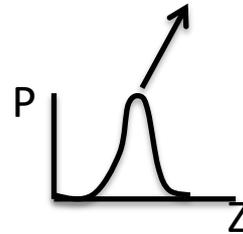
If you know the instantaneous mixture fraction (Z) at a point P, you use these state relations to look up instantaneous temperature and mass fractions at P

Probability density function - used to define a mean value

$P(c) dc$ = probability that c lies in the range between $c - dc/2$ and $c + dc/2$

$$\bar{Y}_{CO}(\bar{Z}, \overline{Z'^2}) = \int_0^1 Y_{CO}(Z) P(Z, \bar{Z}, \overline{Z'^2}) dZ$$

State relation =
Mass fraction of CO
Conditioned on Z



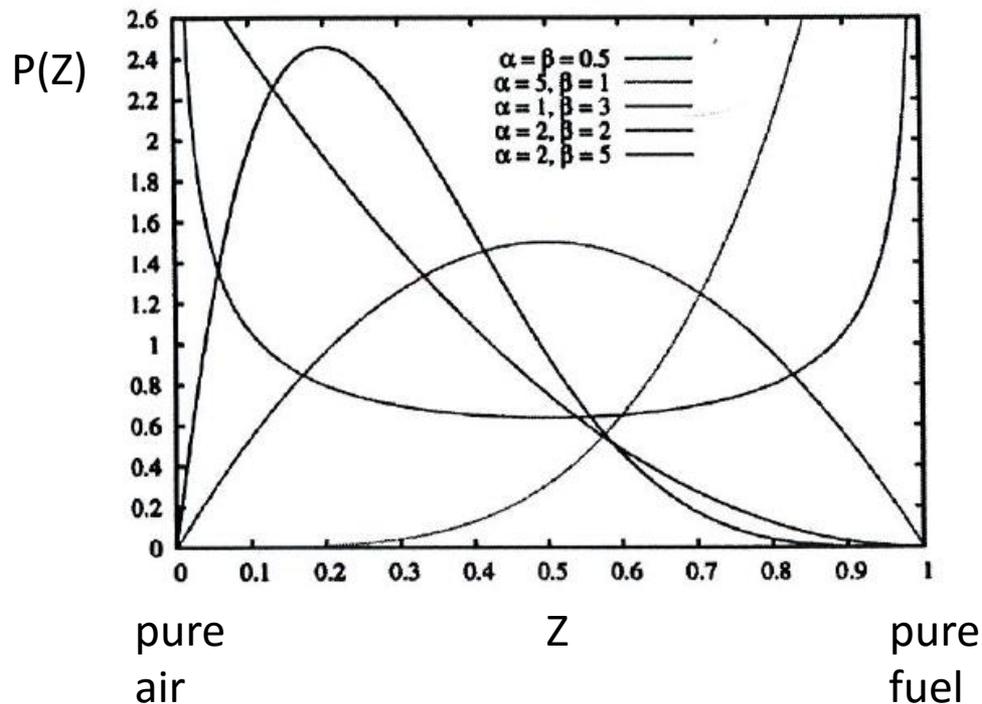
At each point in the flame, we solve conservation equations to get the mean mixture fraction and variance in mixture fraction $\overline{Z'^2}$

Idea: you only have to solve conservation equations for \bar{Z} and $\overline{Z'^2}$
and use above integral to get other mean values;
you avoid solving more conservation equations for each variable

Assume PDF (Z) to be a Beta function for non-premixed flames

$$P(x; \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} du}$$

here $x = Z =$ mixture fraction
 α and β are related to mean
and variance of Z



Importance of state relations

We could solve partial differential equations for every variable, but then we would need source terms for each. Instead use Schvab-Zeldovich approach:

Assume turbulent diffusivity of mass (D_T) equals that of momentum (ν_T) and that of heat (α_T)

- Premixed flames: solve only the differential equation for non-dimensional temperature, called reactedness (this is the energy eqn) and its variance
- Non-premixed flames: solve only the differential equations for mean mixture fraction (the Z equation) and its variance
- Then use **state relations** to compute mean values of ρ , T, Y_i and turbulent reaction rates, using:

$$\bar{Y}_{CO}(\bar{Z}, \overline{Z'^2}) = \int_0^1 Y_{CO}(Z) P(Z, \bar{Z}, \overline{Z'^2}) dZ$$

- Where to get the state relations? From equilibrium chemistry, or from strained flamelet (non-equil) chemistry – we will discuss



Turbulent reaction rate $\overline{\dot{\omega}}$ - the biggest unknown

The conservation equation for time-averaged CO mass fraction is:

$$\bar{\rho} \bar{u} \frac{d\bar{Y}_{CO}}{dx} + \bar{\rho} \bar{v} \frac{d\bar{Y}_{CO}}{dy} = \bar{\rho} \alpha_T \frac{d^2 \bar{Y}_{CO}}{dy^2} + \overline{\dot{\omega}_{CO}} \quad \leftarrow \text{kg/s/m}^3 \text{ CO produced} = ?$$

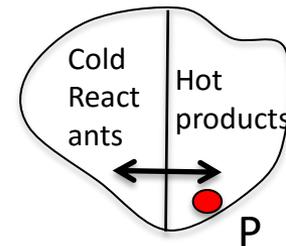
Turbulence-chemistry interaction – why do we need PDFs ?

Reaction rate depends on the JOINT PROBABILITY that:
sufficient fuel, sufficient O₂ and sufficient temperature are simultaneously present

You could ignore interactions and say the reaction rate is:

$$\overline{\dot{\omega}_{CO}} = \bar{Y}_{fuel} \bar{Y}_{O_2} \exp(-E/RT)$$

That would be WRONG. Suppose a glob of pure reactants (cold) and a glob of pure products (hot) oscillate over point P, as shown:



Why we need PDFs, continued

The actual temperature at P is 300 K when the reactants are present = no reaction

When products are present at P, temperature is 2100 K but no reactants = no reaction

If you use the time-average temperature at P, which is $2400/2 = 1200$ K, and the time-average fuel-air ratio at P and plug into the above Arrhenius eqn, you compute a large reaction rate at P ! But reaction rate is nearly zero ! This is totally wrong.

Reaction occurs at P only when the thin line between the reactants and products is on P; then you have **simultaneously** the proper fuel, O₂ and temperature

The correct reaction rate of CO is the following, which has the PDF in it:

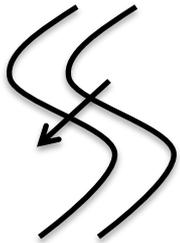
$$\overline{\dot{\omega}_{CO}}(\bar{Z}, \overline{Z'^2}) = \int_0^1 \dot{\omega}_{CO}(Z) P(Z, \bar{Z}, \overline{Z'^2}) dZ$$

Turbulent reaction rate of a premixed flame - proportional to FSD

For a premixed turbulent flame, the turbulent reaction rate at any point, in kg/sec reactants consumed/volume, is needed because it is the source term in the mean conservation of energy equation. Reaction rate is given by:

$$\overline{\dot{\omega}_R} = \rho_R S_L \Sigma$$

Proof: Consider a wrinkled flame that at time t_1 moves normal to itself at speed S_L to new position at time t_2



The volume/sec of reactants overtaken by the wave = (area of wave A_T)
Times the (distance moved /sec) of each segment of the wave
distance moved / sec = S_L laminar burning velocity

mass/sec of reactants overtaken by the wave = ρ_R (volume/sec overtaken)

So: mass/sec of reactants overtaken = $\rho_R S_L A_T$

But: $\overline{\dot{\omega}} = \text{mass/sec/volume} = \rho_R S_L (A_T/\text{volume}) = \rho_R S_L \Sigma$

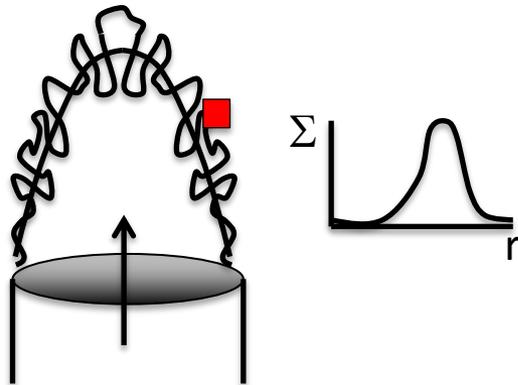
Premixed flame reaction rate depends on flame surface density

(FSD or Σ) = flame area per unit volume

$$\Sigma = \lim_{\Delta x \rightarrow 0} \frac{A_f}{(\Delta x)^3} = \lim_{\Delta x \rightarrow 0} \frac{P_f}{(\Delta x)^2}$$

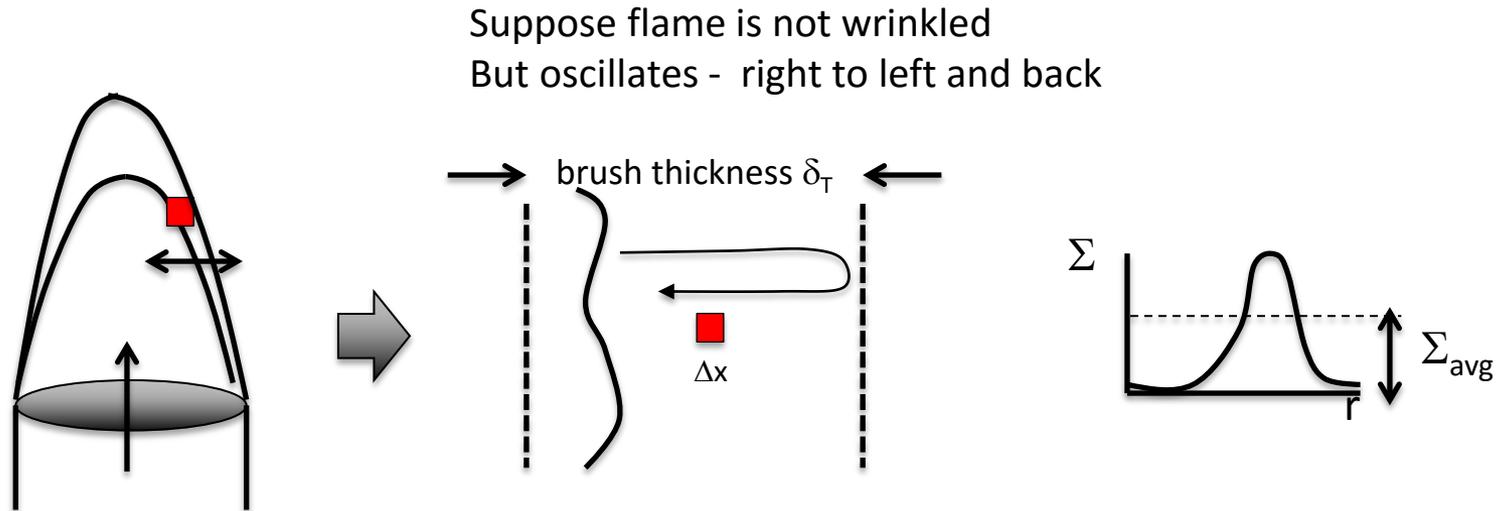
A_f = average surface area of a premixed turbulent flame inside a small 1 mm^3 interrogation box

P_f = average perimeter of flame boundary inside a 1 mm^2 box in laser sheet
= average perimeter of flame when it is inside the box, which is approximately Δx , times the probability that it is inside box



FSD is a Gaussian-like function in space

Physical meaning of FSD = flame surface density



a. perimeter of flame inside of interrogation box,
when flame is inside box = approx. Δx

b. fraction of time flame is inside box is: $\Delta x / \delta_T$

Time avg perimeter in box is $a \times b = (\Delta x)^2 / \delta_T$
Average FSD = avg perimeter / $(\Delta x)^2$ so:

FSD should be
independent of
box size Δx

Average FSD = approx. $1 / \delta_T$ (= typically 0.2 mm^{-1})

How to measure FSD ?

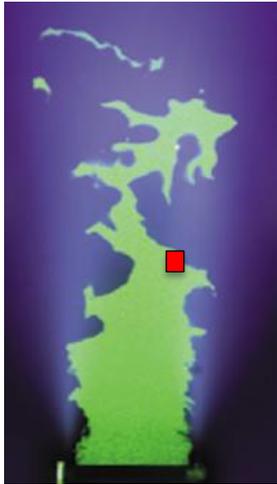


Image the flame boundary - using PLIF of OH or Mie scattering

Binarize the signal: green = 1, blue = 0

Canny edge detection - to obtain coordinates of a continuous contour, infinitely thin, fit to the flame boundary

Determine the average perimeter of this contour in the 1 mm^3 interrogation box; vary the box location

Resulting value of FSD must be independent of interrogation box size

$$\Sigma = [\text{area of flame when it is in box}] (\text{prob. flame is in box}) / (\Delta x)^3$$

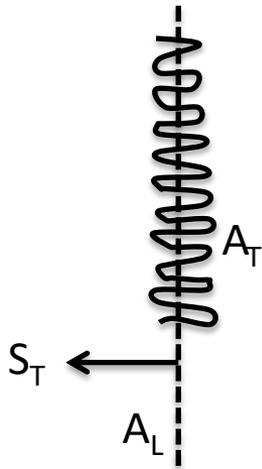
$$\Sigma = \text{approx. } [(\Delta x)^2 (\Delta x / \delta_T)] / (\Delta x)^3 \quad \text{so it should be independent of } (\Delta x)$$

$\delta_T = \text{brush thickness}$

Turbulence increases propagation speed - of a premixed flame

Damkohler first concept- moderate turbulence increases flame area by wrinkling

Consider this thin wrinkled flamelet; its wrinkled area is A_T , and the area of the straight dotted line is A_L



Each point on the wrinkled line propagates normal to the wrinkled line at a speed S_L , so the mass per second of reactants overtaken by the wrinkled line is $\rho_R S_L A_T$.

→ **So larger wrinkled area = more reactants consumed /sec**

The time-averaged wave is the dotted line; it propagates to left at S_T so the mass/sec of reactants dotted line overtakes = $\rho_R S_T A_L$

Equating these two mass flow rates, we get:

$$\frac{S_T}{S_L} = \frac{A_T}{A_L}$$

Turbulent flames propagate faster because they have more wrinkled surface area to consume the reactants



First definition of turbulent burning velocity S_T

Bray: assumed that thin flamelets propagate at S_L normal to themselves

Contribution of turbulent burning velocity due to thin flamelets is:

$$S_{T,F} / S_L = A_T / A_L$$

Where:

$$A_T = \int \Sigma dV = \iint \Sigma d\eta d\xi W$$

And $A_L = W L$ so:

$$S_{T,F} = S_L I_0 \frac{1}{L} \iint \Sigma d\eta d\xi$$

“flamelet contribution
To turbulent burning
Velocity” = total burning
Velocity if in the thin
Flamelet regime

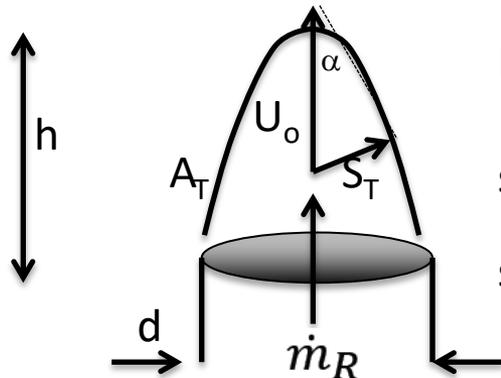
To measure $S_{T,F}$ measure Σ everywhere and
integrate it over the flame brush

Second definition of turbulent burning velocity S_T

$S_{T,GC}$ = Global consumption speed

$$S_{T,GC} = \frac{\dot{m}_R}{\rho_R A_T} = \text{mass flow reactants} / (\text{density reactants}) (\text{area of } \bar{c} = 0.5 \text{ surface})$$

How to measure $S_{T,GC}$?



From the triangle drawn:

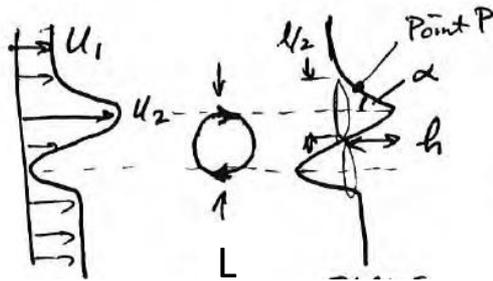
$$\sin \alpha = [d/2] [h^2 + (d/2)^2]^{-1/2} = S_T/U_o$$

so:

$$S_T = U_o (d/2) [h^2 + (d/2)^2]^{-1/2}$$

large burning velocity S_T = short flame

Damkohler's first concept - flame area A_T determines burning velocity



Imagine an eddy of diameter L moving at a stationary laminar flame at speed S_L ; the eddy causes reactant to move at higher speed U_2 at one place, and at lower speed U_1 at another place

Suppose the flame wrinkles into two Bunsen cones, where α is the cone half-angle. Similar to a Bunsen burner, the velocity normal to the wave must be S_L , and the velocity normal to the cone is $(U_2 - S_L) \sin \alpha$, so equating these gives:

$$\sin \alpha = S_L / (U_2 - S_L) \quad \text{and we define } u' = (U_2 - S_L)$$

the cone has a radius of $L/4$ and height h , so: $\sin \alpha = (L/4) [h^2 + (L/4)^2]^{-1/2}$

Equating these (and neglecting $L/4$ wrt h) yields: $h = (u' L) / (4 S_L)$

Now the surface area of a cone is $A_T = \pi/4 (L/2)^2 [(h/(L/2))^2 + 1]^{1/2}$
 and the area of the base of the cone is $A_L = \pi/4 (L/2)^2$ so:

$$\frac{S_T}{S_L} = \frac{A_T}{A_L} = \left[1 + c_1 \left(\frac{u'}{S_L} \right)^2 \right]^{1/2}$$

Predicted turbulent burning velocity
 see Kuo, Turb Combustion



Damkohler's second concept - small eddies increase thermal diffusivity

$$S_L \sim \sqrt{\alpha RR}$$

$$S_T \sim \sqrt{(\alpha + \alpha_T) RR}$$

If eddies get inside preheat zone, we assume turbulent flames propagate faster because Eddies create larger thermal diffusivity α_T

$$\alpha_T = c_2 u' L$$

turbulent diffusivity = velocity fluctuation times integral scale L ; ν = kinematic viscosity

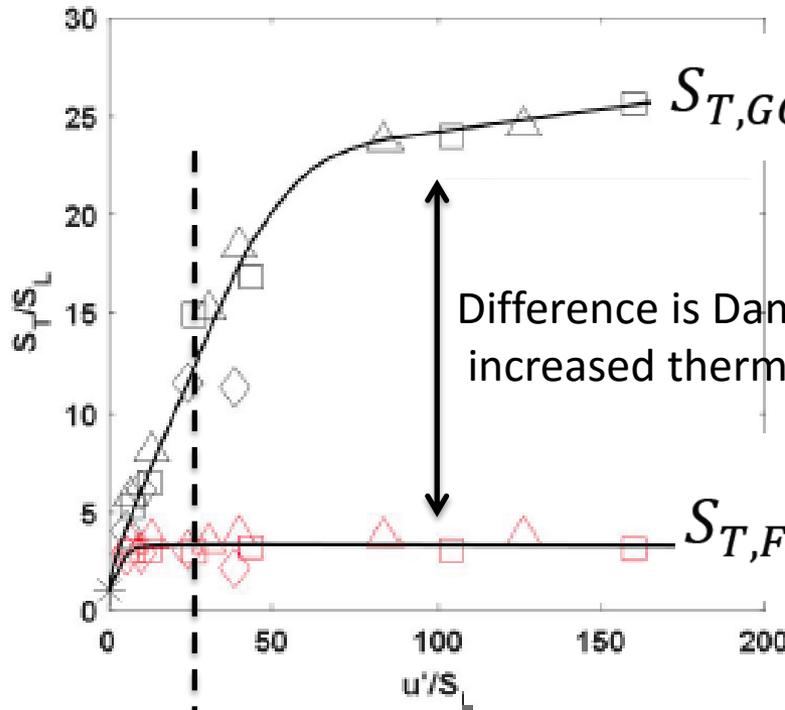
$$\frac{S_T}{S_L} = \sqrt{1 + c_2 \frac{u' L}{\nu}} \quad \text{where} \quad \frac{u' L}{\nu} = Re_T$$

Turbulent flames propagate faster because turbulence diffuses the heat upstream to preheat the reactants faster than laminar flames



Turbulent burning velocity at “extreme” turbulence levels

Wabel, Skiba, Driscoll PROCI 36



$$S_{T,GC} = \frac{\dot{m}_R}{\rho_R A_T}$$

Global consumption speed

Difference is Damkohler second idea - increased thermal diffusivity

$$S_{T,F} = S_L I_0 \frac{1}{L} \iint \Sigma d\eta d\xi$$

Contribution from Flame surface density (wrinkling)

← previous turbulence levels

→ extreme turbulence levels

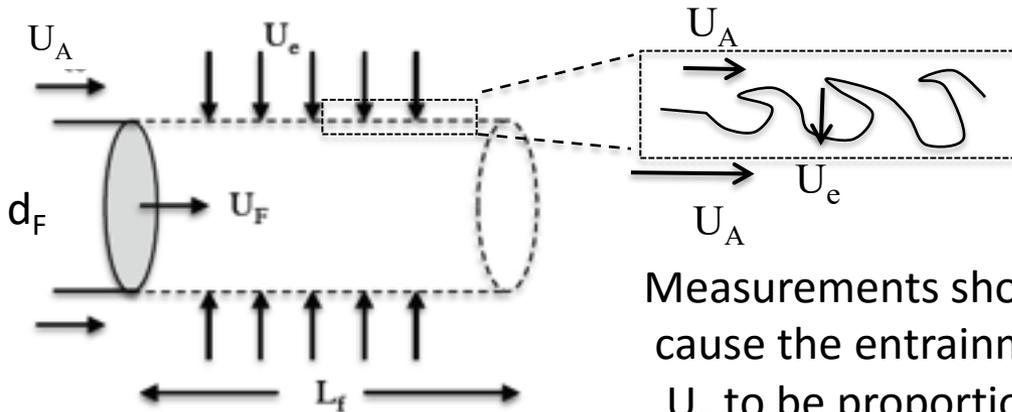
Turbulent burning velocity – what do we know ?

1. Six major canonical geometries for premixed turbulent flames
Bunsen, jet, low-swirl, V, spherical and swirl (Gas Turbine)
2. Burning velocity formula is different for each
3. Residence time (x/U) is important, bunsen flame tip becomes more wrinkled than flame base, spherical flame gets more wrinkled in time
4. Reactant temperature, Reynolds number , Karlovitz number are important
5. Role of integral scale different for each geometry - can we correlate burning velocity with Reynolds number Re_T ?

Turbulence Causes Faster Mixing = shorter flame length

Consider a non-premixed turbulent jet flame. Suppose we simplify by saying the fuel from the fuel tube - stays within the cylinder shown

Air enters at an entrainment velocity U_e that is perpendicular to the cylinder wall



(H. Rehab,
J Fluid Mech
345, 357)

Measurements show that the eddies rotate to cause the entrainment velocity

$$U_e \text{ to be proportional to } |c_1 U_F - U_A| (\rho_F / \rho_A)^{1/2}$$

The length of the flame L_f is where the mass/sec of fuel, divided by the mass/sec of entrained air, equals the stoichiometric fuel-air ratio f_s , which is 0.055 for methane

$$\rho_F U_F \left(\pi \frac{d_F^2}{4} \right) / (\rho_A U_e \pi d_F L_f) = f_s$$

Combine the above relations to eliminate U_e and solve for L_f to give :

$$\frac{L_f}{d_F} = \frac{c_2}{f_s} \left(\frac{\rho_F}{\rho_A} \right)^{1/2} \frac{U_F}{|c_1 U_F - U_A|}$$

Length of a jet flame
with co-axial air

If there is no co-flow (U_A is zero) the turbulent eddies at the edge of the cylinder are created by the jet velocity U_F so U_e is prop. to U_F and the above reduces to:

$$\frac{L_f}{d_F} = \frac{c_3}{f_s} \left(\frac{\rho_F}{\rho_A} \right)^{1/2}$$

Length of a jet flame
with NO co-axial air
 H_2-O_2 has large f_s = short

If there is strong co-flow air velocity, as in a gas turbine engine or rocket, then We neglect $c_1 U_F$ with respect to U_A so the above reduces to:

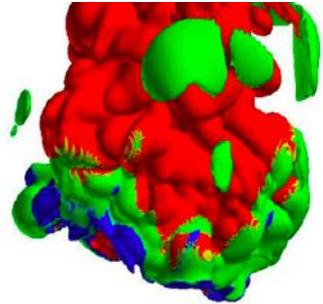
$$\frac{L_f}{d_F} = \frac{c_2}{f_s} \left(\frac{\rho_F}{\rho_A} \right)^{1/2} \frac{U_F}{|U_A|}$$

Length of a jet flame with
STRONG co-axial air
larger air velocity = shorter



Flame Liftoff and Blowout

Base of a lifted, jet flame
that is initially non-premixed



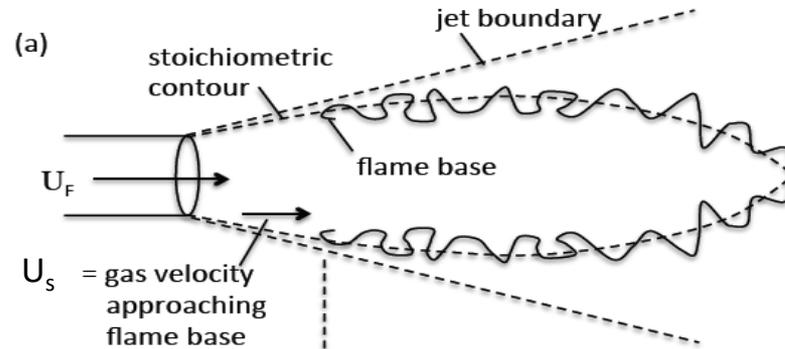
A lifted initially non-premixed jet flame
has a partially-premixed flame base

DNS of Mizobuchi, Takeno
red= rich premix, blue = lean
premix green = non-premix
PROCI 30, 2005

Cases of interest:

- | | |
|--------------------------------|--|
| Fuel jet with no coflow air | (flame base is partially premixed) |
| Fuel jet with cold coflow air | (base is partially premixed) |
| Fuel jet w very hot coflow air | (base is not a flame, is auto ignition) |
| Fuel jet in hot cross flow | (base is not a flame, is auto ignition) |
| Fuel jet w swirled air | (base is partially premixed) |
| Cavity stabilized flame | (premixed in shear layer at top of cavity) |

Definitions



Consider a fuel jet issuing into air (initially non-premixed) with

U_F = jet exit velocity Z_s = stoichiometric mixture fraction = 0.055 for methane

U_s = axial velocity of gas along the stoichiometric contour; theory says:

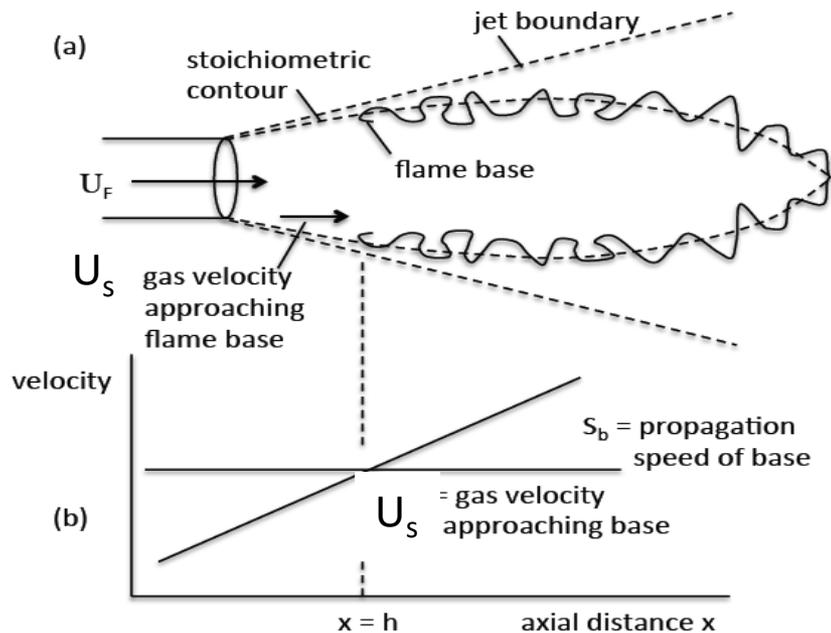
$U_s = \text{constant}$ - along the stoichiometric contour and this constant is $= U_F Z_s$

Why ? See Kuo, conservation equations for mixture fraction (Z) and (u/U_F) are identical for a jet flame

Shear layer at fuel/air boundary is **premixed** upstream of lifted flame base

S_{base} = turbulent burning velocity – propagation speed of flame base against incoming reactants

Stability criterion (VanTiggelen, Comb Flame 10, p. 59)



Two requirements
For flame to be stable:

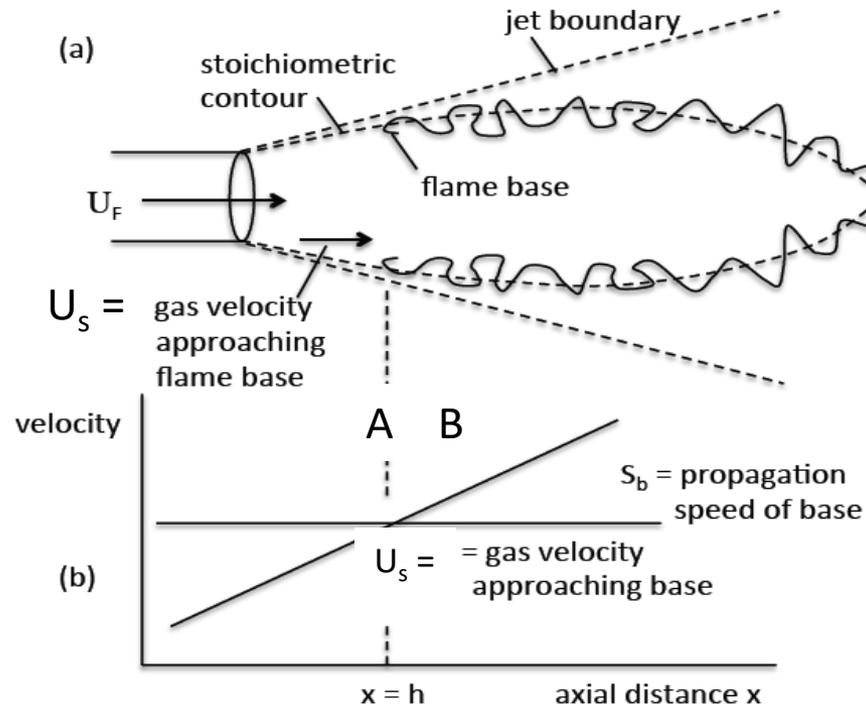
$$U_s = S_{\text{base}}$$

$$\partial S_{\text{base}}/\partial x > \partial U_s/\partial x$$

Experiments show: The propagation speed (S_{base}) decreases in the negative- x direction since strain rate on the flame increases as you approach the fuel tube
In last slide we argue that U_s is constant in x -direction

notice that the slope $dS_{\text{base}}/dx > dU_s/dx$ in this plot

Stability



Flame normally is at location A where the two curves cross ($U_s = S_{\text{base}}$)

Suppose the flame base was perturbed to move downstream to location B
 Since $\partial S_{\text{base}}/\partial x > \partial U_s/\partial x$, it follows that $S_{\text{base}} > U_s$ at location B, this means
 That the flame will propagate upstream at speed $(S_{\text{base}} - U_s)$ until it reaches A

This flame is stable – a perturbation causes it to return to its original position !

Scaling Analysis – for Liftoff height, Blowout velocity jet flame

we said: $U_s = S_{\text{base}}$ and $U_s = U_F Z_s$

now we need a formula to estimate S_{base}

Define Karlovitz number = nondim strain rate = $(U_s / x) / (S_L / \alpha)$

Assume: $S_{\text{base}} = S_L [1 + Ka]^{-2}$ if Ka is large, S_{base} is small

S_L = laminar burning velocity, stoichiometric

Liftoff height $h = x$ and we said: $U_s = U_F Z_s$

Combine above, liftoff height is: $h = \{c_1 U_F / (S_L^2 / \alpha)\} F$

where function F is nearly constant

Larger jet velocity U_F – higher liftoff height



Blowout velocity of a jet flame

There is no stoichiometric location downstream of $x = L_f =$ mixing length to stoich.

So when liftoff height (h) of flame base exceeds (L_f) – blowout occurs

$$h = L_f \quad L_f = c_2 d_F Z_s^{-1} \quad \text{from previous slide}$$

$$h = c_1 U_F / (S_{L,0}^2 / \alpha) \quad \text{from previous slide}$$

Combine these three equations and solve for U_F

Blowout fuel velocity:
$$U_F = c_3 d_F / (S_L^2 / \alpha) / F$$

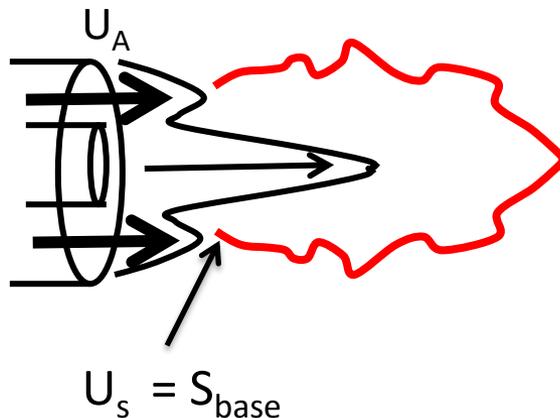
RESULT: Liftoff and blowout formulas agree with measurements !
Don't make fuel injector diameter d_F too small

Co-flowing air is de-stabilizing

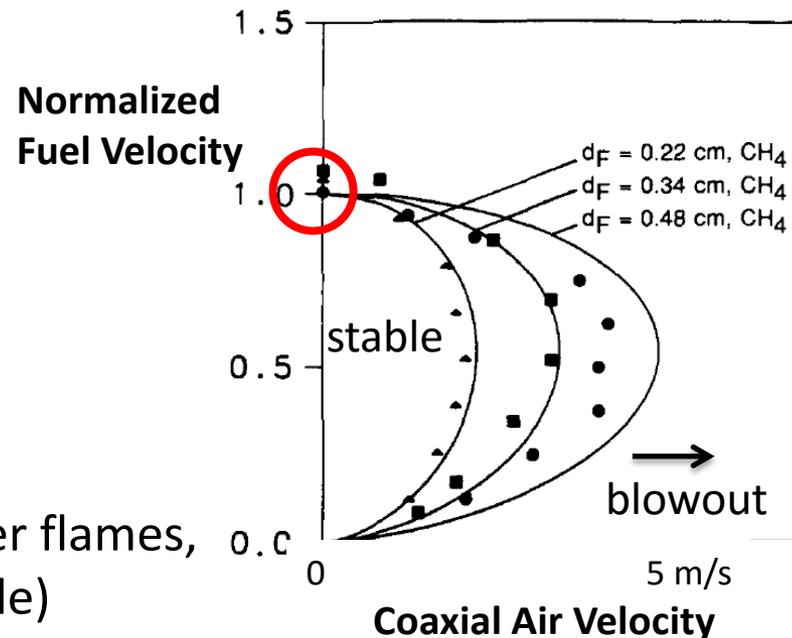
Co-flow air flows at velocity U_A , as shown below

Co-flow adds a velocity of U_A to U_s , at every point along the stoichiometric contour (red line)

Since $U_s = S_{\text{base}}$, the flame base must propagate against a larger velocity, so you must reduce the fuel velocity to avoid blowout



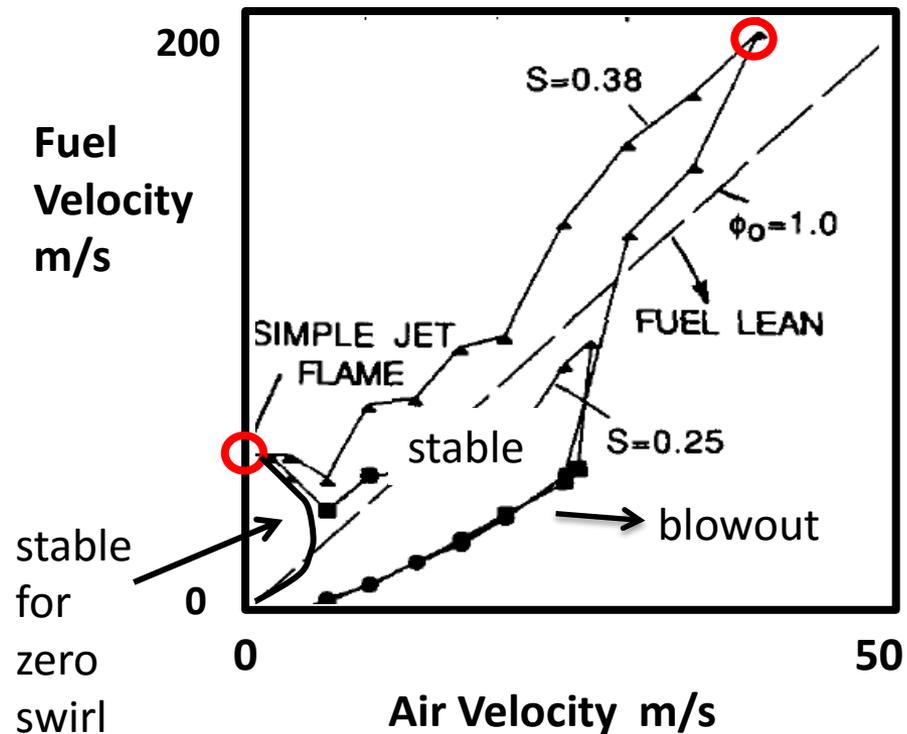
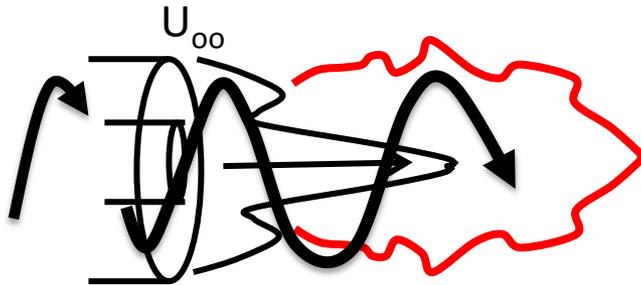
But co-flow air promotes shorter flames, faster mixing (see previous slide)



Swirl is both STABILIZING and promotes faster mixing

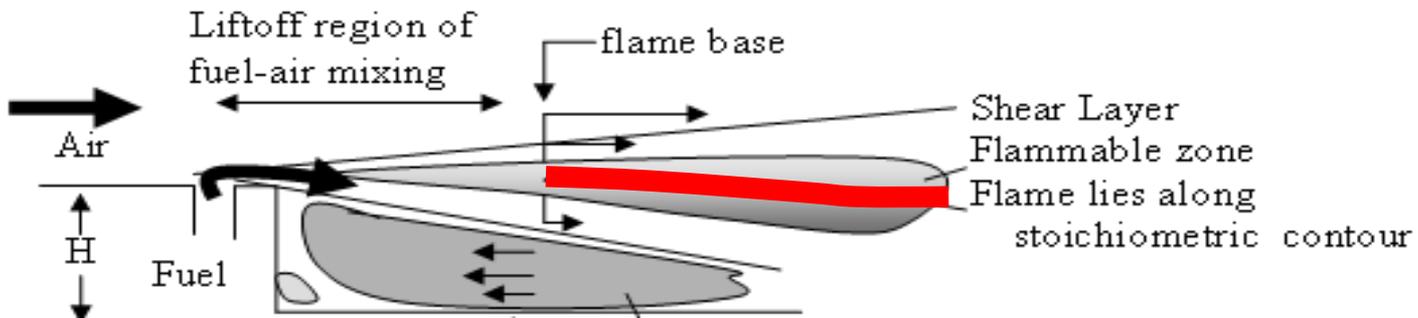
so all gas turbine designs employ swirl
swirl creates internal recirculation = low gas velocities

Much larger fuel
Velocity achieved
with swirl



Feikema, Chen, Driscoll, Combust. Flame 80, 1990

Scramjets use wall cavities to stabilize the flame

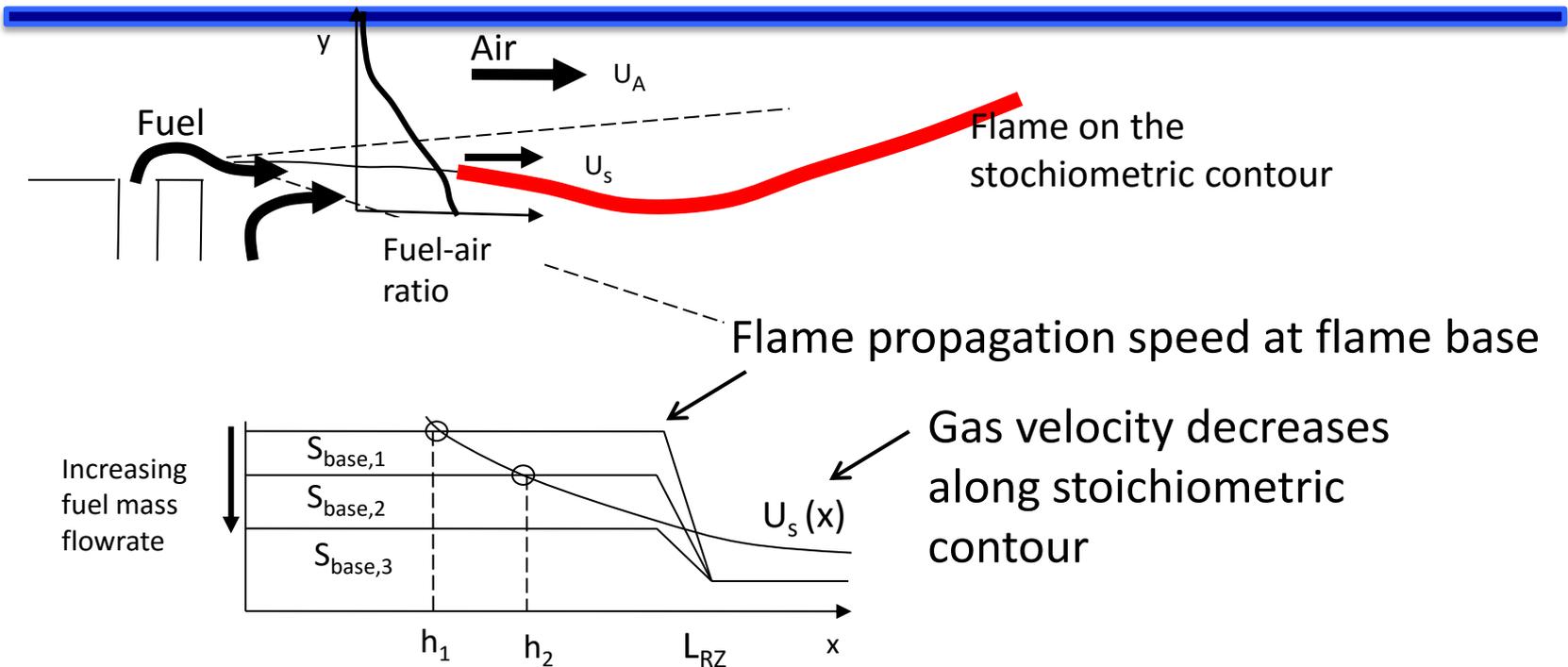


Air enters across upper side of shear layer

Fuel and hot products from recirculation zone enter across lower side of shear layer

Flame exists along stoichiometric contour

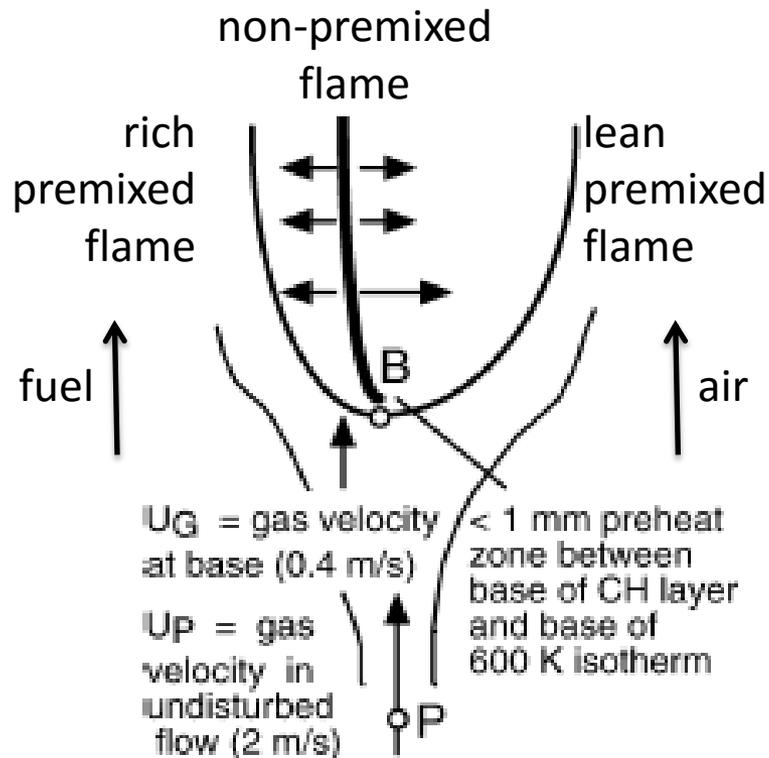
Blowout of cavity-stabilized flame – same ideas as before



1. Liftoff height (h) where propagation speed (S_{base}) = gas velocity U_g
2. Temperature at base is elevated due to hot recirculation zone
3. Too much cold fuel into RZ lowers temperatures, flame speed
4. When liftoff height h exceeds L_{RZ} = "rich blowout"
5. Stable if $\partial S_{base} / \partial x > \partial U_s / \partial x$

Base of a lifted flame is a “triple flame”

How fast does a triple flame propagate ?



References - Flame blowout

Vanquickenborne L, van Tiggelen A. Combust Flame 1966;10, p. 59

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Brown C, Lyons K Studies on lifted jet flames in coflow. Flow Turb Comb 1999, p. 249.

Driscoll, J. F. Correlation and Analysis of Blowout Limits of Flames in High-Speed Airflows, J. Propulsion Power 21, 6,1035, 2005

Mizobuchi, Takeno, PROCI 30, p. 611, 2005

