Course Outline

• A) Introduction and Outlook
• B) Flame Aerodynamics and Flashback
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• E) Flame Response to Harmonic Excitation

• Constraints and metrics
• Emissions
• Autoignition
• Future outlook for needed research

Gas Turbine Cycle

“Brayton Cycle”
– Inlet » Compressor » Combustor » Turbine » Nozzle
– \( Pr = \text{Compressor Pressure Ratio} \)

Role of Combustor within Larger Energy System

• Example: Ideal Brayton Cycle
  \[ \eta_{th} = 1 - (Pr)^{(r-1)/\gamma} \]
  \( Pr = \text{compressor pressure ratio} \)
  \( \gamma = C_p/C_v, \text{ ratio of specific heats} \)

• Conclusions
  – Combustor has little effect upon cycle efficiency (e.g. fuel \( \rightarrow \) kilowatts) or specific power
  – Combustor does however have important impacts on
    • Realizability of certain cycles
      – E.g., steam addition, water addition, EGR, etc.
    • Engine operational limits and transient response
    • Emissions from plant
Combustor Performance Metrics

• What are important combustor performance parameters?
  – Burns all the fuel
  – Ignites
  – Pattern Factor
  – Operability
    • Blow out
    • Combustion instability
    • Flash back
    • Autoignition
  – Low pollutant emissions
  – Fuel flexibility
  – Good turndown
  – Transient response

Combustor Architectures

• Different approaches for distributing a given \( m_{\text{air}} \) and \( m_{\text{fuel}} \) to achieve a specified global stoichiometry and exit temperature:
  – Lean, premixed
    – Fuel and air premixed ahead of flame to control stoichiometry
    – Method used in low NO\(_x\) gas turbines
  – Non-premixed
    – Mixture burns at \( \phi=1 \); produces more NO\(_x\), soot
    – More robust, higher turndown, fuel flexible, simpler
  – RQL (Rich-Quick/Quench-Lean)
    – Fuel and primary air premixed ahead of rich flame; secondary air/fuel injection to lower \( \phi<1 \)
    – Standard design for aircraft engines (lots of turndown) and combusting fuels with nitrogen

Conventional Diffusion/Non-Premixed Flame Combustor

• Global fuel/air ratio controlled by turbine inlet temperature requirements
• Staging used to achieve turndown and stable flame
  – Air is axially staged in this image
  – Non-premixed flame in “primary zone”

Combustor Configurations

Dry, Low NO\(_x\) (DLN) Systems

• Premixed operation
  – If liquid fueled, must prevaporize fuel (lean, premixed, prevaporized, LPP)
• Almost all air goes through front end of combustor for fuel lean operation – little available for cooling
• Multiple nozzles required for turndown
**Can Combustion Layout**
- Needs cross-fire tubes
- Useful testing can be done with limited air supplies

**Annular Combustor Layout**
- Aircraft engines
- Aero-derivatives
- Siemens V-series
- Alstom GT24

**Frame Engine Layouts**
- Can access combustors without requiring engine disassembly
- Silo combustors

**Aero-Derivative Combustors**
Combustor Configurations
Dry, Low NOx (DLN) Systems

- More complicated staging schemes required for turndown

Tradeoffs and Challenges
Cost/Complexity ↔ Turndown

Combustion Instabilities ↔ Blowoff

Emissions
NOX, CO, CO2

Natural Gas Composition Variability

Source: C. Carson, Rolls Royce Canada
Useful Fuel Grouping

- Higher Hydrocarbons
  - \( \text{C}_2\text{H}_6 \) - ethane
  - \( \text{C}_3\text{H}_8 \) - propane
  - \( \text{C}_4\text{H}_{10} \), ….
  - \( \text{C}_{10}\text{H}_{22} \) (decane, large constituent of jet fuel)
  - \( \text{C}_{12}\text{H}_{26} \) (dodecane – large constituent of diesel fuel)
- \( \text{H}_2 \) content
- "Inerts"
  - \( \text{N}_2 \) - Nitrogen
  - \( \text{CO}_2 \) – Carbon Dioxide
  - \( \text{H}_2\text{O} \) – Water

autoignition, combustion instabilities, \( \text{NO}_2 \) emissions

flashback, combustion instabilities

blowoff, \( \text{CO} \) emissions, combustion instabilities

Combustion Instabilities

- Single largest issue associated with development of low \( \text{NO}_X \) GT’s
- Designs make systems susceptible to large amplitude acoustic pulsations

Operability issues of low \( \text{NO}_X \) technologies

- Power
  - Example: Broken part replacement largest non-fuel related cost for F class gas turbines
- Industrial
- Residential
  - Example: issues in EU with deployment of low \( \text{NO}_X \) water heaters, burners


Turndown

- Operational flexibility has been substantially crimped in low \( \text{NO}_X \) technologies
- Significant number of combined cycle plants being cycled on and off daily
Transient Response Needs

- Locations with high penetration of wind and photovoltaic solar are seeing significant transient response needs
- Avoiding blowoff and flashback are key issues

Blowoff

- Low NO\textsubscript{X} designs make flame stabilization more problematic

Autoignition

- Liquid fuels
- Higher hydrocarbons in natural gas
- Poor control of dewpoint

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Emissions

- NOX – Reactions with nitrogen in air and/or fuel
- CO – Incomplete or rich combustion
- UHC – Incomplete combustion
- SOX – sulfur in fuel
- Particulates (soot, smoke)
- CO2 and H2O? – Major project of hydrocarbon combustion

Equilibrium Hydrocarbon/Air Combustion Products

- Major products:
  - Lean: CO2, H2O, O2
  - Rich: CO2, CO, H2O, H2, O2

NOx Emissions

- NOx stands for Nitrogen Oxides
  - NO, N2O, NO2
- Different mechanisms for NOx formation
  - $\text{Nox}=\text{NOx}_{\text{flame}}+\text{NOx}_{\text{post-flame}}$
  - $=a+b\tau_{\text{residence}}$
- Flame generated NOx
  - N2O
  - Prompt NOx
  - NNH
  - Fuel NOx
- Post-flame NOx
  - Zeldovich reaction (Thermal NOx)
Equilibrium Pollutant Concentrations, NO and NO₂

- NO levels pressure independent
- Most NOₓ formed at combustion conditions is NO, not NO₂
  - NO converted to NO₂ in atmosphere (note crossover at low temps)
- NO emissions from lean, premixed combustors strongly influenced by non-equilibrium phenomenon
  - NO usually increases with pressure, \( p_n \) (n~0.5-0.8)
  - Non-equilibrium NO values less than equilibrium values

Pollutant Trends, Thermal NOₓ

- Primarily formed at high temperatures (>1800 K), due to reaction of atmospheric oxygen and nitrogen
  - Water/steam injection used to cool flame in nonpremixed combustors
  - Fuel lean operation to minimize flame temperature is a standard strategy in DLN combustors

Zeldovich Reaction

- Reaction 1: \( O + N₂ \Rightarrow NO + N \)
- Reaction 2: \( N + O₂ \Rightarrow NO + O \)
- Net reaction: \( N₂ + O₂ \Rightarrow 2NO \)
- Reaction rate increases exponentially with flame temperature
- Often called “thermal” NOₓ

CH₄/Air, varying \( T_{ad} \), \( p=15\text{atm} \), Tin=635K (t = 0, taken at T = 640K)
CO Emissions

- A simple 2 step conceptualization of CO formation and oxidation is
  - Step 1: Fuel reacts to form “intermediate species”, including CO
  - Step 2: CO reacts to form CO2
- Without “step 2”, you get CO emissions!

Quenching Leads to CO

- Step 2 will not happen if the combustion products are “quenched” or cooled prematurely
  - Occurs at low temperatures where insufficient residence time to oxidize CO
  - Occurs where cooling air is mixed into the flow
- CO levels relax down toward equilibrium – i.e., longer residence time is better
- Step 2 will also not happen during fuel-rich combustion

CH4/Air, varying Tad, p=15atm, Tin=635K (t = 0, taken at T = 640K)

Equilibrium Pollutant Concentrations, CO

Equilibrium CO levels for reaction
Equilibrium Pollutant Concentrations, CO

• CO emissions from lean, premixed combustors strongly influenced by non-equilibrium effects
  – Near equilibrium for range of \( \phi \) values
  – Rapid departure from equilibrium for low \( \phi \)
  – Occurs due to quenching of reactions
  – Thus, non-equilibrium effects cause CO levels to exceed their equilibrium values

KO

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SOx Emissions

• SOx (SO2 and SO3)
• SO3 reacts with water to form sulfuric acid
  \[ \text{SO}_3 + \text{H}_2\text{O} \rightarrow \text{H}_2\text{SO}_4 \]
  – Occurs with fuels containing sulfur, such as coal or residual oils
  – Very high conversion efficiency of fuel bound sulfur to SOx
    • i.e., can’t minimize SOx emissions through combustion process (as can be done for NOx), it must be removed in pre- or post-treatment stage

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NOx-CO Tradeoff

• Almost always
  – Low power operation limited by CO
  – High power limited by NOx
  – Competing trends in terms of temperature and residence time

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Particulate Matter

• Fine carbon particles formed in flame
• Particles may or may not make it through flame
• Competition between soot formation and soot ‘burn-out’
• Nearly zero in lean, premixed flames
• Occurs in fuel-rich flames and diffusion flames
• Cause of yellow luminosity in flames
• Increases radiative heat transfer loading to combustor liners
• Particulate matter in exhaust related to respiratory ailments in humans
• Small particles ingested into lungs
• May contain adsorbed carcinogens

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NOx-Efficiency (CO2) Tradeoffs

- Future turbine efficiency improvements may be NOx rather than turbine inlet temperature limited!

Autoignition

- In premixed systems, premature ignition is a significant concern
  - Temperature above which a fuel-air mixture can spontaneously ignite is called the “autoignition temperature”
  - Amount of time it takes to spontaneously ignite is known as “ignition delay time”

- Competes with need for good premixing for NOx reduction

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Operability: Autoignition

- Methane has significantly higher autoignition temperatures than higher hydocarbons
  - Important consideration for LNG, particularly with high pressure ratio aeroderivatives
Auto-ignition Behavior as a function of Fuel Type

Typical compressor discharge temperatures

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Petersen’s Data – Ethane Effects

- CO2 emissions set by fuel and cycle choice
  - Sets combustion configuration and challenges
  - High pressure combustion
  - Exhaust gas recirculation
  - Pre-combustion carbon capture
  - Post-combustion carbon capture
  - Bio-fuels (near zero net CO2 emitting fuels)

Combustion challenges in a CO2 constrained world


Fig. 5 Ignition delay times for the methane/ethane blends in comparison to the methane-only data at similar pressures.
Pre-combustion Carbon Capture

- Carbon removed prior to combustion, producing high H2 fuel stream
  - IGCC
- High H2 introduces significant combustion issues
  - VERY high flame speed – causes flashback
    - Warranties generally limit H2 <5% by volume
  - Plants burning high H2 fuels use older, high NOx technology

80% H2, 20% CH4, flashback at 281 K, 1 atm, nozzle velocity of 58.7 m/s, and φ = 0.426

Significant Issues associated with generating a sequesterable exhaust

- Air:
  - O2/N2 ratio fixed
  - Stoichiometry varied to control flame temperature
- Emissions:
  - NOx a major pollutant
  - CO to a lesser extent

<table>
<thead>
<tr>
<th>Component</th>
<th>Canyon Reef</th>
<th>Weyburn pipeline</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO2</td>
<td>&gt;95%</td>
<td>96%</td>
</tr>
<tr>
<td>CO</td>
<td>-</td>
<td>0.1%</td>
</tr>
<tr>
<td>H2O</td>
<td>No free water &lt; 0.489 m³ in the vapor phase, &lt;20ppm</td>
<td>&lt;300ppm</td>
</tr>
<tr>
<td>H2S</td>
<td>&lt;1500 ppm</td>
<td>0.9%</td>
</tr>
<tr>
<td>N2</td>
<td>4%</td>
<td>&lt;300ppm</td>
</tr>
<tr>
<td>O2</td>
<td>&lt;10ppm (weight)</td>
<td>&lt;50ppm</td>
</tr>
<tr>
<td>CH4</td>
<td>0.7%</td>
<td>-</td>
</tr>
<tr>
<td>Hydrocarbons</td>
<td>&lt;5%</td>
<td>-</td>
</tr>
<tr>
<td>Temperature</td>
<td>&lt;49°C</td>
<td>-</td>
</tr>
<tr>
<td>Pressure</td>
<td>-</td>
<td>15.2 MPa</td>
</tr>
</tbody>
</table>

Table 1. Specifications for two CO2 transport pipelines for EOR

Post Combustion Carbon Capture

- Sequesterable stream preferably composed primarily of CO2 and H2O
  - Oxy-combustion
    - Control flame temperature by diluting oxygen with recycled steam or CO2
  - Exhaust gas recirculation

Kimberlina Power Plant

Challenges: Emissions

- Emissions:
  - CO: high CO2 levels lead to orders of magnitude increase in exhaust CO
  - O2: normally, a major exhaust effluent; requires operating slightly rich to minimize

CO ppm

O2 ppm
Concluding Remarks

• Many exciting challenges associated with
  – Fuel flexibility
  – Air quality emissions and CO2
  – Operational flexibility
  – Reliability
  – Low cost
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Flashback and Flameholding

• Flashback:
  – Upstream propagation of a premixed flame into a region not designed for the flame to exist
  – Occurs when the laminar and/or turbulent flame speed exceeds the local flow velocity
    • Reference flow speed and burning velocity?

• Flameholding:
  – Flame stabilizes in an undesired region of the combustor after a flashback/autoignition event
  – Problem has hysteretic elements
    • Wall temperature effects
    • Boundary layer and swirl flow stability effects

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Boundary Layer Flashback - Classical Treatment

• Neglects effects of
  – Heat release (changes approach flow)
  – Stretch (changes burning velocity)

• Flashback occurs if flame speed exceeds flow velocity at distance, \( \delta_q \), from the wall
  \[ u_x(y = \delta_q) = s_d(y = \delta_q) \]
  – Expanding velocity in a Taylor series, establish flashback condition:
  \[ u_x(y = \delta_q) \approx \frac{\partial u_x}{\partial y} \bigg|_{y=0} \delta_q = \frac{g_u \delta_q}{s_d} = 1 \]
  – Assuming \( \delta_q \ll \delta_f \), define flashback Karlovitz number
  \[ K_d = \frac{g_u \delta_f}{s_d} \]

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Boundary Layer Flashback

• Turbulent Boundary Layers
  – Multi-zoned
    • Near wall \( \rightarrow \) laminar sublayer, \( \delta_q \)
    – Basic scaling developed for laminar flows holds if:
      \[ \delta_q < \delta \]
  – Most literature data shows
    \[ g_{u, turbulent} \gg g_{u, laminar} \]
  – Significant space-time variation during flashback
    • Images suggest flame interactions with boundary layer instabilities

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Boundary Layer Flashback

• Flashback Karlovitz number approach is well validated for open flames, such as Bunsen burners
  – Performed detailed kinetics calculations to determine flame speed and thickness for several data sets
  – Shows how prior burning velocity, flame thickness tendencies can be used to understand tendencies
    • Pressure
    • Preheat temperature
    • Stoichiometry

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Coupled Effects of Flame Curvature and Gas Expansion

• Flame bulging into reactants
  – Approach flow decelerates
  – Streamlines diverge
  – Adverse pressure gradient

• Implications:
  – Boundary layers – adverse pressure gradients lead to separation
  – Swirl flows – adverse pressure gradients can lead to vortex breakdown
  – Triple flames – flame can propagate into region with velocity that is higher than flame speed
  – Flame stability – flame spontaneously develops wrinkles

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Heat Conduction Influences on Boundary Layers

- Important implications for
  - Scaling velocity gradients in shear layers
  - Flame stretch rates
  - Shear layer instability frequencies – acoustic sensitivities

Heat Release and Stretch Effects

- Particularly important in explaining flameholding phenomenon
- Once a flashback event has occurred, difficult to expel flame from combustor
- Leading point of advancing flashback event subject to positive curvature
- Effect of gas expansion due to heat release on local flow velocity

Heat Release and Stretch Effects

- Heat release modifies approach flow
- Stretch modifies burning velocity

Stretch Effects

- Leading point of advancing flashback event subject to positive curvature
  - For $Ma < 0$, this can cause:
    \[ s_d^y(y = \delta_q) >> s_d^{u,0} \]
  - $s_d^{u,0}$ can be a significant underestimate of flame speed
Heat Release Effects

- Gas expansion across a curved flame alters the approach flow
  - Resulting adverse pressure gradient ahead of flame decelerates flow
    - In extreme cases, can cause boundary layer separation
    - Approach flow “sucks” flame back into nozzle

Flow Stability and Vortex Breakdown

- The degree of swirl in the flow, \( S \), has profound influences on the flow structure
- Most prominent feature of high swirl number flows is the occurrence of “vortex breakdown”, which is manifested as a stagnation point followed by reverse flow

Prominent Features of Swirling Flows with Vortex Breakdown: Precessing Vortex Core

- The flow does not instantaneously rotate about the geometric centerline
- The location of zero azimuthal velocity is referred to as the “precessing vortex core” (PVC)
  - The frequency of rotation of the precessing vortex core scales with a Strouhal number based on axial flow velocity and diameter
  - Leads to a helical pattern in instantaneous axial flow velocity
  - Important to differentiate the PVC from the other helical shear flow structures which may also be present

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Prominent Features of Swirling Flows: Shear Layer Instability

- Shear layers exist in both span- and streamwise directions
  - Can be axisymmetric or helical

Huang and Yang, Proc. Comb. Inst., 2005

Flow Stability and Vortex Breakdown

- Vortex breakdown can be described as a “fold catastrophe”
  - Bifurcation of the possible steady state solutions to the Navier-Stokes equations

- In high Re flows, there is an intermediate swirl number range where flow is bi-stable and hysteretic
  - i.e., either vortex breakdown or no vortex breakdown flow state possible

Source: Lopez, Physics of Fluids, 1994

Core Flow Flame Propagation

- Vortex breakdown – flame interaction
  - Can occur even if flame speed everywhere less than flow speed
  - Gas expansion across a curved flame:
    1. Adverse pressure gradient & radial divergence imposed on reactants
    2. Low/negative velocity region generated upstream of flame
    3. Flame advances further into reactants
    4. Location of vortex breakdown region advances upstream
  - Due to bi-stable nature of vortex breakdown boundaries
    - CIVB itself not necessarily bi-stable
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- Introductory Concepts
- Flame Stretch
- Edge Flames
- Flame Stabilization in Shear Layers
- Flame Stabilization by Stagnation Points

Flame Stabilization and Blowoff

- Flame stabilization requires:
  - A point where the local flame speed and flow velocity match: $s_u^d = \bar{u}$

- Typically found in regions with:
  - Low flow velocity
    - Aerodynamically decelerated regions (VBB)
  - High shear
    - Locations of flow separation (ISL & OSL)

Figures:
Natarajan et al. Combustion and Flame 2007
Premixed Flame Stabilization: Basic Effects

- Flame stabilization:
  - Balance combustion wave propagation with flow velocity
    - Burning velocity, edge speed, autoignition front?
  - Suggests that stable flames are rare
    - However, flames have self-stabilizing mechanisms
      - Shear layer stabilized: Upstream flame propagation increases wall quenching
      - Aerodynamic stabilization – velocity profiles

Flame Stabilization and Blowoff

- Stabilization locations determine location/spatial distribution of flame
  - Flame Shape
  - Flame Length

- Combustor operability, durability, and emissions directly tied to these fundamental characteristics affecting:
  - Heat loadings to combustor hardware
  - Combustion instability boundaries
  - Blowoff limits

Flame Anchoring Locations

- Complex flows can provide multiple anchor locations

Review of the Idealized Premixed Flame

- Simplest possible premixed flame configuration
  - 1-dimensional, planar
  - Adiabatic
- \( T_0 = T_{ad} \rightarrow \text{Adiabatic Flame Temperature} \)
- \( \dot{\rho}_u u_u = \dot{\rho}_L S_L^0 = \dot{\rho}_F u_F \rightarrow \text{Mass Burning Flux} \)
- What are the controlling parameters?
  - Thermal and mass diffusivities
  - Reaction rates
  - Temperature of reactants
  - Pressure
  - Exothermicity of fuel/oxidizer
- \( S_L^0 \rightarrow \text{Fundamental property of fuel/oxidizer mixture} \)
What Happens if a Flame isn’t Flat?

- Reaction zone
- Preheat zone
- Streamtube

What Happens if Flow Field isn’t 1-D?

- Reaction zone
- Preheat zone
- Streamtube

• Common theme to 2 problems:
  misaligned convective and diffusive fluxes

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Overview of Flame Stretch

- Consider a stationary flame; focus on a C.V. intersection of a streamtube, and the flame.
- Steady state energy balance
  • (no viscous effects, no body forces)

\[
\rho U \cdot V h_t = -\nabla \cdot \tilde{q}
\]

- Constitutive relation of enthalpy flux
  (Fickian diffusion, no radiative heat transfer, no Soret or DuFour effects)

\[
\tilde{q} = -k_t \nabla T - \rho \sum h_i \nabla Y_i
\]
Overview of Flame Stretch

- Flame stretch effects: is there a net enthalpy loss/gain or change in composition inside the C.V. because of diffusion fluxes through its lateral surface?
- Two mechanisms:
  - Lewis Number effects \( Le = \alpha / \phi \)
    Diffusion of mass and of heat are unbalanced.
  - Differential diffusion effects \( \partial_{\text{Fuel}} / \partial_{\text{Ox}} \)
    Lighter species diffuse faster than heavier species: equivalence ratio or diluent/reactant ratio inside the C.V. can change.

Differential diffusion effects

Consider the temperature at the tip of a Bunsen Flame \( T_{\text{tip}} \):

- \( \text{CH}_4/\text{Air} \):
  - \( \text{CH}_4 \) is lighter than \( \text{O}_2 \), thus diffuses faster (\( \partial_{\text{Fuel}} > \partial_{\text{Ox}} \)) and its concentration inside the C.V. decreases.
  - If overall equivalence ratio is lean, then locally in the C.V. \( \phi \) is made leaner: \( T_{\text{tip}} \) is lower then \( T_{\text{ad}} \) calculated at the overall \( \phi \).
  - If overall equivalence ratio is rich, then locally in the C.V. \( \phi \) pulled toward stoichiometric: \( T_{\text{tip}} \) is then higher then \( T_{\text{ad}} \) calculated at the overall \( \phi \).
- \( \text{C}_3\text{H}_8/\text{Air} \):
  - \( \text{C}_3\text{H}_8 \) is heavier than \( \text{O}_2 \), thus diffuses more slowly (\( \partial_{\text{Fuel}} < \partial_{\text{Ox}} \)) and its concentration inside the C.V. increases.
  - The dependence of \( T_{\text{tip}} \) on the overall \( \phi \) is opposite to that of \( \text{CH}_4/\text{Air} \).

Lewis number effects

- If \( Le = 1 \) then \( \alpha = \phi \): no net enthalpy loss through the lateral surface
  
  Energy Eq.: \( \rho u \cdot \nabla h_p = \nabla \cdot (\rho \phi \nabla h_p) \)

- If \( Le > 1 \) then \( \alpha > \phi \): Heat flux > Mass flux
  - Positive stretch: net enthalpy flux out of the C.V.
  - Negative stretch: net enthalpy flux into the C.V.

- If \( Le < 1 \) then \( \alpha < \phi \): Heat flux < Mass flux
  - Positive stretch: net enthalpy flux into the C.V.
  - Negative stretch: net enthalpy flux out of the C.V.
Example: Tips of Bunsen Flames

Propane(C₃H₈)  Methane(CH₄)

\[ d = 10 \text{ mm} \]

\[ \phi = 1.38 \quad 0.53 \quad 1.52 \quad 0.58 \]


Mathematical expressions of stretch \( \kappa \)

\[ \kappa = \frac{1}{A} \frac{dA}{dt} \quad \text{Williams (1975)} \]

- Flame stretch rate is defined as the normalized differential change with respect to time of an infinitesimal flame surface area element.
- Flame stretch rate quantifies the degree of stretch imposed on a differential flame surface element.
  - Lagrangian quantity.
  - Units of flame stretch are 1/s – i.e. an inverse time scale.

Mathematical expressions of stretch \( \kappa \)

- Expression for \( \kappa \) in terms of flow velocity, \( u \), and flame sheet velocity, \( v_F \):
  \[ \kappa = \frac{\partial u}{\partial A} + \frac{\partial u}{\partial A} + (\nabla \cdot \bar{u})(\nabla \cdot \bar{v}) = \nabla \cdot \bar{u} + (\nabla \cdot \bar{v})(\nabla \cdot \bar{v}) \]

  - Hydrodynamic stretch \( \kappa_c \): variation of tangential flow velocity in the tangential direction (\( \theta \), \( \phi \)) or, equivalently (by continuity), variation of normal flow velocity in the direction normal (\( n \)) to the flame.

  - Unsteady Curvature stretch \( \kappa_b \): non-stationary flames.
    - Positive \( \kappa \): divergent tangential velocities or expanding flame \( \rightarrow \) flame area increases.
    - Negative \( \kappa \): convergent tangential velocity or contracting flame \( \rightarrow \) flame area decreases.
**Mathematical expressions of stretch $\kappa$**

- **Stationary flames**
  \[
  \kappa = \nabla \cdot u = -\hat{n} \cdot \nabla (\hat{n} \cdot \hat{u})
  \]
  since $u \cdot \hat{n} = \hat{n} \times (\hat{u} \times \hat{n})$

Hydrodynamic stretch can be interpreted as a variation of the angle between flow velocity and flame normal or, equivalently, as a variation in tangential flow magnitude along the flame surface.

- **Alternative flame stretch expression**
  \[
  \kappa = \nabla \cdot \nabla = -\hat{n} \cdot \nabla (\hat{n} \cdot \hat{u})
  \]
  \[
  S = \frac{1}{2} \left( \nabla \cdot \hat{u} + \nabla \cdot \hat{u} ^{t} \right) \text{ Flow Strain, } s \cdot \hat{n} = (\nabla \cdot \hat{u}) \hat{n}
  \]
  - $\kappa _{F}$: stretch due to flow non uniformities
  - $\kappa _{curv}$: stretch of a curved flame in a uniform approach flow
  - $\kappa$ can be non zero also when the flow strain is zero

**Weak stretch effects**

- Asymptotic analysis shows that in the linear limit of weak stretch the effect of various type of stretch ($\kappa$, $\kappa _{F}$, $\kappa _{curv}$) on flame characteristics ($s^{u}$, $\delta_{F}$) is the same.

- For the flame speed measured in a reference frame attached to the unburned gases, $s^{u}$, we can write:
  \[
  s^{u} = s^{u} (\kappa) \quad s^{u} \bigg|_{\kappa=0} + \frac{\partial s^{u}}{\partial \kappa} \bigg|_{\kappa=0} \kappa = s^{u,0} - \delta_{M} \kappa
  \]
  - $\delta_{M}$: Markstein length

- Definition of Markstein length is not unique but depends on the isosurface used to define it; e.g. for the flame speed measured in a reference frame attached to the burned gasses $s^{b}$ we have:
  \[
  s^{b} = s^{b,0} - \delta_{M} \kappa
  \]
  - No dimensionless quantities
    - Markstein number $Ma$
      \[
      Ma^{a} = \frac{\delta_{M}^{a}}{\delta_{M}^{b}} \quad Ka^{a} = \frac{\delta_{M}^{b}}{s^{a,0}^{b}} \rightarrow \frac{s^{a}}{s^{a,0}} = 1 - Ma^{a} Ka
      \]
    - Karlovitz number $Ka$

**Unsteady Effects – Motion of Curved Flames**

- Curvature is present but flow velocities align with flame surface normal
- Stationary spherical flame would be stretchless

\[
\kappa = \frac{\partial u_{t}}{\partial t} + \frac{\partial u_{F}}{\partial t} + (\nabla_{F} \cdot \hat{n})(\nabla \cdot \hat{n}) = \nabla \cdot u + (\nabla_{F} \cdot \hat{n})(\nabla \cdot \hat{n})
\]

*Combustion Physics* by C.K. Law (Cambridge University Press, 2006)
**Application – Stoichiometry Effects**

\[ C_3H_8/\text{Air} \]

\[ p = 1 \text{ atm}, T^\text{u} = 300 \text{ K} \]

*Tseng et al., Comb. Flame, 95(2), 1993*

**Application – Pressure Effects**

H\(_2\)/CO 30/70 (by vol.) in air, \( T^\text{u} = 300 \text{ K} \)

Counterflow twin flame

\( \phi \) adjusted at different \( p \) to maintain \( s_v^0 = 34 \text{ cm/s} = \text{const} \)

**Application – Fuel Effects**

n-alkanes/air

\[ p = 1 \text{ atm}, T^\text{u} = 300 \text{ K} \]

*Ref: Tseng et al., Comb. Flame, 95(2), 1993*

*Ref: Halter et al. Comb and Flame, 157 (2010)*

**PREMIXED FLAME CONCEPTS**

**FLAME STRETCH AND FLAME EXTINCTION**

**Strong Stretch Effects**
Displacement Speed $s_d$ and Consumption Speed $s_c$

- **Displacement speed** $s_d$: speed at which the flame is moving along its normal relative to the flow
  - The value of $s_d$ depends on the reference surface:
    - Low $\kappa$: the approach flow varies weakly upstream of the flame and the iso-surface choice is not to problematic
    - High $\kappa$: velocity gradients occurs on a scale comparable to the flame thickness; $s_d$ definition becomes ambiguous
  - Displacement speed can also become negative when diffusive fluxes are strong enough to contrast the bulk convection in the opposite direction

- **Consumption speed** $s_c$: spatial integral of chemical rates
  - Can be obtained integrating along a streamline the heat production rate and normalizing by the total change in sensible enthalpy across the flame
  - Alternatively, the integrated quantity can be a reactant species consumption rate normalized by the total change in reactant species mass density

Example: 1D steady flame sensible enthalpy balance (no viscous and body forces)

\[
\int_{x}^{2} \left( \frac{\partial(h_{sens} - h_{sens,\infty})}{\partial x} \right) dx = \int_{x}^{2} \dot{q} dx - \int_{x}^{2} \left( \sum_{i} \dot{r}_{i} \frac{\partial \rho_{i} \gamma_{i}}{\partial x} \right) dx \rightarrow \rho^{0} s^{*} = \rho^{0} s^{0} = \int_{x}^{2} \dot{q} dx \left( h_{sens} - h_{sens,\infty} \right)
\]

Similarly from species eq. $s_{c} = \int_{x}^{2} \dot{w} dx / \rho (Y_{sens} - Y_{sens,\infty})$

\[
H_{2}/CO 30/70 \text{ by vol.) in air } \phi = 0.75, T^{*} = 300 \text{ K, } p = 5 \text{ atm}
\]

Extinction stretch rate $\kappa_{ext}$

$\kappa_{ext}$: maximum stretch that a flame can sustain before extinguishing

\[
H_{2}/CO 30/70 \text{ by vol.) in air } \phi = 0.75, T^{*} = 300 \text{K}
\]
**Example: Pressure Effects**

- Most of the available data are for steady symmetric opposed flow flames:
  - $\kappa_{\text{ext}}$ depends on flame chemical time $\tau_{\text{chem}} = \delta_T / s^{u,0}$
    - (eg. $p$ effects at fixed $s^{u,0}$)

H$_2$/CO 30/70 (by vol.) in air $T^* = 300$ K

$\phi$ adjusted at different $p$ to maintain $s^{u,0} = 34$ cm/s = const

**Example: Fuel and Stoichiometry Effects**

H$_2$/air, $\phi = 0.37$, $T^* = 298$ K, $p = 1$ atm

**Example: Preheat Effects**

- At high dilution/preheating levels, the flame does not "extinguish"
  - increases in reactant temperature are equivalent to a reduction in dimensionless activation energy
  - Example: calculation of CH$_4$/air flame stagnating against hot products, whose temperature is indicated on the plot

**Caveats on Stretch Sensitivity of Highly Stretched flames**

Stretch sensitivities and $\kappa_{\text{ext}}$ values are not intrinsic to mixture but also depend on manner in which stretch is applied

- Example: it depends also on flame geometry and configuration:
  - Velocity profile across the flame thickness (eg. $\kappa_{\text{ext}}$ for opposed flow flames depends on jets distance)
  - Type of stretch ($\kappa$, $\kappa_{\alpha}$, $\kappa_{\beta}$, $\kappa_{\gamma}$, $\kappa_{\text{curv}}$)
  - Length and time scale of flame/stretch interaction
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Overview

- Real flames have edges
- Structure is different than “continuous” flames previously considered:
  - Non-premixed flames do not propagate, but their edges do;
  - Premixed flame edge velocity is different from the laminar burning rate (e.g., can be negative)
- Applications
  - Stabilization of non-premixed (a) and premixed flames (b)
  - Propagation of an ignition front (c)
  - Flame propagation after local extinction (d)
- Edge flame can be advancing, retreating or stationary
  - Attention has to be paid to the observer reference frame.

Edge flame examples

- Piloted Bunsen flame
  - A retreating flame edge that is stationary in lab coordinate

- Premixed bluff body stabilized flame near blow-off
  [Chaudhury et al., Comb. Flame (158), 2011]
Edge flame examples

• Cabra burner

Dunn et. al, Comb. Flame (151) 2007

Vitiated coflow 1500K, 0.8m/s

Edge Flame Concepts

Illustrative Model Problem

Buckmaster’s Edge flame model problem

• Generalize the one dimensional non-premixed chambered flame (z→+∞).

\[
\rho c_p \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial z^2} = \left[ k_f \frac{\partial^2 T}{\partial x^2} - \rho c_p \mu_f \frac{\partial T}{\partial x} \right] - Q + \frac{\dot{W}_{\text{fuel}}}{\rho \dot{m}_{\text{fuel}}} - \frac{\dot{W}_{\text{fuel}}}{\rho \dot{m}_{\text{fuel}}} \frac{v}{L}\]

-Approximate transverse flux terms by convective loss-like term
-L characterizes the scale of gradients normal to the flame, such as due to strain.
**Edge flame model problem**

- **Dimensionless equation** ($Le=1$)

\[
E = \frac{E_L}{c_p T_0}, \quad \tau = \frac{c_p T_0}{T}, \quad \frac{T}{T_0}, \quad \frac{\dot{Q}}{\dot{Q}_{flow}}, \quad Da = \frac{\tau_{flow} \rho c_l^2}{k_i} \quad (1)
\]

\[
\bar{v}_F \frac{d \bar{T}}{d \bar{z}} - \frac{d^2 \bar{T}}{d \bar{z}^2} = F(\bar{T}, Da)
\]

where \( F(\bar{T}, Da) = 1 - \bar{T} + \frac{Y_{Fuel,s} Da}{\dot{Q}} \left( 1 - \bar{T} + \frac{\dot{Q}}{\dot{Q}_{flow}} \right) \exp \left( - \frac{E}{T} \right) \) and \( \bar{v}_F = \frac{v_F \rho c_p L}{k_F} \)

**Solution Limit for Edge-less Flame**

- **Steady state solution with no z-direction variation:** \( F(\bar{T}, Da) = 0 \)
  - the problem becomes the same as the steady well stirred reactor;
  - recover the same S-curve behavior.

\( F(\bar{T}, Da) = 1 - \bar{T} + \frac{Y_{Fuel,s} Da}{\dot{Q}} \left( 1 - \bar{T} + \frac{\dot{Q}}{\dot{Q}_{flow}} \right) \exp \left( - \frac{E}{T} \right) = 0 \)

- **We will focus on** \( Da_{II} < Da < Da_{II} \)
- **range**, where propagating flame edges can occur
  - **Three possible behaviors:**
    - propagates ($v_F > 0$)
    - retreats ($v_F < 0$)
    - stays stationary ($v_F = 0$)

**Edge flame model problem**

- **Edge flame velocity**

\[
\bar{v}_F = \frac{\tau_{flow}}{\tau_{chem}} \left( \int_{\bar{T}}^{\infty} \frac{d \bar{T}'}{d \bar{z}} \right) d\bar{z}
\]

- **Physical meaning:**
  - **Bi-stable steady state region further divided into 2 regions if an edge exists**
  - for \( Da > Da_{II} \) the flame edge acts as an ignition source; if the flame locally develops an hole this will close;
  - for \( Da < Da_{II} \) the unburned gasses quench the flame; if the flame develops an hole this will spread until the whole flame is extinguished.
**Edge Flame Concepts**

*Edge Structure and Velocity*

**Edge flames structure**

- **Non-premixed flames:**
  - Advancing (ignition front, $v_F > 0$) have often a triple flame structure;
  - Retreating (extinction front, $v_F < 0$) generally consist of a single edge.

  $v_F > 0 \quad v_F \approx 0 \quad v_F < 0$

- **Premixed flames:** generally have a single edge but can have significant hook-like structures

- Also stretch and heat losses influence the flame edge structure

**Flame edge velocity**

*Gas Expansion Effects*

- Velocity of edge flames $v_F$ depends on density ratio $\sigma = \rho_i/\rho_f$ across the flame, Damkohler number $Da$, heat losses, $Le$, $\dot{Q}_{Fuel}/\dot{Q}_{Ox}$, and $Z_{st}$.

- **Dependence on $\sigma$:** for a convex flame the deceleration $\Delta u$ of the approach flow in front of the flame monotonically increases with $\sigma$, (to be discussed later)

  \[ v_F \propto \sqrt[3]{\sigma} \]

- Reutsch et al.’s nonpremixed flame scaling for $Da \rightarrow \infty$

**Flame edge velocity**

*Heat Loss Effects*

- Heat losses may be important process in edge flames stabilized near metal surfaces

- Heat losses can cause $v_F < 0$ at high $Da$

- **Example:** Non-Premixed counterflow flame
Conditions at the flame edge

• Consider a premixed flame subject to a spatially varying stretch rate $\kappa$ at $t=0$.
  – Hole will form at points where $\kappa > \kappa_{\text{ext}}$
  – Edges will retreat to points where $\kappa = \kappa(v_p=0) = \kappa_{\text{edge}}$

Experiments show additional physics:
– Tangential flows of hot gases (e.g., in a lab stationary, retreating edge flame) can increase $v_F$ and cause $\kappa_{\text{edge}} > \kappa_{\text{ext}}$
– When a hole forms in a premixed flames, reactants and products can mix:
  • Mass burning rate increases because of presence of radicals and increased initial temperature. Peak heat release rate changes little across different dilution levels;
  • The flame loses its S-curve character.

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• Introductory Concepts
• Flame Stretch
• Edge Flames
• Flame Stabilization in Shear Layers
• Flame Stabilization by Stagnation Points
**Stretch Effects on Shear Layer Flames**

- Flame will extinguish when flame stretch rate exceeds $\kappa_{\text{ext}}$
  - As expected, higher flow velocities result in flame extinction occurring at higher values of $\kappa_{\text{ext}}$

**Shear Layer Stabilized Flames**

- In high speed flows, although locally low velocities exist within the shear layer, flame extinction typically leads to liftoff/blowoff
  - Limited by the amount of flame stretch which they can withstand before extinction
  - E.g., 50 m/s jet with 1 mm shear layer thickness shear~$du_z/dx \sim 50 \times 10^3$ s$^{-1}$
    - Much greater than typical extinction strain rates, $\kappa_{\text{ext}}$
- How is **flame stretch** related to **flow strain** in a shear layer?

**Sources of Flame Stretch**

1. Flame curvature
2. Unsteadiness in flame and flow
3. Hydrodynamic strain:
   - For reference, fluid strain rate given by tensor:
     \[ S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

**Flame Stretch Due to Fluid Strain**

- General expression can be reduced by assuming 2-D flow and incompressibility (upstream of flame):

\[
\kappa_z = (n_z^2 - n_x^2) \frac{\partial u_y}{\partial z} - n_x n_z \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)
\]

---

*Q. Zhang et al. J. Eng. for Gas Turbines & Power 2010*
Flame Stretch Due to Fluid Strain

Shear Strain Contribution

- Flame strain occurs due to variations in tangential velocity
- Leads to positive stretch

\[ \kappa_{s,\text{shear}} = -n_s n_x \left( \frac{\partial u_x^u}{\partial z} + \frac{\partial u_z^u}{\partial x} \right) \]

Normal Strain Contribution

- Jet flows typically decelerate producing normal strain
- Leads to negative stretch

\[ \kappa_{s,\text{normal}} = (n_x^2 - n_z^2) \frac{\partial u_z^u}{\partial z} \]

For high speed flows:

\[ u_z^u \ll s_d^u \]
\[ \therefore \theta \ll 1 \]
\[ \Rightarrow \kappa_s = -1 + O(\theta^2) \quad \& \quad \kappa_s = O(\theta) \]

Also assume:

\[ \frac{\partial u_z^u}{\partial z} \ll \frac{\partial u_x^u}{\partial x} \]

Expressions for shear and normal stretch can be simplified as follows:

\[ \kappa_{s,\text{shear}} \approx \theta \frac{\partial u_x^u}{\partial x} \]
\[ \kappa_{s,\text{normal}} \approx \frac{\partial u_z^u}{\partial z} \]

Stretch from Shear Strain

- \( \kappa_{s,\text{shear}} \approx \theta \frac{\partial u_x^u}{\partial x} \)

- If \( \theta \approx \frac{s_d^u}{u_z^u} \) then \( \kappa_{s,\text{shear}} \approx \frac{s_d^u}{u_z^u} \frac{\partial u_z^u}{\partial x} \)

\[ \Rightarrow \text{Stretch rate due to shear } \sim \text{indep. of } u \? \]

\text{increasing } u \Rightarrow \text{shear } \uparrow \text{ but } \theta \downarrow \]

\text{but } u \text{ can influence shear layer: } \delta_{sh} \propto u^{-1/2} \]
**Stretch from Normal Strain**

\[ \kappa_{s,\text{normal}} \approx \frac{\partial u_z''}{\partial z} \approx \frac{u_z''}{L_{\text{char}}} \]

- Obtain traditional flow time scaling approach

**Flame Stretch Distribution in Swirl Flame**

- Dimensionless value of \(1 = 4,125 \text{ l/s}\)
- Shows dominance of deceleration term in first 10 mm
- Shear term dominates farther downstream

**Total Stretch from Strain**

\[ \kappa_s \approx \frac{\partial u_z''}{\partial z} + \theta \frac{\partial u_z''}{\partial x} \]

\(\Rightarrow\) Opposite signs if decelerating flow which term dominates - \(\theta\) small?

Even if \(\frac{\partial u_z''}{\partial z} \ll \frac{\partial u_z''}{\partial x} \quad \text{vs.} \quad \theta \frac{\partial u_z''}{\partial x}\)

**Piloting or Flow Recirculation Effects**

- Flame stabilization can be enhanced through:
  - Pilot flames
  - Recirculation zones
  - Transport hot products to the attachment point of a flame
Dilution/Liftoff Effects

- At high dilution/preheating levels, the flame does not "extinguish"
  - Increases in reactant temperature are equivalent to a reduction in dimensionless activation energy
  - Example: calculation of CH₄/air flame stagnating against hot products with indicated temperature

Blowoff of Bluff Body Stabilized Flames

- Stages of blowoff
  - Stage 1: Flame is continuous but marked by local extinction events
  - Stage 2: Changes in wake dynamics as large scale structures become visible
  - Stage 3: Blowoff of flame

- Da Approach:
  - Scaling captures onset of flame extinction events
  - Ability to capture blowoff dependent on link between extinction events and blowoff physics

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Flame Anchoring Locations and Flame Shapes in Swirling Flows

- Images courtesy of D. R. Noble

Flame Anchoring Locations and Flame Shapes in Annular Nozzle Geometries

- Flame stabilizes in front of stagnation point of vortex breakdown bubble
  - Stagnation point apparently precesses, probably also moves up and down
  - i.e., flame anchoring position highly unsteady, in contrast to stabilization at edges/corners
- Under what circumstances can such flames exist?
  - Not always observed; flames may blowoff directly without reverting to a “free floating” configuration
  - Flow must have interior stagnation point

Vortex Breakdown in Annular Geometries

- Nature of centerbody wake/ VBB changes with geometry, swirl #, and Reynolds #

Recirculation zone with vortex tube
Recirculation zone with bubble-like breakdown above
Merged recirculation zones

Shear et al., Phys. Fluids, 1996

Swirl number/ Centerbody Diameter
### Vortex Breakdown in Annular Geometries

- Nature of centerbody wake/ VBB changes with geometry, swirl #, and Reynolds #

### Flame Stabilization Influenced by Downstream Boundary Conditions

- Fluid rotation introduces inertial wave propagation mechanism
  - “sub-” and “supercritical” flow distinction
- Exit boundary condition has significant influence on vortex breakdown bubble topology
Disturbance Propagation and Generation in Reacting Flows

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Motivation – Combustion Instabilities

• Introduction
• Decomposition of Disturbances into Fundamental Disturbance Modes
• Disturbance Energy
• Nonlinear Behavior
• Acoustic Wave Propagation Primer
• Unsteady Heat Release Effects and Thermoacoustic Instability

• Large amplitude acoustic oscillations driven by heat release oscillations
• Oscillations occur at specific frequencies, associated with resonant modes of combustor

Video courtesy of S. Menon
Resonant Modes – You can try this at home

- **Helmholtz Mode**
  - 190 Hz
- **Longitudinal Modes**
  - 1,225 Hz
  - 1,775 Hz
- **Transverse Modes**
  - 3,719 Hz
  - 10,661 Hz

*Slide courtesy of R. Mihata, Alta Solutions*

Key Problem: Combustor System Sensitive to Acoustic Waves

**Rubens Tube**

**Key Problem: Flame Sensitive to Acoustic Waves**

Video from Ecole Centrale – 75 Hz, Courtesy of S. Candel

Thermo-acoustics

- **Rijke Tube** (heated gauze in tube)
- Self-excited oscillations in cryogenic tubes
- Thermo-acoustic refrigerators/heat pumps

*Purdue's Thermoacoustic Refrigerator*
Liquid Rockets

- **F-1 Engine**
  - Used on Saturn V
  - Largest thrust engine developed by U.S.
  - Problem overcome with over 2000 (out of 3200) full scale tests

Solid Rockets

- **Examples:**
  - Space shuttle booster- 1-3 psi oscillations (1 psi = 33,000 pounds of thrust)

  **Adverse effects**—thrust oscillations, mean pressure changes, changes in burning rates

Small Amplitude Propagation in Uniform, Inviscid Flows

- Assume infinitesimal perturbations superposed upon a spatially homogenous background flow.
- We will also introduce two additional assumptions:
  1. The gas is non-reacting and calorically perfect, implying that the specific heats are constants.
  2. Neglect viscous and thermal transport.

**Decomposition Approach**
- Decompose variables into the sum of a base and fluctuating component; e.g.,

\[
p(x,t) = p_0 + p_1(x,t) \\
\rho(x,t) = \rho_0 + \rho_1(x,t) \\
\bar{u}(x,t) = \bar{u}_0 + \bar{u}_1(x,t) \quad (2.9)
\]
Perturbing the Continuity Equation

Consider the continuity equation:
\[
-\frac{1}{\rho} \frac{D \rho}{Dt} + \frac{1}{c_p} \frac{D s}{Dt} - \frac{1}{\gamma p} \frac{D p}{Dt} = \nabla \cdot \vec{u} \tag{2.10}
\]

\(Ds/Dt\) is identically zero because of the neglect of molecular transport terms and chemical reaction. Therefore …
\[
-\frac{1}{\gamma p} \frac{D p}{Dt} = \nabla \cdot \vec{u} \tag{2.11}
\]

Expanding each variable into base and fluctuating components yields:
\[
-\frac{1}{\gamma (p_0 + p_1)} \left( \frac{\partial p_1}{\partial t} + (\vec{u}_0 + \vec{u}_1) \cdot \nabla p_1 \right) = \nabla \cdot \vec{u}_1 \tag{2.12}
\]

Motivating Model Problem

- Continuity:
\[
\frac{\partial \rho_1}{\partial t} + u_{x,0} \frac{\partial \rho_1}{\partial x} + \rho_0 \frac{\partial u_{x,1}}{\partial x} = 0
\]

- X momentum
\[
\frac{\partial u_{x,1}}{\partial t} + u_{x,0} \frac{\partial u_{x,1}}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p_1}{\partial x}
\]

- Y Momentum
\[
\frac{\partial u_{y,1}}{\partial t} + u_{x,0} \frac{\partial u_{y,1}}{\partial x} = 0
\]

- Energy
\[
\frac{\partial p_1}{\partial t} + u_{x,0} \frac{\partial p_1}{\partial x} = -\gamma p_0 \frac{\partial u_{x,1}}{\partial x}
\]

Linearizing Continuity

- Continuity Equation Expanded:
\[
-\frac{1}{\gamma} \left( \frac{\partial p_1}{\partial t} + \vec{u}_0 \cdot \nabla p_1 + \vec{u}_1 \cdot \nabla p_1 \right) = p_0 \nabla \cdot \vec{u}_1 + p_1 \nabla \cdot \vec{u}_1 \tag{2.13}
\]

- Linearize by neglecting products of perturbations. If perturbations are small in amplitude then products of perturbations are negligible.
\[
-\frac{1}{\gamma p_0} \frac{D p_1}{Dt} = \nabla \cdot \vec{u}_1 = \Lambda_1 \tag{2.14}
\]

- Definition of the substantial derivative, \(D_0 / Dt\).
\[
\frac{D_0}{Dt} \left( \vec{v} \right) = \frac{\partial}{\partial t} \left( \vec{v} \right) + \vec{u}_0 \cdot \nabla \left( \vec{v} \right) \tag{2.15}
\]

Solution of Motivating Model Problem: Assumed Solution Form

\[
\begin{align*}
\rho_1(x,t) &= \hat{\rho}_1(x,\omega)e^{-i\omega t} & p_1(x,t) &= \hat{p}_1(x,\omega)e^{-i\omega t} & T_1(x,t) &= \hat{T}_1(x,\omega)e^{-i\omega t} \\
u_{x,1}(x,t) &= \hat{u}_{x,1}(x,\omega)e^{-i\omega t} & u_{y,1}(x,t) &= \hat{u}_{y,1}(x,\omega)e^{-i\omega t} & \Omega_{z,1}(x,t) &= \hat{\Omega}_{z,1}(x,\omega)e^{-i\omega t}
\end{align*}
\]
Solution of Motivating Model Problem

\[
\begin{bmatrix}
\rho_0(x) \\
\rho_0(x) \\
\frac{\gamma - 1}{\rho_0} T(x) \\
\rho_0 c_0 \\
0 \\
0 \\
\frac{i \omega}{\rho_0 c_0 (c_0 + u)} - \frac{i \omega}{\rho_0 c_0 (c_0 - u)} \\
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 0 & 0 \\
\frac{\gamma - 1}{\rho_0} & \frac{\gamma - 1}{\rho_0} & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
A e^{i \omega (v - u)} \\
B e^{i \omega (v - u)} \\
\end{bmatrix}
\]

Summary of Disturbance Equations

- Vorticity:
\[
\frac{D_0 \tilde{\Omega}_I}{Dt} = 0
\]  
(2.18)

- Acoustic:
\[
\frac{D_0^2 p_I}{Dt^2} - c_0^2 \nabla^2 p_I = 0
\]  
(2.20)

- Entropy:
\[
\frac{D_0 \tilde{\varepsilon}_I}{Dt} = 0
\]  
(2.27)

Canonical Decomposition

- Further decompose perturbations by their origin
\[
\tilde{\Omega}_I = \tilde{\Omega}_{I\Lambda} + \tilde{\Omega}_{I\varsigma} + \tilde{\Omega}_{I\Omega}
\]
\[
\tilde{\varepsilon}_I = \tilde{\varepsilon}_{I\Lambda} + \tilde{\varepsilon}_{I\varsigma} + \tilde{\varepsilon}_{I\Omega}
\]  
(2.28)

\[
p_{I\varsigma} = p_{I\Lambda} + p_{I\varsigma} + p_{I\Omega}
\]

- Examples:
  - \( \tilde{\Omega}_{I\varsigma} \) vorticity fluctuations induced by entropy fluctuations.
  - \( p_{I\Omega} \) pressure fluctuations induced by vorticity fluctuations.

- Dynamical equations are linear and can be decomposed into subsystems

Oscillations from Vorticity

- Oscillations associated with vorticity mode:
\[
\frac{D_0 \tilde{\Omega}_I}{Dt} = 0
\]  
(2.29)

\[
p_{I\Omega} = \tilde{\varepsilon}_{I\Omega} = T_{I\Omega} = \rho_{I\Omega} = 0
\]  
(2.30)

\[
\frac{D_0 \tilde{\varepsilon}_I}{Dt} = 0
\]  
(2.31)

- Vorticity, and induced velocity, fluctuations are convected by the mean flow.
- Vorticity fluctuations induce no fluctuations in pressure, entropy, temperature, density, or dilatation.
Oscillations from Acoustics

- Oscillations associated with acoustic mode:

\[
\frac{D_0^2 p_{1\lambda}}{D t^2} - c_0^2 \nabla^2 p_{1\lambda} = 0 \quad (2.32)
\]

\[
\dot{\Omega}_{1\lambda} = \dot{4}_{1\lambda} = 0 \quad (2.33)
\]

\[
\rho_{1\lambda} = \frac{p_{1\lambda}}{c_0^2} = \frac{\rho_0 c_p}{c_0^2} T_{1\lambda} \quad (2.34)
\]

\[
\rho_0 \frac{D \tilde{u}_{1\lambda}}{D t} = -\nabla p_{1\lambda} \quad (2.35)
\]

- The density, temperature, and pressure fluctuations are locally and algebraically related through their isentropic relations.

Oscillations from Entropy

- Oscillations associated with entropy mode:

\[
\frac{D_0 \dot{4}_{1\varepsilon}}{D t} = p_{1\varepsilon} = \dot{\Omega}_{1\varepsilon} = \tilde{u}_{1\varepsilon} = 0 \quad (2.37)
\]

\[
\rho_{1\varepsilon} = \frac{\rho_0}{c_p} \dot{4}_{1\varepsilon} = -\frac{\rho_0}{T_0} T_{1\varepsilon} \quad (2.38)
\]

- Entropy oscillations do not excite vorticity, velocity, pressure, or dilatational disturbances.

- They do excite density and temperature perturbations.

Comments on the Decomposition

- In a homogeneous, uniform flow, these three disturbance modes propagate independently in the linear approximation.

- Three modes are decoupled within the approximations of this analysis - vortical, entropy, and acoustically induced fluctuations are completely independent of each other.

- For example, velocity fluctuations induced by vorticity and acoustic disturbances, \( \tilde{u}_{1\lambda} \) and \( \tilde{u}_{1\varepsilon} \) are independent of each other and each propagates as if the other were not there.

- Moreover, there are no sources or sinks of any of these disturbance modes. Once created, they propagate with constant amplitude.

Comments on Length Scales

- Acoustic disturbances propagate at the speed of sound.

- Vorticity and entropy disturbances convect at bulk flow velocity, \( u_0 \).

- Acoustic properties vary over an acoustic length scale, given by \( \lambda_A = c_0 / f \).

- Entropy and vorticity modes vary over a convective length scale, given by \( \lambda_c = u_0 / f \).

- Entropy and vortical mode “wavelength” is shorter than the acoustic wavelength by a factor equal to the mean flow Mach number.

\[
\frac{\lambda_c}{\lambda_A} = \frac{u_0}{c_0} = M
\]
Comments on Acoustic Disturbances

- Acoustic disturbances, being true waves, reflect off boundaries, are refracted at property changes, and diffract around obstacles.

![Image of instantaneous pressure field and flame front of a sound wave incident upon a turbulent flame from the left at three successive times. Courtesy of D. Thévenin.](image1)

![Flame Acoustic wave Image](image2)

![Illustration of acoustic refraction effects. Data courtesy of J. O'Connor](image3)

### Summary

<table>
<thead>
<tr>
<th></th>
<th>Acoustic</th>
<th>Entropy</th>
<th>Vorticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>X</td>
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<tr>
<td>Density</td>
<td>X</td>
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<tr>
<td>Temperature</td>
<td>X</td>
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<td>Vorticity</td>
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<td></td>
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<td>Entropy</td>
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</table>

### Example: Effects of Simultaneous Acoustic and Vortical Disturbances

- Consider superposition of two disturbances with different phase speeds:

\[
u_t(x,t) = u_{1_t} + u_{2_t} = A_1 \cos\left(\omega t - \frac{x}{c_0}\right) + A_2 \cos\left(\omega t - \frac{x}{u_{s,0}}\right)
\]

- For simplicity, assume \( A_1 = A_2 = A \):

\[
u_t(x,t) = 2A \cos\left(\frac{c_0 - u_{s,0}}{2c_0u_{s,0}}x\right) \cos\left(\omega t - \frac{c_0 - u_{s,0}}{2c_0u_{s,0}}x\right)
\]

- Velocity field oscillates harmonically at each point as \( \cos(\omega t) \)
- Amplitude of these oscillations varies spatially as \( \cos\left(\frac{c_0 - u_{s,0}}{2c_0u_{s,0}}x\right) \) due to interference.
Some Data Illustrating This Effect

- Fit parameter: \( \frac{\alpha_1}{\alpha_2} = 0.6 \)

 Modal Coupling Processes

- We just showed that the small amplitude canonical disturbance modes propagate independently within the fluid domain in a homogeneous, inviscid flow.
- These modes couple with each other from:
  - Boundaries
    - e.g., acoustic wave impinging obliquely on wall excites vorticity and entropy
  - Regions of flow inhomogeneity
    - e.g., acoustic wave propagating through shear flow generates vorticity
    - Accelerating an entropy disturbance generates an acoustic wave
  - Nonlinearities
    - e.g., large amplitude vortical disturbances generate acoustic waves (jet noise)

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A) Introduction and Outlook

B) Flame Aerodynamics and Flashback

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D) Disturbance Propagation and Generation in Reacting Flows

E) Flame Response to Harmonic Excitation

Energy Density and Energy Flux Associated with Disturbance Fields

- Although the time average of the disturbance fields may be zero, they nonetheless contain non-zero time averaged energy and lead to energy flux whose time average is also non-zero.
  - Ex: 
    \[ E_{\text{kin}} = \frac{1}{2} \rho (\bar{u}_i \cdot \bar{u}_i) \]
- Consider the energy equation.
  \[ \frac{\partial E}{\partial t} + \nabla \cdot \mathbf{T} = \Phi \]
  \[ \frac{d}{dt} \iiint V E dV + \iiint A \mathbf{T} \cdot \mathbf{n} dA = \iiint A \Phi dV \]
Acoustic Energy Equation

- Consider the acoustic energy equation, assuming:
  - Entropy and vorticity fluctuations are zero
  - Zero mean velocity, homogenous flow.
  - Combustion process is isomolar

\[
\frac{\partial}{\partial t}\left(\frac{1}{2 \rho_0} (\bar{u} \cdot \bar{u}) + \frac{1}{2 \rho_0 c_0^2} \right) + \nabla \cdot (p_0 \bar{u}) = \frac{(\gamma - 1)}{\gamma P_0} p_0 \dot{q}_1
\]  

(2.52)

\[
E_\Lambda = \frac{1}{2} \rho_0 (\bar{u} \cdot \bar{u}) + \frac{1}{2 \rho_0 c_0^2}
\]

(2.53)

\[
I_\Lambda = p_0 \bar{u}
\]

(2.54)

\[
\Phi_\Lambda = \frac{(\gamma - 1)}{\gamma P_0} p_0 \dot{q}_1
\]

(2.55)

Time Average of Products

- Time average of product of two fluctuating quantities depends on their relative phasing
  
- Example \( \sin(\omega t) \sin(\omega t + \theta) = \frac{1}{2} \cos \theta \)

Rayleigh’s Criterion

- Rayleigh’s Criterion states that unsteady heat addition locally adds energy to the acoustic field when the phases between the pressure and heat release oscillations is within ninety degrees of each other.
  
- Conversely, when these oscillations are out of phase, the heat addition oscillations damp the acoustic field.
  
- Figure illustrates that the highest pressure amplitudes are observed at conditions where the pressure and heat release are in phase.

Data courtesy of K. Kim and D. Santavicca.
Course Outline

A) Introduction and Outlook
   • Introduction

B) Flame Aerodynamics and Flashback
   • Decomposition of Disturbances into Fundamental Disturbance Modes
   • Disturbance Energy

C) Flame Stretch, Edge Flames, and Flame Stabilization Concepts
   • Nonlinear Behavior
   • Acoustic Wave Propagation Primer
   • Unsteady Heat Release Effects and Thermoacoustic Instability

D) Disturbance Propagation and Generation in Reacting Flows
   • Introduction
   • Decomposition of Disturbances into Fundamental Disturbance Modes
   • Disturbance Energy
   • Nonlinear Behavior
   • Acoustic Wave Propagation Primer
   • Unsteady Heat Release Effects and Thermoacoustic Instability

E) Flame Response to Harmonic Excitation

Perturbation Method Example

\[
O(\varepsilon) \quad \frac{p_1(t)}{\bar{p}_1} = \varepsilon \gamma \sin(\omega t) \quad (2.3)
\]

\[
O(\varepsilon^2) \quad \frac{p_2(t)}{\bar{p}_1} = \frac{\varepsilon^2 \gamma (\gamma - 1)(1 - \cos(2\omega t))}{4} \quad (2.4)
\]

\[
O(\varepsilon^3) \quad \frac{p_3(t)}{\bar{p}_1} = \frac{\varepsilon^3 \gamma (\gamma - 1)(\gamma - 2)(3\sin\omega t - \sin 3\omega t)}{24} \quad (2.5)
\]

Intro to Perturbation Methods

• Consider the following example problem where \( \varepsilon \) is a small parameter.

\[
p(t) = \bar{p}_1 (1 + \varepsilon \sin(\omega t))^\gamma \quad (2.1)
\]

• We will denote the “base” or “nominal” flow as the value of the quantity in the absence of perturbations with the subscript “0”.

• It is often useful to expand the quantity about this base state in a Taylor’s series; i.e.,

\[
p = p_0 + p_1 + p_2 + \ldots \quad (2.2)
\]

where by assumption, \( p_0 \gg p_1 \gg p_2 \gg \ldots \).

General Results

• 1\textsuperscript{st} order term, \( p_1 \), is linear in disturbance amplitude, \( \varepsilon \), and oscillates with the same frequency as the disturbance.

• 2\textsuperscript{nd} order term is quadratic in disturbance amplitude, \( \varepsilon^2 \), and has two different terms, one which does not vary in time and the other that oscillates at \( 2\omega \).

• 3\textsuperscript{rd} order term is cubic in disturbance amplitude, \( \varepsilon^3 \), and has one term that oscillates at \( \omega \) and the other at \( 3\omega \).

• Even order terms contribute time invariant terms and higher harmonics that are even multiples of the disturbance frequency.

• Odd order terms contribute at the disturbance frequency and higher harmonics that are odd multiples of the disturbance frequency.
**Experimental Comparison Considerations**

- Time average of a quantity denoted with an overbar, e.g., \( \bar{p} \). Fluctuations of quantities about the time average are denoted with a \( (\cdot)' \).

\[
p'(t) = p(t) - \bar{p}
\]

(2.6)

- The time average is not equal to its nominal value; i.e., \( \bar{p} \neq p_0 \).

\[
\bar{p} = p_0 + \varepsilon^2 \gamma (\gamma - 1) + \frac{\varepsilon^4 \gamma (\gamma - 1)(\gamma - 2)(\gamma - 3)}{64} + O(\varepsilon^6)
\]

(2.7)

\[
p'(t) = (\varepsilon \gamma + \varepsilon^3 \gamma (\gamma - 1)(\gamma - 2)) \sin(\omega t) - \varepsilon^2 \gamma (\gamma - 1) \cos(2\omega t)
\]

\[- \frac{\varepsilon^3 \gamma (\gamma - 1)(\gamma - 2)}{24} \sin 3\omega t + O(\varepsilon^5)
\]

(2.8)

---

**Linear and Nonlinear Stability of Disturbances**

- The disturbances which have been analyzed arise because of underlying instabilities, either in the local flow profile or to the coupled flame-combustor acoustic systems (such as thermoacoustic instabilities).

- Consider a more general study of stability concepts by considering the time evolution of a disturbance

\[
\frac{d \mathcal{A}(t)}{dt} = F_A - F_D
\]

(2.63)

\( F_A \) and \( F_D \) denote processes responsible for amplification and damping of the disturbance, respectively.

---

**Pressure Measurements During Azimuthal Instability (courtesy of J.W. Kim)**

- In a linearly stable/unstable system, infinitesimally small disturbances decay/grow, respectively.

- To illustrate these points, we can expand the functions \( F_A \) and \( F_D \) around their \( \mathcal{A}=0 \) values in a Taylor’s series:

\[
F_A = \varepsilon_A \mathcal{A} + F_{A,NL}
\]

(2.64)

\[
F_D = \varepsilon_D \mathcal{A} + F_{D,NL}
\]

(2.65)

\[
\varepsilon_A = \left. \frac{\partial F_A}{\partial \mathcal{A}} \right|_{\mathcal{A}=0}
\]

(2.66)
Comments on Linear Behavior

- The amplification and damping curves intersect at the origin, indicating that a zero amplitude oscillation is a potential equilibrium point.

- However, this equilibrium point is unstable, since $\varepsilon_A > \varepsilon_D$ and any small disturbance makes $F_A$ larger than $F_D$, resulting in further growth of the disturbance.

- If $\varepsilon_A < \varepsilon_D$ the $A=0$ point is an example of an “attractor” in that disturbances are drawn toward it.

- Linearized solution:

$$A(t) = A(t=0) \exp((\varepsilon_A - \varepsilon_D)t)$$

(2.67)

Comments on Nonlinear Behavior

- This linearized solution may be a reasonable approximation to the system dynamics if the system is linearly stable (unless the initial excitation, $A(t=0)$ is large).

- However, it is only valid for small time intervals when the system is unstable, as disturbance amplitudes cannot increase indefinitely.

- In this situation, the amplitude dependence of system amplification/damping is needed to describe the system dynamics.

- The steady state amplitude is stable at $A_c=0$ because amplitude perturbations to the left (right) causes $F_A$ to become larger (smaller) than $F_D$, thus causing the amplitude to increase (decrease).

Example of Stable Orbits: Limit Cycles

- In many other problems, $A(t)$ is used to describe the amplitude of a fluctuating disturbance; for example,

$$\frac{\partial p}{\partial t}$$

- Limit cycle is example of orbit that encircles an unstable fixed point
  - A stable limit cycle will be pulled back into this attracting, periodic orbit even when it is slightly perturbed.

$$p(t) - p_0 = A(t) \cos(\omega t)$$

(2.69)

Variation in System Behavior with Change in Parameter

- Consider a situation where some combustor parameter is systematically varied in such a way that $\varepsilon_A$ increases while $\varepsilon_D$ remains constant.
Supercritical Bifurcations

- The $e_A = e_D$ condition separates two regions of fundamentally different dynamics and is referred to as a **supercritical bifurcation point**.
  - Note smooth monotonic variation of limit cycle amplitude with parameter.

Nonlinearly Unstable Systems

- Small amplitude disturbances decay in a nonlinearly unstable system, but disturbances with amplitudes exceeding a critical value, $A_c$, will grow.
- This type of instability is sometimes referred to as **subcritical**.
- Other examples:
  - Hydrodynamic stability of shear flows without inflection points
  - Certain kinds of thermoacoustic instabilities in combustors.
    - Historically referred to as “triggering” in rocket instabilities

Subcritical Instability

- Figure provides example of amplitude dependence of $F_A$ and $F_D$ that produces **subcritical** instability.
- In this case, the system has three equilibrium points where the amplification and dissipation curves intersect.
- All disturbances with amplitudes $A < A_T$ return to the stable solution $A=0$ and disturbances with amplitudes $A > A_T$ grow until their amplitude attains the value $A = A_C$.
- Consequently, two stable solutions exist at this operating condition. The one observed at any point in time will depend upon the history of the system.
Subcritical Bifurcation Diagram

- If a system parameter is monotonically increased to change the sign of \( \epsilon_{A} - \epsilon_{D} \) from a negative to a positive value, the system’s amplitude jumps discontinuously from \( \mathcal{A} = 0 \) to \( \mathcal{A} = \mathcal{A}_{LC} \) at \( \epsilon_{A} - \epsilon_{D} = 0 \).

- Note hysteresis as well

\[ \text{Amplitude, } \mathcal{A} \]

\[ \text{Pressure amplitude, } \frac{p'}{p} \]

\[ \text{Nozzle velocity, m/s} \]

\[ 13 \quad 13.5 \quad 14 \quad 14.5 \quad 15 \quad 15.5 \]

Course Outline

A) Introduction and Outlook

B) Flame Aerodynamics and Flashback

C) Flame Stretch, Edge Flames, and Flame Stabilization Concepts

D) Disturbance Propagation and Generation in Reacting Flows

E) Flame Response to Harmonic Excitation

- Introduction

- Decomposition of Disturbances into Fundamental Disturbance Modes

- Disturbance Energy

- Nonlinear Behavior

- Acoustic Wave Propagation Primer

- Unsteady Heat Release Effects and Thermoacoustic Instability

Overview 1 of 2

- Acoustic Disturbances:
  - Propagate energy and information through the medium without requiring bulk advection of the actual flow particles.
  - Details of the time averaged flow has relatively minor influences on the acoustic wave field (except in higher Mach number flows).
  - Acoustic field largely controlled by the boundaries and sound speed field.

- Vortical disturbances
  - Propagate with the local flow field.
  - Highly sensitive to the flow details.
  - No analogue in the acoustic problem to the hydrodynamic stability problem.

Overview 1 of 2

• Introduction

• Flashback and Flameholding

• Flame Stabilization and Blowoff

• Combustion Instabilities
  - Motivation
  - Disturbance propagation, amplification, and stability
  - Acoustic Wave Propagation Primer
  - Unsteady Heat Release Effects and Thermoacoustic Instability

• Flame Dynamics

Overview 1 of 2
Some distinctives of the acoustics problem:

- Sound waves reflect off of boundaries and refract around bends or other obstacles.
- Vortical and entropy disturbances advect out of the domain where they are excited.
- An acoustic disturbance in any region of the system will make itself felt in every other region of the flow.
- Wave reflections cause the system to have natural acoustic modes; oscillations at a multiplicity of discrete frequencies.

Traveling and Standing Waves

- The acoustic wave equation for a homogeneous medium with no unsteady heat release is a linearized equation describing the propagation of small amplitude disturbances.

We will first assume a one-dimensional domain and neglect mean flow, and therefore consider the wave equation:

\[
\frac{\partial^2 p_1}{\partial t^2} - c_0^2 \frac{\partial^2 p_1}{\partial x^2} = 0
\]  

(5.1)

Harmonic Oscillations

- We will use complex notation for harmonic disturbances. In this case, we write the unsteady pressure and velocity as

\[
p_1 = \text{Real}\left( \hat{p}_1(x, y, z) \exp(-i\omega t) \right)
\]

\[
\vec{u}_1 = \text{Real}\left( \hat{\vec{u}}_1(x, y, z) \exp(-i\omega t) \right)
\]

(5.13)  

(5.14)

- As such, the one-dimensional acoustic field is given by:

\[
p_1 = \text{Real}\left( (A \exp(ikx) + B \exp(-ikx)) \exp(-i\omega t) \right)
\]

\[
u_{x,1} = \frac{1}{\rho_0 c_0} \text{Real}\left( (A \exp(ikx) - B \exp(-ikx)) \exp(-i\omega t) \right)
\]

(5.15)  

(5.16)

Time Response of Harmonically Oscillating Traveling Waves

- The disturbance field consists of space-time harmonic disturbances propagating with unchanged shape at the sound speed.

- An alternative way to visualize these results is to write the pressure in terms of amplitude and phase as:

\[
\hat{p}_1(x) = |\hat{p}_1(x)| \exp(-i\theta(x))
\]

(5.17)
Amplitude and Phase of Traveling Wave Disturbances

- The magnitude of the disturbance stays constant but the phase decreases/increases linearly with axial distance.
- The slope of these lines is \( \frac{d\phi}{dx} = \omega / c_0 \).
- Harmonic disturbances propagating with a constant phase speed have a linearly varying axial phase dependence, whose slope is inversely proportional to the disturbance phase speed.

Standing Waves

- Consider next the superposition of a left and rightward traveling wave of equal amplitudes, \( \mathcal{A} = \mathcal{B} \), assuming without loss of generality that \( \mathcal{A} \) and \( \mathcal{B} \) are real.

\[
p_1(x,t) = 2\mathcal{A}\cos(kx)\cos(\omega t) \quad (5.18)
\]

\[
u_{x,1} = \frac{1}{\rho_0 c_0} 2\mathcal{A}\sin(kx)\sin(\omega t) \quad (5.19)
\]

- Such a disturbance field is referred to as a “standing wave”.

- Observations:
  - amplitude of the oscillations is not spatially constant, as it was for a single traveling wave.
  - phase does not vary linearly with \( x \), but has a constant phase, except across the nodes where it jumps 180 degrees.
  - pressure and velocity have a 90 degree phase difference, as opposed to being in-phase for a single plane wave.

One Dimensional Natural Modes

- Consider a duct of length \( L \) with rigid boundaries at both ends, \( u_{x,1}(x=0,t) = u_{x,1}(x=L,t) = 0 \). Applying the boundary condition at \( x=0 \) implies that \( \mathcal{A} = \mathcal{B} \), leading to:

\[
u_{x,1} = \frac{1}{\rho_0 c_0} \text{Real} \left( 2i\mathcal{A} \sin(kx) \exp(-i\omega t) \right) \quad (5.62)
\]

\[
k = \frac{n\pi}{L} \quad (5.63)
\]

\[
f_n = \frac{\omega}{2\pi} = \frac{k\rho_0 c_0}{2\pi} = \frac{nc_0}{2L} \quad (5.64)
\]

- These natural frequencies are integer multiples of each other, i.e., \( f_2 = 2f_1, f_3 = 3f_1, \) etc.

\[
p_1(x,t) = 2\mathcal{A} \cos \left( \frac{n\pi x}{L} \right) \cos(\omega t) \quad (5.65)
\]

\[
u_{x,1}(x,t) = \frac{2\mathcal{A}}{\rho_0 c_0} \sin \left( \frac{n\pi x}{L} \right) \sin(\omega t) \quad (5.66)
\]
Course Outline

A) Introduction and Outlook

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Unsteady Heat Release Effects

- Oscillations in heat release generate acoustic waves.
  - For unconfined flames, this is manifested as broadband noise emitted by turbulent flames.
  - For confined flames, these oscillations generally manifest themselves as discrete tones at the natural acoustic modes of the system.

- The fundamental mechanism for sound generation is the unsteady gas expansion as the mixture reacts.

- To illustrate, consider the wave equation with unsteady heat release
  \[
  \frac{\partial^2 p_i}{\partial t^2} - c_0^2 \nabla^2 p_i = (\gamma - 1) \frac{\partial \dot{q}_i}{\partial t} + \gamma p_0 \left[ \frac{n}{n_i} \right]
  \]  

- The two acoustic source terms describe sound wave production associated with unsteady gas expansion, due to either heat release (first term) or non isomolar combustion (second term).

Unsteady Heat Release Effects – Confined Flames

- Unsteady heat release from an acoustically compact flame induces a jump in unsteady acoustic velocity, but no change in pressure:
  \[
  u_{x,lb} - u_{x,la} = \frac{1}{A} \frac{\gamma - 1}{\gamma p_0} \dot{Q}_l
  \]  

- Unsteady heat release causes both amplification/damping and shifts in phase of sound waves traversing the flame zone.
  - The relative significance of these two effects depends upon the relative phase of the unsteady pressure and heat release.
    - The first effect is typically the most important and can cause systems with unsteady heat release to exhibit self-excited oscillations.
    - The second effect causes shifts in natural frequencies of the system.

Unsteady Heat Release Effects – Confined Flames

- Assume rigid and pressure release boundary conditions at \( x=0 \) and \( x=L \), respectively, and that the flame is acoustically compact and located at \( x=L_F \).

- Unsteady pressure and velocity in the two regions are given by:

Region I:

\[
\begin{align*}
  p_i(x,t) &= \left( \phi_i e^{i\omega(x-L_F)} + \phi_{ii} e^{i\omega(x-L_F)} \right) e^{-i\omega t} \\
  u_{x,li}(x,t) &= \frac{1}{\rho_{li}} \left( \phi_i e^{i\omega(x-L_F)} - \phi_{ii} e^{i\omega(x-L_F)} \right) e^{-i\omega t}
\end{align*}
\]  

Region II:

\[
\begin{align*}
  p_{ii}(x,t) &= \left( \phi_{i} e^{i\omega(x-L_F)} + \phi_{ii} e^{i\omega(x-L_F)} \right) e^{-i\omega t} \\
  u_{x,li}(x,t) &= \frac{1}{\rho_{li}} \left( \phi_i e^{i\omega(x-L_F)} - \phi_{ii} e^{i\omega(x-L_F)} \right) e^{-i\omega t}
\end{align*}
\]
Matching Conditions

- Applying the boundary/matching conditions leads to the following algebraic equations:

  (Left BC) \( \mathcal{A}_i e^{-\beta_i L_F} - \mathcal{B}_i e^{\beta_i L_F} = 0 \)

  (Right BC) \( \mathcal{A}_j e^{\beta_j (L-L_F)} + \mathcal{B}_j e^{-\beta_j (L-L_F)} = 0 \)

  (Pressure Matching) \( \mathcal{A}_t + \mathcal{B}_t = \mathcal{A}_j + \mathcal{B}_j \)

  (Velocity Matching) \( u_j \left( L_F^+, t \right) - u_j \left( L_F^-, t \right) = (\gamma - 1) \frac{\dot{Q}_i}{\rho_{0,j} c_{0,j}} \)


Heat Release Modeling Simplifications

- Solving the four boundary/matching conditions leads to:

  \[
  \left( \frac{\rho_{0,j} c_{0,j}}{\rho_{0,i} c_{0,i}} \right) \cos(k_i L_F) \cos(k_{ij} (L - L_F))
  - (1 + \kappa e^{i\omega \tau}) \sin(k_i L_F) \sin(k_{ij} (L - L_F)) = 0
  \]

- In order to obtain analytic solutions, we will next assume that the flame is located at the midpoint of the duct, i.e., \( L = 2L_F \) and that the temperature jump across the flame is negligible.

  \[
  \cos kL = \kappa e^{i\omega \tau} \sin^2 \left( \frac{kL}{2} \right)
  \]

Unsteady Heat Release Model

- Model for unsteady heat release is the most challenging aspect of combustion instability prediction

- We will use a “velocity coupled” flame response model

- Assumes that the unsteady heat release is proportional to the unsteady flow velocity, multiplied by the gain factor, \( \kappa \), and delayed in time by the time delay, \( \tau \).

  \[
  \dot{Q}_i = A \frac{\rho_{0,j} c_{0,j}^2}{\gamma - 1} k \kappa_{\omega \tau} \left( x = L_F, t - \tau \right)
  \]

- This time delay could originate from, for example, the convection time associated with a vortex that is excited by the sound waves.

Approximate Solution for Small Gain Values

- We can expand the solution around the \( \kappa = 0 \) solution by looking at a Taylor series in wavenumber \( k \) in the limit \( \kappa \ll 1 \).

  \[
  k = k_{\kappa=0} \left( 1 - \kappa \exp \left( i\omega_{\kappa=0} \tau \right) \right) + O(\kappa^2)
  \]

  \[
  k_{\kappa=0} = \frac{(2n - 1)\pi}{2L}
  \]
Complex Wavenumbers

- Wavenumber and frequency have an imaginary component. Considering the time component and expanding $\omega$:
  \[ \exp(-\imath \omega t) = \exp(-\imath \omega_r t) \exp(\imath\omega_i t) \]  
  (75)

- Real component is the frequency of combustor response.
  \[ \omega_r = \omega_{r,0} \left[ 1 + \left(-1\right)^n \frac{\pi}{2} \frac{\cos(\omega_{r,0} \tau)}{(2n-1)\pi} \right] \]

- Imaginary component corresponds to exponential growth or decay in time and space. Thus, $\omega_i > 0$ corresponds to a linearly unstable situation, referred to as combustion instability.
  \[ \omega_i = \left(-1\right)^n \frac{n c_v \sin(\omega_{r,0} \tau)}{2L} \]  
  (77)

Rayleigh’s Criterion

- **Rayleigh’s criterion**: Energy is added to the acoustic field when the product of the time averaged unsteady pressure and heat release is greater than zero.

  \[ p_i (x = L/2, t) \dot{Q}_i (t) \propto \frac{\mu}{2} \sin \left( \frac{(2n-1)\pi}{2} \right) \cos(\omega_{r,0} \tau) \sin(\omega_{r,0} (t - \tau)) \]

  \[
  \overline{p_i (x = L/2, t) \dot{Q}_i (t)} \propto \frac{\mu}{4} \sin \left( \frac{(2n-1)\pi}{2} \right) \sin(\omega_{r,0} \tau)
  \]

  \[ p_i (x = L/2, t) \dot{Q}_i (t) > 0 \Rightarrow \left(-1\right)^{n-1} \sin(\omega_{r,0} \tau) > 0 \]

Stability of Combustor Modes

- Unstable 1/4 wave mode ($n=1$)
  \[ m - 1/2 < \frac{\tau}{\tau_{1/4}} < m \]

- Unstable 3/4 wave mode ($n=2$)
  \[ \frac{m}{3} < \frac{\tau}{\tau_{1/4}} < \frac{m + 1/2}{3} \]

- Sign of $p_i \dot{Q}_i$ alternates with time delay
  - Important implications on why instability prediction is so difficult: no monotonic dependence upon underlying parameters

- Largest frequency shifts occur at the values where oscillations are not amplified and that the center of instability bands coincides with points of no frequency shift.

Thermo-acoustic Instability Trends

- Two parameters, the heat release time delay, $\tau$, and acoustic period, $\tau_{1/4}$ control instability conditions.

- Data clearly illustrate the non-monotonic variation of instability amplitude with $\tau / \tau_{1/4}$.

Data illustrating variation of instability amplitude with normalized time delay. Image courtesy of D. Santavicca
Understanding Time Delays

- Data from variable length combustor

Measured instability amplitude (in psi) of combustor as a function of fuel/air ratio and combustor length. Data courtesy of D. Santavicca.

More Instability Trends 1 of 2

- Note non-monotonic variation of instability amplitude with axial injector location, due to the more fundamental variation of fuel convection time delay, $\tau$.

- Biggest change in frequency is observed near the stability boundary.

Measured dependence of instability amplitude and frequency upon axial location of fuel injector. Data obtained from Lovett and Uznanski.

More Instability Trends 2 of 2

- Measured dependence of the excited instability mode amplitude upon the mean velocity in the combustor inlet. Obtained from measurements by the author.

More Instability Trends 2 of 2

- As amplitudes grow nonlinear effects grow in significance and the system is attracted to a new orbit in phase space, typically a limit cycle.

- This limit cycle oscillation can consist of relatively simple oscillations at some nearly constant amplitude, but in real combustors the amplitude more commonly "breathes" up and down in a somewhat random or quasi-periodic fashion.
Combustion Process and Gas Dynamic Nonlinearities

- Gas dynamic nonlinearities introduced by nonlinearities present in Navier-Stokes equations
- Combustion process nonlinearities are introduced by the nonlinear dependence of the heat release oscillations upon the acoustic disturbance amplitude.

Graph generated from data obtained by Bellows et al.

Boundary Induced Nonlinearities

- The nonlinearities in processes that occur at or near the combustor boundaries also affect the combustor dynamics as they are introduced into the analysis of the problem through nonlinear boundary conditions.
- Such nonlinearities are caused by, e.g., flow separation at sharp edges or rapid expansions, which cause stagnation pressure losses and a corresponding transfer of acoustic energy into vorticity.
- These nonlinearities become significant when \( u' / \bar{u} \sim O(1) \).
- Also, wave reflection and transmission processes through choked and unchoked nozzles become amplitude dependent at large amplitudes.
Course Outline

A) Introduction and Outlook
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D) Disturbance Propagation and Generation in Reacting Flows

E) Flame Response to Harmonic Excitation

- Governing Equations
- Premixed Flame Dynamics
  - General characteristics of excited flames
  - Wrinkle convection and flame relaxation processes
  - Excitation of wrinkles
  - Interference processes
  - Destruction of wrinkles
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Basic Problem

- Wave Equation:
  \[ p_{tt} - c^2 p_{xx} = (\gamma - 1) q'_t \]
- Key issue – combustion response
  - How to relate \( q' \) to variables \( p', u' \), and etc., in order to solve problem
  - Focus of this talk is on sensitivity of heat release to flow disturbances

Flame Response to Harmonic Disturbances

- Combustion instabilities manifest themselves as narrowband oscillations at natural acoustic modes of combustion chamber

Response of Global Heat Release to Flow Perturbations

\[ \dot{Q}(t) = \int_{\text{flame}} \dot{m}_F' \kappa_R dA \]

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- Premixed Flame Dynamics
  - General characteristics of excited flames
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**Analytical Tools – Z Equation**

- Key assumptions
  - $Le=1$ assumption
  - Flame sheet at $Z=Z_{st}$ surface
  - Imposed flow field
  - Equal diffusivities

\[
\rho \frac{D Y_F}{D t} - \nabla \cdot (\rho \mathcal{G}_F \nabla Y_F) = \dot{\phi}_F
\]

\[
- \dot{\phi}_F = \frac{\dot{\phi}_F}{(1 + \nu)}
\]

\[
\rho \frac{D(Y_v/\nu + 1)}{D t} - \nabla \cdot (\rho \mathcal{G}_F \nabla(Y_v/(\nu + 1))) = \frac{\dot{\phi}_v}{(\nu + 1)}
\]

Add these species equations:

\[
\rho \frac{D(Y_v + Y_{pr}/(\nu + 1))}{D t} - \nabla \cdot (\rho \mathcal{G}_F \nabla(Y_v + Y_{pr}/(\nu + 1))) = 0
\]

**Analytical Tools/Governing Equations**

- Work within fast chemistry, flamelet approximation and use G- and Z- equations to describe flame dynamics

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Premixed Flame Sheets: G-Equation

Flame fixed (Lagrangian) coordinate system:
\[ \frac{D}{Dt} G(\bar{x},t) = 0 \]

Coordinate fixed (Eulerian) coordinate system:
\[ \frac{\partial G}{\partial t} + \bar{v}_f \cdot \nabla G = 0 \]
\[ \bar{v}_f = \bar{u} - s_f \bar{h} \]
\[ \bar{h} = \nabla G / |\nabla G| \]
\[ \frac{\partial G}{\partial t} + \left( \bar{u} - s_f \frac{\nabla G}{|\nabla G|} \right) \cdot \nabla G = 0 \]

G-equation for single valued flame front

Two-dimensional flame front

Position is single valued function, \( \xi \), of the coordinate \( y \)

Define and substitute \( G(x,y,t) = x - \xi(y,t) \)

\[ \frac{\partial \xi}{\partial t} - u_x + u_y \frac{\partial \xi}{\partial y} = -s_f \sqrt{1 + \left( \frac{\partial \xi}{\partial y} \right)^2} \]

Governing Equations

- Left side:
  - Same convection operator
  - Wrinkles created on surface by fluctuations normal to iso- \( G \) or \( Z \) surfaces

\[ \frac{\partial Z}{\partial t} + \bar{u} \cdot \nabla Z = \nabla \cdot ( D \nabla Z ) \]

\[ \frac{\partial G}{\partial t} + \bar{u} \cdot \nabla G = s_d |\nabla G| \]

- Right side:
  - Non-premixed flame – diffusion operator, linear
  - Premixed flame – flame propagation, nonlinear
  - Right side of both equations becomes negligible in \( \text{Pe} = \frac{uL}{D} \gg 1 \) or \( \frac{u}{s_d} \gg 1 \) limits

Governing Equations

- G-equation only physically meaningful at the flame surface, \( G=0 \)
  - Can make the substitution, \( G(x,y,z,t) = x - \xi(y,z,t) \)

- Z-equation physically meaningful everywhere
  - Cannot make analogous substitution
Governing Equations

- Reflects fundamental difference in problem physics
  - Premixed flame sheet only influenced by flow velocity at flame
  - Non-premixed flame sheet influenced by flow disturbances everywhere

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- Governing Equations
  - Premixed Flame Dynamics
    - General characteristics of excited flames
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Excited Bluff Body Flames
(Line of sight luminosity)

Overlay of Instantaneous Flame Edges

Quantifying Flame Edge Response

Spatial Behavior of Flame Response

- Strong response at forcing frequency
  - Non-monotonic spatial dependence

Convective wavelength:
\[ \lambda_c = \frac{U_0}{f_0} \]
- distance a disturbance propagates at mean flow speed in one excitation period
Flame Wrinkling Characteristics

1. Low amplitude flame fluctuation near attachment point, with subsequent growth downstream.
2. Peak in amplitude of fluctuation, $L' = L'_{\text{peak}}$
3. Decay in amplitude of flame response farther downstream.

Typical Results – Other Flames

- 50 m/s, 644K
- 1.8 m/s, 150Hz

- Magnitude can oscillate with downstream distance.

Analysis of Flame Dynamics

1. Wrinkle convection and flame relaxation processes.
2. Excitation of wrinkles.
3. Interference processes.
4. Destruction of wrinkles.

Level Set Equation for Flame Position

$$ G\text{-equation}: \frac{\partial L}{\partial t} + \left( u_f \frac{\partial L}{\partial x} - v_f \right) = S_L \sqrt{1 + \left( \frac{\partial L}{\partial x} \right)^2} $$
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Wrinkle Convection

Model problem: Step change in axial velocity over the entire domain from $u_a$ to $u_b$, both of which exceed $s_d$:

$$ u = \begin{cases} 
  u_a & t < 0 \\
  u_b & t \geq 0 
\end{cases} $$

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Wrinkle Convection

- Flame relaxation process consists of a “wave” that propagates along the flame in the flow direction.

Harmonically Oscillating Bluff Body


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Phase Characteristics of Flame Wrinkle

Convection speed of Flame wrinkle, \( u_{c,f} \)

\( \neq \)

Mean flow velocity, \( u_0 \)

\( \neq \)

Disturbance Velocity, \( u_{c,v} \)


Harmonically Oscillating Bluff Body

- Linearized, constant burning velocity formulation:
  - Excite flame wrinkle with spatially constant amplitude
  - Phase: linearly varies

- Wrinkle convection is controlling process responsible for low pass filter character of global flame response

Excitation of Wrinkles on Anchored Flames

\[
\frac{\partial L^\prime(x,t)}{\partial x} = \frac{1}{u_j} \int_0^x \frac{\partial u'_n}{\partial x} \left( x - \frac{x - x'}{u_j} \right) dx' + \frac{1}{u_j} u'_n(x = 0, t = \frac{x}{u_j})
\]

- Linearized solution of G Equation, assume anchored flame

- Wrinkle convection can be seen from delay term

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- Governing Equations
- Premixed Flame Dynamics
  - General characteristics of excited flames
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**Excitation of Flame Wrinkles – Spatially Uniform Disturbance Field**

\[
\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u'_r} \int_0^x \frac{\partial u'}{\partial x}(x',t-x/u_r) \, dx' + \frac{1}{u'_r} u'_r(x=0,t=t-x/u_r)
\]

- Wave generated at attachment point \((x=0)\), convects downstream
- If excitation velocity is spatially uniform, flame response exclusively controlled by flame anchoring “boundary condition”
  - Kinetic /diffusive/heat loss effects, though not explicitly shown here, are very important!

**Near Field Behavior - Predictions**

- Can derive analytical formula for nearfield slope for arbitrary velocity field:

\[
\frac{\partial L'(x,t)}{\partial x} = \frac{1}{\cos^2 \theta} \frac{u'_r}{u_r}
\]

- Flame starts with small amplitude fluctuations because of attachment
  \(L'(x=0, t) = 0\)
- Nearfield dynamics are essentially linear in amplitude

**Comparisons With Data**

\[
\frac{1}{\cos^2 \theta} \frac{u'_r}{u_r} = \frac{\partial L'}{\partial x}
\]

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D) Disturbance Propagation and Generation in Reacting Flows
E) Flame Response to Harmonic Excitation

- Governing Equations
- **Premixed Flame Dynamics**
  - General characteristics of excited flames
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Excitation of Flame Wrinkles – Spatially Varying Disturbance Field

\[
\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t} \int_0^1 \frac{\partial u'_t}{\partial x}(x',t) \, dx' \, dx + \frac{1}{u_t} u'_t(x = 0, t = t - \frac{x}{u_t})
\]

- Flame wrinkles generated at all points where disturbance velocity is non-uniform, \(du'/dx \neq 0\)
  - Flame disturbance at location \(x\) is convolution of disturbances at upstream locations and previous times
- Convecting vortex is continuously disturbing flame
  - Vortex convecting at speed of \(u_{cv}\)
  - Flame wrinkle that is excited convects at speed of \(u_t\)

Solution Characteristics

- Note interference pattern on flame wrinkling
- Interference length scale:
  \[
  \frac{\lambda_{int}}{(\lambda_j \sin \theta)} = \frac{1}{\left| \frac{u_j}{u_{cv}} - 1 \right|}
  \]

---

Model Problem: Attached Flame Excited by a Harmonically Oscillating, Convecting Disturbance

- Model problem: flame excited by convecting velocity field,

\[
\frac{u_{t,0}'}{u_{j,0}} = \varepsilon_n \cos(2\pi f(t - x / u_{cv}))
\]

- Linearized solution:

\[
\frac{\xi_i}{u_{j,0}f} = \text{Real} \left\{ -i \cdot \frac{\varepsilon_n}{\sin \theta} \cdot \frac{1}{2\pi \left( u_{j,0} \cos \theta / u_{cv} - 1 \right)} \cdot e^{i2\pi f \left( \frac{u_j}{u_{cv}} \tan \theta \right)} - e^{i2\pi f \left( \frac{u_j}{u_{cv}} \sin \theta \right)} \right\}
\]
Interference Patterns

- Result emphasizes “wave-like”, non-local nature of flame response
- Can get multiple maxima/minima if excitation field persists far enough downstream

D. Shin et al., AIAA Aerospace Science Meeting, 2011.
V. Acharya et al., ASME Turbo Expo, 2011.

### Comparison with Data

\[ \frac{x_{\text{peak}}}{\lambda_c} = \frac{\cos^2 \theta}{2 \left( \frac{u_0}{u_{c,v}} \cos^2 \theta - 1 \right)} \]

- Space/time coherence of disturbances key to interference patterns
- Example: convecting random disturbances to simulate turbulent flow disturbances

Aside: Randomly Oscillating, Convecting Disturbances

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E) Flame Response to Harmonic Excitation

- Governing Equations
- Premixed Flame Dynamics
  - General characteristics of excited flames
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Flame Wrinkle Destruction Processes: Kinematic Restoration

- Flame propagation normal to itself smoothes out flame wrinkles
- Typical manifestation: vortex rollup of flame
- Process is amplitude dependent and strongly nonlinear
  - Large amplitude and/or short length scale corrugations smooth out faster

Kinematic Restoration Effects: Oscillating Flame Holder Problem

- Leads to nonlinear farfield flame dynamics
- Decay rate is amplitude dependent

Kinematic Restoration Effects: Velocities, Numerical Calculation, Experimental Result

Multi-Zone Behavior of Kinematic Restoration

- Near flame holder
  - Higher amplitudes and shorter wavelengths decay faster
- Farther downstream
  - Flame position independent of wrinkling magnitude
  - Flame position only a function of wrinkling wavelength
  - is determined by the leading points

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Flame Wrinkle Destruction Processes:  
Kinematic Restoration


Flame Stretch Effects

\[
\frac{\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}_L}}{\tilde{\sigma}} \approx \exp\left(-\tilde{\sigma}\tilde{\varepsilon}_L, \tilde{x}\right)
\]

\(\tilde{\sigma}: \text{Normalized Markstein length}\)

Linear in amplitude wrinkle destruction process

Course Outline

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E) Flame Response to Harmonic Excitation

- Governing Equations
- Premixed Flame Dynamics
  - General characteristics of excited flames
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Flame Wrinkle Destruction Processes: 
Flame Stretch in Thermodiffusively Stable Flames

• Conditions
  – Over ventilated flame
  – Fuel & oxidizer forced by spatially uniform flow oscillations
  – Will show illustrative solution in $\text{Pe} \gg 1$ (i.e., $W_{tu_0} \gg \text{Co}$) limit

K. Balasubramanian, R. Sujith, Comb sci and tech, 2008.
Illustrative Result of Flame Front Dynamics

Convective wavelength \( \left( \frac{u_{x,0}}{f} \right) \) = 3.3

Flame length \( (L_f) \)

Oxidizer

Fuel

Illustrative Result of Flame Front Dynamics

Convective wavelength \( \left( \frac{u_{x,0}}{f} \right) \) = 0.5

Flame length \( (L_f) \)

Oxidizer

Fuel
Comparison - similarities

- Non-premixed
  \[ \xi_{i,n}(x,t) = \frac{i \sigma u_{x,0}}{2 \pi f} \sin \theta(x) \left( 1 - \exp \left( i 2 \pi f \frac{x}{u_{x,0}} \right) \right) \exp[-i2\pi ft] \]

- Premixed
  \[ \xi_{i,n}(x,t) = \frac{i \sigma u_{x,0}}{2 \pi f} \sin \theta \cdot \left( 1 - \exp \left( i 2 \pi f \frac{x}{u_{x,0}} \cos \theta \right) \right) \exp[-i2\pi ft] \]

Similarities between space/time dynamics of premixed and non-premixed flames responding to bulk flow perturbations

- Magnitude
- Flame Angle
- Wave Form

Comparison - difference

- Non-premixed
  \[ \xi_{i,n}(x,t) = \frac{i \sigma u_{x,0}}{2 \pi f} \sin \theta(x) \left( 1 - \exp \left( i 2 \pi f \frac{x}{u_{x,0}} \right) \right) \exp[-i2\pi ft] \]

- Premixed
  \[ \xi_{i,n}(x,t) = \frac{i \sigma u_{x,0}}{2 \pi f} \sin \theta \cdot \left( 1 - \exp \left( i 2 \pi f \frac{x}{u_{x,0}} \cos \theta \right) \right) \exp[-i2\pi ft] \]

Convective wave speeds

Spatially Integrated Heat Release

- Unsteady heat release
  \[ \dot{Q}(t) = \int_{\text{flame}} m^*_{\text{R}} \dot{\Lambda}_R \, dA \]
  - Flame surface area (Weighted Area)
  - Mass burning rate (MBR)
  - We’ll assume constant composition

Flame describing function:

\[ \mathcal{T} = \frac{\hat{\dot{Q}}_1}{\tilde{u}_{x,1} / u_{x,0}} = \mathcal{T}_{WA} + \mathcal{T}_{MBR} \]

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E) Flame Response to Harmonic Excitation

- Governing Equations
- Premixed Flame Dynamics
  - General characteristics of excited flames
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### Premixed Flames

- Spatially integrated heat release:
  \[ \dot{Q}(t) = \int_{\text{flame}} \rho \overline{u} S_{\epsilon} \kappa_0 dA \]

- Linearized for constant flame speed, heat of reaction, and density:
  \[ \frac{\dot{Q}(t)}{Q_0} = \int_{\text{flame}} \frac{dA}{A_0} \quad \text{Proportional to flame area} \]
  \[ A(t) = \sin \theta \int_{\text{flame}} W(y) \sqrt{1 + \left( \frac{\partial \xi_f}{\partial y} \right)^2} dy \]

- \( W(y) \) is a geometry dependent weighting factor:
  - Two-dimensional
  - Axisymmetric Cone
  - Axisymmetric Wedge

\[ W(y) = \frac{1}{W_f} \quad \frac{2\pi (W_f - y)}{(\pi W_f^2)} \quad \frac{2\pi y}{(\pi W_f^2)} \]

where: \( W_f = L_F \tan \theta \)

### Premixed Flame TF Gain – Bulk Flow Excitation

- St<<1: \( \mathcal{F} = 1 \)
- St>>1: \( \mathcal{F} \sim 1/St \)

### Why the 1/St Rolloff?

- Flame position \( \sim 1/St \)
  \[ \xi_{i,\epsilon}(x, t) = \frac{i \epsilon u_x}{2 \pi f_l} \sin \theta \exp \left[ \frac{i 2 \pi St_s x}{L_{f,0}} \right] \exp \left[ -i 2 \pi ft \right] \]
  
  - Low pass filter characteristic!

- Flame area/unit axial distance:
  \[ dA = \sqrt{1 + \left( \frac{\partial \xi_f}{\partial x} \right)^2} \]

- Linearized:
  \[ \frac{dA}{dx} = \sqrt{1 + \left( \frac{\partial \xi_f}{\partial x} \right)^2} + \sqrt{1 + \left( \frac{\partial \xi_f}{\partial x} \right)^2} \]

\[ \infty \epsilon \sin \theta \exp \left[ \frac{i 2 \pi St_s x}{L_{f,0}} \right] \exp \left[ -i 2 \pi ft \right] \]
Why the 1/St Rolloff?

- Consider spatial integral of traveling wave disturbance:

\[
\int_{x=0}^{L_F} \cos \left( \omega \left( t - \frac{x}{u} \right) \right) dx = -\frac{u}{\omega} \left\{ \sin \left( \omega \left( t - \frac{L_F}{u} \right) \right) - \sin \omega t \right\}
\]

Traveling Wave

1/St due to interference effects associated with tangential convection of wrinkles

- 1/St comes from the integration!

Premixed Flame Response - Phase

Flame area-velocity relationship for convectively compact flame (low St values):

\[
\frac{A(t)}{A_0} = n \frac{u(t - \tau)}{u_0}
\]

\[\tau = C \frac{L_f}{u_0}\]

Axi-symmetric Wedge: 

\[C = \frac{2(1 + k_C^{-1})}{3 \cos^2 \theta}\]

Axi-symmetric Cone: 

\[C = \frac{2(k_c + 1)}{3 k_c \cos^2 \theta}\]

Two-dimensional: 

\[C = \frac{(k_c + 1)}{2 k_c \cos^2 \theta}\]

Nonpremixed Flames-Bulk Flow Excitation

- Returning to spatially integrated heat release:

\[
\dot{Q}(t) = \int \dot{m}_F^* \kappa_R dA
\]

- Linearize the MBR and area terms:

\[
\frac{\dot{Q}(t)}{\kappa_R} = \int \dot{m}_{F,0}^* dA_0 + \int \dot{m}_{F,0}^* dA_0 + \int \dot{m}_{F,1}^* dA_0
\]

Steady State

Area Fluctuation

MBR Fluctuation

Contribution

Contribution

Contribution
**Non-Premixed Flames: Role of Area Fluctuations**

\[
\mathcal{T}_{WA} = \frac{\int_{\text{flame}} (\dot{\dot{m}}_F)_0^i dA}{\int_{\text{flame}} (\dot{m}_F)_0^i dA_0}
\]

Very strong function of \(x\)!

For the higher velocity,
- Area increases \(\Rightarrow\) Premixed
- Weighted area decreases \(\Rightarrow\) Non-premixed

**Weighted Area cont’d**

At low frequencies
- Non-premixed
  - Weighted Area
- Premixed
  - Area (as weighting is constant)

At low frequencies, area and weighted area are **out of phase**

**Mass Burning Rate**

\[
\mathcal{F}_{MBR} = \frac{\int_{\text{flame}} (\dot{m}_F)_i^i dA_0}{\int_{\text{flame}} (\dot{m}_F)_0^i dA_0}
\]

- Non-premixed: 
  \[
  (\dot{m}_F)_i^i \sim \frac{1}{\cos \theta} \frac{\partial Z}{\partial y}
  \]
  - Fluctuations in spatial gradients of the mixture fraction
- Premixed: 
  \[
  (\dot{m}_F)_i^i \sim \frac{\partial S_L}{\partial \phi}
  \]
  - Stretch sensitivity of the burning velocity

**Flame Transfer Functions - Contributions**

Significant differences in dominant processes controlling heat release oscillations
- Non-premixed: Mass burning rate
- Premixed: Area  
  Magina et al., *Proc of the Comb Inst*, 2012.
Comparisons of Gain and Phase of FTF

Gain Phase

St \ll 1 \quad \sim 1
St \gg 1 \quad \sim 1/St
St \sim O(1) \quad \sim 1/St^{1/2}

- At St \sim 0(1), non-premixed flames are more sensitive to flow perturbations

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Premixed Flame TF’s: More Complex Disturbance Fields

Disturbance convecting axially at velocity of U_c & k_c = U_d/U_c

Axisymmetric wedge flame:

Gain
- fcn (St, k_c)
- Unity at low St
- Gain increases greater than unity
- "Nodes" of zero heat release response

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Closing Remarks

- Flame response exhibits “wavelike”, non-local behavior due to wrinkle convection, leading to:
  - maxima/minima in gain curves, interference phenomenon, etc.
  - 1/f behavior in transfer functions

- Premixed flame wrinkles controlled by different processes in different regions

- Role of area, weighted area, mass burning rate are quite different for premixed and non-premixed flames

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Summary

- Many great fundamental problems in gas turbine combustion
  - Edge flames
  - Reacting swirl flows
  - Differential diffusion effects on turbulent flames
  - Turbulent flames in highly preheated flows
  - Response of flames to harmonic disturbances
  - …and many more!!

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