The 2024 Princeton-Combustion Institute Summer School on Combustion and the Environment:

Combustion Instability

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In the Reacting Flow Dynamics Lab, our research addresses these decarbonization challenges with three research areas:

- **Combustor Operability**
- **Materials and Durability**
- **Alternative Fuels**

Wolfe et al. (2005). *Surface and coatings technology*, 190(1), 132-149.
Who are you?

Introduce yourself with three pieces of information:

— Name

— Institution

— Your thesis work in no more than two words (example: combustion instability)
Learning objectives of this course – at the end of the course, students should:

1. Identify the technologies in which combustion instability is an issue
2. Describe the basic thermoacoustic coupling process
3. Explain the kinematics of flame response to harmonic inputs
4. Assess a flame transfer function to identify the potential coupling processes
5. Use your understanding of flame response and thermoacoustic coupling to explain instability control techniques
Course schedule

Monday:
— 1400-1500: Introduction and combustion instability in technologies
— 1500-1530: Thermoacoustic feedback
— 1530-1545: Break
— 1545-1730: Flame kinematics (with crafts!) and flame response

Tuesday:
— 1400-1530: Flame response and flame transfer functions
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Combustion dynamics is one of the most expensive problems in several industries, including gas turbines, rockets, and boilers

- Damages expensive hardware
- Reduces operability
- Increases emissions ($\text{NO}_x$, CO)

Thermoacoustic combustion instability can occur when a flame is placed in a chamber with an acoustic mode.
Example 1: F-1 rocket engine
Example 2: Rolls-Royce Trent 60 Aeroderivative
Example 3: Babcock and Wilcox package boiler


https://www.babcock.com/home/products/package-boilers
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Acoustics time!
Acoustic wave propagation – to the board!
Acoustic modes in a cylindrical chamber

- **Longitudinal Mode**
  - \( P' \)
  - \( u' \)

- **Radial Mode**

- **Azimuthal Mode**
Back to thermoacoustics
Thermoacoustic instability is driven by a feedback process between acoustic oscillations and heat release rate oscillations of the flame.
The energy transfer is facilitated by a compression/expansion work process.
The Rayleigh criterion provides a way of determining the extent of coupling between the heat release rate oscillations and pressure

\[ RI = \int_V \int_T p'(\tilde{x}, t) \dot{q}'(\tilde{x}, t) \, dt \, dV \]

- If RI > 0 \(\rightarrow\) thermoacoustic feedback drives instability growth
- If RI < 0 \(\rightarrow\) thermoacoustic feedback does not exist
The Rayleigh criterion provides a way of determining the extent of coupling between the heat release rate oscillations and pressure

\[ RI = \int \int_{V,T} p'(\bar{x},t) \dot{q}'(\bar{x},t) dtdV \]

- If \( RI > 0 \) \( \Rightarrow \) thermoacoustic feedback drives instability growth
- If \( RI > \text{Damping} \) \( \Rightarrow \) instability actually grows
- If \( RI < 0 \) \( \Rightarrow \) thermoacoustic feedback does not exist
The Rayleigh criterion provides a way of determining the extent of coupling between the heat release rate oscillations and pressure.

\[
RI = \int \int p'(\bar{x}, t) \dot{q}'(\bar{x}, t) \, dt \, dV
\]

Phase relationship between pressure (solid lines) and three heat release rate oscillations at different phases:

- \(\pi/6\) (dashed)
- \(\pi/4\) (dotted)
- \(\pi/2\) (dash-dot)
Phase effects example #1: impact of fuel injection point

Phase effects example #2: impact of bulk flow velocity

Key Point: Because instability is non-monotonic, changes to any number of operating parameters will never always suppress instability. It always depends.
Illustration of key point – flame speed variation by equivalence ratio, fuel composition, preheat temperature, etc.

→ Making the flame shorter makes instability worse

Example courtesy of Tim Lieuwen, Georgia Tech
Illustration of key point – flame speed variation by equivalence ratio, fuel composition, preheat temperature, etc.

→ Making the flame shorter makes instability better

Example courtesy of Tim Lieuwen, Georgia Tech
As a result, you will always, somewhere, have the possibility of an instability

Data courtesy of D. Santavicca
The amplitude of the instability is determined by the balance between damping and driving, producing a limit cycle oscillation.

\[ RI = \int_V \int_T p'(\bar{x},t) \dot{q}'(\bar{x},t) \, dt \, dV \]

- If \( RI > 0 \) → thermoacoustic feedback drives instability growth
- If \( RI > \) Damping → instability actually grows
- If \( RI < 0 \) → thermoacoustic feedback does not exist
Coupling mechanisms

Acoustic oscillation

Heat release rate oscillation

Coupling mechanism

$p'$, $u'$
Velocity coupling was identified in some of the earliest work on combustion instability as a critical mode for coupling.

Velocity coupling can occur through multiple sources of velocity, although the most common two sources are acoustic and vortical.

**Acoustic**

Source: S. Ducruix, D. Durox, S. Candel, Centre National de la Recherche Scientifique and Ecole CentraleSupelec, Paris-Saclay

**Vortical**

SIDEBAR

Disturbance decompositions – to the board!
SIDEBAR OVER

Back to velocity coupling
Velocity coupling works through a variety of mechanisms, though the area component is largely responsible

\[ Q(t) = \int_{A_{\text{flame}}} \rho_u S_L h_R dA \]

Source: Shreekrishna, *Response mechanisms of attached premixed flames to harmonic forcing*, Aerospace Engineering, Georgia Institute of Technology (2011)
Equivalence ratio coupling occurs when fluctuations in the fuel and/or oxidizer drive local fluctuations in the equivalence ratio – sometimes called “mixture coupling”.

\[ P_{\text{air}} = \overline{P}_{\text{air}} + P'_{\text{acoustic}} \]

\[ P_{\text{fuel}} \]

\[ \dot{m}_{\text{fuel}}(t) \sim c_v \sqrt{2\left( P_{\text{fuel}} - P_{\text{air}}(t) \right)} \]
Equivalence ratio fluctuations drive heat release rate fluctuations through a complex web of pathways

\[ Q(t) = \int_{A_{\text{flame}}} \rho_u S_L h_R \, dA \]

Source: Shreekrishna, *Response mechanisms of attached premixed flames to harmonic forcing*, Aerospace Engineering, Georgia Institute of Technology (2011)
Phase delays sum additively in realistic systems, resulting in interference.

Key Point: Keep track of your phase lags – they are the driver of instability.
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Flames respond to acoustic forcing in really beautiful ways and the physics behind the response is referred to as “flame kinematics”.

Videos courtesy of D. Durox, EM2C Laboratory
Flames propagate normal to themselves at a speed $S_L$, the flame speed, which is not that fast for most mixtures.

Keyword: Kinematic condition is where $S_L=u$
Flame propagation in two dimensions follows the same basic concept, but now you have to do some trig to determine the flame angle.

\[ S_L = u \sin(\alpha) \]
As you change the velocity of the flow, you change the angle at which the flame stabilizes relative to the flow where the kinematic condition is met.
Now with crafts!
Let’s try this with our straws and pipe cleaners
Let’s explore what happens when the velocity varies very slowly

\[ u(t) = \bar{u} + u' \cos(\omega t) \]

Would nominally result in this flame shape

Is a very low frequency
SIDEBAR

Convective wavelength – to the board!
SIDEBAR OVER

Back to flame response
Harmonic forcing – convective wavelength < Lf

\[ u(t) = \bar{u} + u' \cos(\omega t) \]

Would nominally result in this flame shape

Is no longer a small number...
Harmonic forcing – convective wavelength $\ll L_f$

\[ u(t) = \bar{u} + u'\cos(\omega t) \]

Would nominally result in this flame shape

Is a large number...
When flames propagate normal to themselves, they first cusp and then destroy the wrinkles imposed by an oscillating field.
The flame can only withstand a certain frequency of oscillation before kinematic restoration destroys wrinkles faster than they can cause heat release rate oscillation.
Put a boundary condition on this problem and you can now explain the flame area pathway of velocity-coupled combustion instability.

Source: Shreekrishna, Response mechanisms of attached premixed flames to harmonic forcing, Aerospace Engineering, Georgia Institute of Technology (2011)
We can use a level-set formulation to better explain this – to the board!
The base shape of the flame in addition to the frequency of excitation determines the kinematic response of the flame to perturbations.

\[
\frac{u_o}{S_L} = \sqrt{\left(\frac{L_f}{R}\right)^2 + 1}
\]

\[\beta = \frac{L_f}{R}\]

Key Point: Flames act as low-pass filters and their shape (and robustness to stretch) determines what the cut-off frequency is.
What have we learned so far?

1. Identify the technologies in which combustion instability is an issue
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Day 2: Tuesday, June 18

Welcome!
Review of key points

— Thermoacoustic instability is driven by a feedback process between pressure and heat release rate oscillations – the phase between these two determines whether the thermoacoustic coupling is driving or damping

— The final amplitude of the instability is a balance between driving and damping, located at a limit cycle amplitude that is determined by the level of nonlinearity in the flame response

— Several different coupling mechanisms exist to couple the acoustics and flame response – the mechanism of heat release rate oscillation is largely through area fluctuations where velocity fluctuations are present

— Flame kinematics determines these area fluctuations, where the flame responds as a low-pass filter to incoming oscillations
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Flame response is often described in terms of “flame transfer functions,” which describe the response of a flame to an input disturbance.

\[
G(f) = \frac{\hat{Q}(f)}{\hat{u}(f)}/\bar{u}
\]

where \(G(f)\) is the flame transfer function, \(\hat{Q}(f)\) is the Fourier transform of the output heat release rate fluctuation, \(\hat{u}(f)\) is the Fourier transform of the input velocity fluctuation, and \(\bar{u}\) is the mean input velocity.

\[
g(t) = \bar{u} + u' \cos(\omega t)
g(t) = \bar{u} + u' \cos(2\pi ft)
\]

\[
Q(t) = \bar{Q} + Q' \cos(\omega t)
Q(t) = \bar{Q} + Q' \cos(2\pi ft)
\]
We typically plot the transfer function using a Bode Diagram, which shows the frequency dependence of the gain and phase (thank you Stack Exchange).
Flame transfer functions are a useful construct for understanding thermoacoustic response as feedback occurs at specific resonant frequencies.

Response of a flame at a given frequency:

\[
G(f) = \frac{\hat{Q}(f)}{\hat{u}(f)} / \frac{\bar{Q}}{\bar{u}}
\]

If \( G(f) > 1 \) → amplification

If \( G(f) < 1 \) → not amplified

Knowledge of the acoustic modes of a system (pretty easy to calculate):

Information about what acoustic modes could couple with the flame.
Flame transfer functions are typically measured using chemiluminescence as a marker of heat release rate.

Acoustic velocity fluctuation is calculated using a multi-microphone method, which uses conservation of momentum from pressure transducer measurements.

\[ P(x, t) = Re\{[P_+ e^{ikx} + P_- e^{-ikx}]e^{-i2\pi ft}\} \]
\[ \hat{P}(x, f) = \hat{P}_+ e^{ikx} + \hat{P}_- e^{-ikx} \]

Momentum conservation says:
\[ \hat{u} = \frac{1}{i\omega \rho} \frac{d\hat{P}}{dx} = \frac{1}{\rho c} [\hat{P}_+ e^{ikx} - \hat{P}_- e^{-ikx}] \]
\[ \begin{bmatrix} \hat{P}_1 \\ \hat{P}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ e^{ikL} & e^{-ikL} \end{bmatrix} \begin{bmatrix} \hat{P}_+ \\ \hat{P}_- \end{bmatrix} \]
The measurement is done at a stable condition and a linear perturbation (~5%) is provided at a range of frequencies, measuring pressure and chemiluminescence.
Resulting flame transfer function shows response of the flame to coherent oscillations at a range of frequencies.

A few interesting features:

Flame acts as amplifier

At zero frequency, FTF gain will equal 0

Presence of nodes suggests cancellation process

Impact of variation in velocity:

Nodes move to higher frequency as the velocity increases – does this make sense?

Phase difference decreases with increasing velocity – does this make sense?

Impact of equivalence ratio:


Phase difference decreases with increasing equivalence ratio – does this make sense?
Impact of preheat temperature:

Nodes move to higher frequency as the temperature increases – does this make sense?

Phase difference decreases with increasing temperature – does this make sense?

Non-dimensionalizing the frequency into Strouhal number tells you whether the mechanisms responsible at these different conditions are the same.

\[ St = \frac{f L_f}{\bar{u}} \]

So what’s up with this cancellation phenomenon?

Back to theory!
The level-set formulation can provide direct insight into the cancellation process – to the board!

One of the best starter references on this type of work:
Two different disturbance sources have varying amplitudes and phases as a function of frequency.

Fig. 6 Strouhal number dependence of the magnitude of the ratio of the transfer functions due to the flow-nonuniformity and boundary-condition terms for different values of $\eta$.

Fig. 7 Strouhal number dependence of the phase of the ratio of the transfer functions due to the flow-forcing and boundary-condition terms for wedge flames. Shaded regions indicate points at which boundary-condition and flow-nonuniformity terms are in phase.

Superimposed, these disturbances result in nodes and anti-nodes in the total flame area fluctuation transfer function.

Fig. 8 Axisymmetric conical linear transfer-function $G_c(St_2, \eta)$ magnitude dependence upon the reduced Strouhal number $St_2$ for different values of $\eta$.

Fig. 10 Axisymmetric wedge linear transfer-function $G_w(St_2, \eta)$ amplitude dependence upon the reduced Strouhal number $St_2$ for different values of $\eta$.

Theory also shows the impact that the nonlinear kinematic restoration effects can have as a function of disturbance amplitude on FTF gain.

![Diagram showing Strouhal number dependence of the magnitude of the flame-area velocity transfer function for the axisymmetric wedge flame; \( \beta = 2 \) and \( \eta = 2 \).]

Key Point: The nodes and anti-nodes of your flame transfer functions tell you something about interference of multiple disturbances.
What does this look like in experiments?
Example 1: Swirl-stabilized flame from Palies et al.

They measured flame describing functions for two different flames and saw significant constructive/destructive interference in the FDF.

Flame describing functions
A flame describing function is a function of both the frequency and amplitude of the input disturbance and so captures nonlinear behavior.

\[
\begin{align*}
G(f, \varepsilon) &= \frac{\hat{Q}(f, \varepsilon)}{\hat{u}(f, \varepsilon)} / \bar{Q}
\end{align*}
\]

\[
\begin{align*}
\hat{u}(f, \varepsilon) &= \hat{u} + \hat{u}'(\varepsilon) \cos(\omega t) \\
\hat{Q}(f, \varepsilon) &= \hat{Q} + \hat{Q}'(\varepsilon) \cos(\omega t) \\
\end{align*}
\]

\[
\begin{align*}
Q(t) &= \bar{Q} + Q'(\varepsilon) \cos(\omega t) \\
Q(t) &= \bar{Q} + Q'(\varepsilon) \cos(2\pi f t) \\
\end{align*}
\]
Careful measurement of the velocity field and theory from Cumpstsy showed that there are fluctuations in axial velocity and swirling velocity.

Destructive interference

60 Hz

Constructive interference

90 Hz

Example 2: Impact of time delay on disturbance interference in a swirl-stabilized flame (Bunce and Santavicca)

Flame location modification led to changes in the location of the nodes and anti-nodes in the flame transfer function.
So what do I do with this FTF?
Network models are a common way that flame transfer functions can be used to predict instability in complex combustion systems.

\[ p_k(x, t) = \bar{p}_k + p^*_k(x, t) = \bar{p}_k + A^+_k(t - \tau^+_k) + A^-_k(t - \tau^-_k) \]

\[ u_k(x, t) = \bar{u}_k + u'_k(x, t) = \bar{u}_k + \frac{1}{\rho_k c_k} \left[ A^+_k(t - \tau^+_k) - A^-_k(t - \tau^-_k) \right] \]

\[ \rho_k(x, t) = \bar{\rho}_k + \rho'_k(x, t) = \bar{\rho}_k + \frac{1}{c_k^2} \left[ A^+_k(t - \tau^+_k) + A^-_k(t - \tau^-_k) \right] - \frac{1}{c_k^2} E_k(t - \tau^*_k) \]

Flame treated as an interface:

\[ \rho_{k+1}(x_{k+1}, t)u_{k+1}(x_{k+1}, t) = \rho_{k+\frac{1}{2}}(x_k, t)u_{k+\frac{1}{2}}(x_k, t) \]

\[ p_{k+1}(x_{k+1}, t) + \rho_{k+1}(x_{k+1}, t)u_{k+1}^2(x_{k+1}, t) = p_{k+\frac{1}{2}}(x_k, t) + \rho_{k+\frac{1}{2}}(x_k, t)u_{k+\frac{1}{2}}^2(x_k, t) \]

\[ \rho_{k+1}(x_{k+1}, t)u_{k+1}(x_{k+1}, t)H_{k+1}(x_{k+1}, t) = \rho_{k+\frac{1}{2}}(x_k, t)u_{k+\frac{1}{2}}(x_k, t)H_{k+\frac{1}{2}}(x_k, t) + \dot{q}(t) \]

\[ p_k(x_k, t) = \rho_k(x_k, t)R_{g,1}T_k(x_k, t), \quad p_{k+\frac{1}{2}}(x_k, t) = \rho_{k+\frac{1}{2}}(x_k, t)R_{g,2}T_{k+\frac{1}{2}}(x_k, t) \]

\[ \bar{H} = \int_{T_0}^{T} C_p dT + \frac{1}{2} \bar{u}^2_k \]

Very cool, open-source tool:
https://www.oscilos.com/
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The Rayleigh Index can tell us a lot about the potential methods of instability control.

\[ RI = \int \int_{VT} p'(\bar{x}, t) \dot{q}'(\bar{x}, t) \, dtdV \]

- Increase the damping so that it suppresses the instability.
- Move the pressure anti-node relative to the flame.
- De-phase the p' and q' over an acoustic cycle.
- Change the heat release rate spatial distribution.
The Rayleigh Index can tell us a lot about the potential methods of instability control.

\[ RI = \int\int_{V,T} p'(\bar{x},t)q'(\bar{x},t)\,dtdV \]

- Increase the damping so that it suppresses the instability.
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- Change the heat release rate spatial distribution.
Increase damping: damping is achieved through both direct damping devices as well as combustor design

Damping devices:

- Helmholtz resonators

- Quarter wave tubes

Helmholtz Frequency:

\[ f = \frac{c}{4L} \]

Increase damping: damping is achieved through both direct damping devices as well as combustor design.


Wikipedia: Rolls-Royce Nene turbojet

Wikipedia: P&W JT9D

https://atshouston.com/project/siemens-westinghouse-gas-turbine-combustor-basket/
The Rayleigh Index can tell us a lot about the potential methods of instability control.

\[ RI = \int_{V} \int_{T} p' (\bar{x}, t) \dot{q}' (\bar{x}, t) \, dt \, dV \]

- Increase the damping so that it suppresses the instability
- Move the pressure anti-node relative to the flame
- De-phase the \( p' \) and \( q' \) over an acoustic cycle
- Change the heat release rate spatial distribution
Example 1: Circumferential modes were broken by a technique called “symmetry breaking,” where the symmetry of Q’ is changed relative to acoustic field.
Example 2: Piloting can change both the shape and strength of the flame, making it less susceptible to combustion instability.
Example 3: Fuel staging is commonly used in systems with multiple flames to redistribute heat release rate and de-phase heat release rate oscillations.

GE DLN2.6

Source: http://www.ccj-online.com/bg/companies/allied/
Example 3: Fuel staging is commonly used in systems with multiple flames to redistribute heat release rate and de-phase heat release rate oscillations.

Diagram:
- Combustor Can
- Five Swirled Nozzles
- Manifold
- Premixed NG-air mixture
- Staging fuel enters combustor here
- Control valve
As we increased the equivalence ratio of the inner nozzle, the equivalence ratio of the outer nozzles decrease and the instability is suppressed.

High-speed imaging allows us to calculate the local Rayleigh Index, showing the regions of driving in the flame.

We noticed that through time, there seemed to be an imbalance in the phasing of the flame oscillations during the staging transient.
High-speed PLIF imaging allowed us to look at the flame edge oscillation in three cases: unstable, meta-stable, and stable.

The phase oscillation between the flame edges suggest that staging changes the phase of oscillation between adjacent flames, leading to $Q'$ cancellation.

Diagram explaining boxplot: $Q_1$, $Q_3$, IQR, min/max (IQR +/− 1.5), median, outliers, etc.

The Rayleigh Index can tell us a lot about the potential methods of instability control.

\[ RI = \int_{V} \int_{T} p'(\vec{x}, t) q'(\vec{x}, t) dt dV \]

- Increase the damping so that it suppresses the instability.
- Move the pressure anti-node relative to the flame.
- De-phase the \( p' \) and \( q' \) over an acoustic cycle.
- Change the heat release rate spatial distribution.
Baffles are commonly used to change the shape of the acoustic mode compared to the flame by impeding its oscillation in certain directions.


Example: Annular combustor with transverse mode and baffles
The Rayleigh Index can tell us a lot about the potential methods of instability control

\[ RI = \int_0^V \int_{-T}^{T} p'(\bar{x}, t) q'(\bar{x}, t) dtdV \]

- Increase the damping so that it suppresses the instability
- Move the pressure anti-node relative to the flame
- Change the heat release rate spatial distribution
- De-phase the \( p' \) and \( q' \) over an acoustic cycle
Active control is the way to active de-phase the oscillations between $p'$ and $Q'$ in a combustor, but is very difficult to achieve in practice.

Active control is the way to active de-phase the oscillations between $p'$ and $Q'$ in a combustor, but is very difficult to achieve in practice.
Bonus suppression method!
What if we could stop the feedback loop not by changing the flame or acoustics, but changing the flow?

Suppression through changes in *flow receptivity*
Flow receptivity
In a stable flow, disturbances are damped and do not grow in space or time.
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Some flows are disturbance amplifiers, where input disturbances grow in space as they’re convected.
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Some flows are disturbance amplifiers, where input disturbances grow in space as they’re convected.
These disturbance amplifiers require continual excitation for oscillations to occur.
Finally, some flows are self-excited and display oscillations without the need for an input disturbance.
If you add additional disturbances to these self-excited flows, it’s not always clear what’s going to happen...
SIDEBAR OVER

Back to bonus suppression mechanism
Experiments done at DLR-Stuttgart showed both hydrodynamic and thermoacoustic instability as a function of air split through the nozzle.

Parameter | Range
--- | ---
Combustion mode | Non-reacting
 | Reacting
Equivalence ratio | 0.67
Air split | 0:0.1:1 (NR)
 | 0.2:0.05:0.5 (R)
Reynolds number | 19,490
Pressure | 5 atm

At the non-reacting conditions, increasing the air split moves the flow to the center, results in stronger central recirculation/higher backflow velocity.
At the non-reacting conditions, we see a PVC develop as the air split increases and the strength of vortex breakdown increases.
In the reacting conditions, there are three separate oscillations – two thermoacoustic modes and the PVC mode.
In cases like these, SPOD is significantly better than POD because the POD would not necessarily separate the different frequency oscillations.
By tracking the mode amplitudes, it appeared as though the PVC mode was suppressing the thermoacoustic oscillation – is this possible?
Yes! It’s possible! Nonlinear theory showed the mechanism by which the precessing vortex core could suppress shear layer response:

\[
\left( \mathcal{B} + \mathcal{B}^S \{\tilde{q}\} \right) \frac{\partial \tilde{q}}{\partial t} = -\mathcal{N} \{\tilde{q}\} \tilde{q} - \mathcal{S} \mathcal{N}^S \{\tilde{q}\} \tilde{q} - \mathcal{S}^2 \mathcal{N}^{SS} \{\tilde{q}\} \tilde{q} \\
\quad + \mathcal{L}_T \tilde{q} + \mathcal{S} \mathcal{L}^S_T \tilde{q} + \varepsilon^3 \hat{q}_a(r, z) \cos(\omega_a t)
\]

Solution expansion:

\[
\tilde{q}(r, \theta, z, t_1, t_2) = q_0(r, z) + \epsilon q_1(r, \theta, z, t_1, t_2) + \epsilon^2 q_2(r, \theta, z, t_1, t_2) + \epsilon^3 q_3(r, \theta, z, t_1, t_2) + \ldots
\]

\[
q_1(r, \theta, z, t_1, t_2) = A_1(t_2) \hat{q}_1(r, z)e^{i(\theta - \omega t_1)} + A_0(t_2) \hat{q}_0(r, z)e^{-i\omega t_1} + c.c.
\]

Oscillation amplitude:

\[
A_{TH} = \left| \frac{\beta_{A_0 f}}{i[\omega_a - \omega_0 - i(S - S_c)\alpha_{A_0}] - \beta_{A_0 A_1} A_{PVC}^2} \right|
\]

\[
\beta_{A_0 f} = \frac{\langle \hat{q}_0^\dagger, \hat{q}_a \rangle}{2\langle \hat{q}_0^\dagger, \mathcal{B}_0 \hat{q}_0 \rangle}
\]
What have we learned so far?

1. Identify the technologies in which combustion instability is an issue
2. Describe the basic thermoacoustic coupling process
3. Explain the kinematics of flame response to harmonic inputs
4. Assess a flame transfer function to identify the potential coupling processes
5. Use your understanding of flame response and thermoacoustic coupling to explain instability control techniques
Course schedule

Monday:
— 1400-1500: Introduction and combustion instability in technologies
— 1500-1530: Thermoacoustic feedback
— 1530-1545: Break
— 1545-1730: Flame kinematics (with crafts!) and flame response

Tuesday:
— 1400-1530: Flame response and flame transfer functions
— 1530-1545: Break
— 1545-1630: Instability control
— 1630-1730: Advanced topics
Advanced Topics

— Transverse modes
— Nonlinear instability
— Entropy coupling
— What do you want to hear about?
Advanced Topics

— Transverse modes (this was the subject of my PhD, so buckle up...)

— Nonlinear instability

— Entropy coupling

— What do you want to hear about?
Transverse modes most often occur in annular combustor configurations where the circumferential and longitudinal length scales are similar.

Can combustor geometry

Annular combustor geometry

This instability mode can cause anti-symmetric and time-varying disturbance fields for the nozzles as they oscillate, which is more complex than a longitudinal mode.
The flame response pathways are more complex in the transverse mode than in the longitudinal mode and is more geometry dependent.

We designed an experiment that allowed us to probe the extremes of the acoustic modes – the pressure node and pressure anti-node – and resultant flame response.
The flame response pathways are more complex in the transverse mode than in the longitudinal mode and is more geometry dependent.

- Pressure node
- Velocity anti-node
- Anti-symmetric flame response
- No global heat release rate oscillation

- Pressure anti-node
- Velocity node
- Symmetric flame response
- Finite global heat release rate oscillation

Experiments in full annular combustors allow for investigation of spinning vs. standing modes and how flow couples with mode structure.

Experiments in full annular combustors allow for investigation of spinning vs. standing modes and how flow couples with mode structure.

All nozzles co-swirling, three spacings (S)

Alternating swirl nozzles, three spacings (S)

Advanced Topics

— Transverse modes

— Nonlinear instability

— Entropy coupling

— What do you want to hear about?
Nonlinear instability refers to the type of instability and how it starts, not the response of the flame (which typically always becomes nonlinear).

As $A \rightarrow 0$, $H(A) > D(A)$

A finite-amplitude $A$ is required for $H(A) > D(A)$.
Example: Space Shuttle booster rockets

Example: gas turbine combustor from Kim and Hochgreb (it is rare to see this in gas turbines, so this is a cool paper)

Advanced Topics

— Transverse modes

— Nonlinear instability

— Entropy coupling

— What do you want to hear about?
Entropy coupling is a unique instability mode as it relies on an acoustic/convective process and is heavily dependent on the combustor outlet boundary condition.

Heat release rate perturbation leads to entropy oscillation. Entropy oscillation convects downstream and interacts with turbine BC. The interaction of an entropy disturbance with a high-impedance BC leads to an acoustic disturbance.

Acoustic disturbance drives an entropy disturbance at the flame and the cycle starts again.

\[
f = \frac{1}{\tau} = \frac{1}{\tau_{\text{conv}} + \tau_{\text{acoustic}}}
\]

where

\[
\tau_{\text{conv}} = \frac{L_{\text{comb}}}{u_{\text{conv}}}
\]

\[
\tau_{\text{acoustic}} = \frac{L_{\text{comb}}}{c}
\]
Because of the convective timescale, entropy modes tend to be low-frequency modes and typically occur when the engine is cold (often called “rumble”).

Advanced Topics

— Transverse modes
— Nonlinear instability
— Entropy coupling
— What do you want to hear about?
Thank you!

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