Defining the Thermal Boundary Condition for Protective Structures in Fire

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Abstract

Protective structures are designed explicitly to fulfil a function which in many cases is an extreme event, therefore, the explicit design has to properly and precisely account for the nature of the solicitation imposed by the extreme event. Extreme events such as explosions or earthquakes are reduced to design criteria on the basis of either empirical or historical data. To determine the design criteria, the physical data has to be translated into physical variables (amplitudes, pressures, frequencies, etc.) that are then imposed to the protective structure. While there is debate on the precision and comprehensive nature of this translation, years of research have provided strong physical arguments in supporting these methods. Performance is then quantified on the basis of the structure's capability to perform its required function. Classified solicitations may then be used to translate performance into prescribed requirements that provide an implicitly high confidence that the structure performs its function. When addressing fire, performance has been traditionally determined by imposing standardized requirements that only bear a weak relationship with the reality of potential events – the fire performance of a protective structure is thus defined as a fire resistance period. This paper addresses the concept of fire resistance and its relevance to the design of protective structures. The mathematical description of the thermal boundary condition for a fire is of extreme complexity, therefore simplified approaches, that include the Fire Resistance concept, are currently used. By using classical heat transfer and structural engineering arguments, it is demonstrated what is the adequate level of complexity that is appropriate for the thermal boundary condition and when a precise definition of this input parameter is fundamental to adequately understanding the response of a structure to afire event. Simple criteria are presented to qualify the relevance of current approaches and to highlight important issues to be considered.

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Keywords

Fire; fire resistance; protective design; explicit performance; boundary conditions

1. Introduction

The capability of a protective structure to perform its function is defined by a design process that should contemplate the different solicitations that a structure may have. In many cases, this requires understanding the effects of single or potentially multiple solicitations. Moreover, critical infrastructure is generally design to withstand combined hazards, therefore, protective structures are also generally designed to perform their function when affected by combined hazards. While protective structures can be designed to withstand the effects of fire, it is often necessary to introduce fire as part of a multiplicity of hazards. This is the case when designing mines protective seals against explosions, where fires tend to follow explosions, or when designing structures that protect the core of nuclear reactors that could be subject to terrorist attacks or earthquakes followed by fires. When considering the potential of combined hazards, the design of structural protection to fire assumes that the capability of the structure to withstand fire remains intact. This assumption has the potential of not being realistic (or even un-conservative), nevertheless, because of the nature of the fire safety design process, the assumption is often unavoidable. When designing for fire, performance is not explicitly calculated but is calculated on the basis of a presumption that "fire resistance" represents a worst case fire scenario condition that if imposed onto a single element of the structure, will result in a solicitation not exceeded by any realistic potential fire.

Fire is an extremely complex combination of physical phenomena that currently cannot be fully described by means of mathematical models. Thus, some level of simplification is always necessary. In particular, in the case of structural analysis the necessary simplifications are very significant because the structure also needs to be described. When focusing on structural performance, coupling between gas and solid phase is commonly avoided and the fire is treated as a thermal boundary condition. The choice of what is the appropriate complexity necessary for the thermal boundary condition and to what section of the structural system should it be applied remains a matter of current debate. This paper will review some basic concepts to clarify the implications of specific simplifications and establish simple criteria that allow to establish when more or less complexity is needed.

In current practice, the fire solicitation and its manifestation on the behavior of the structure is typically solely defined in the temperature domain and does not require any explicit quantification of heat transfer (energy conservation) or mechanical structural performance. During design, elements of the structure might be subject to the standard "fire resistance" testing procedures – for assuring the compliance of single elements considered in isolation. Single element performance under fire as part of a whole structural system behavior is rarely addressed. Once the requirements for "fire resistance" are met, then it is assumed that all serviceability requirements for the structure will be met independent of any other solicitation or the integral nature of the structural system. This approach implies strong simplifications that assume that global structural behavior can be bounded by single element performance assessment and that heat transfer from the fire to the structure can be adequately characterized by gas phase temperatures and standardization of the thermal environment (i.e, a test furnace).

This paper examines the two stages that must be considered as part of any design process for a structure to withstand the effects of fire. Specifically considered are:

- the assessment of thermal performance i.e. the fire and how thermal energy released during fire is transferred into the structure; and
- the structure. i.e. how the structure responds as a function of the thermal boundary conditions.

The paper evaluates, in very simple terms, the conditions under which certain simplification are valid or invalid allowing to clarify the limitations of current performance assessment procedures. Given the complexity of the fire-structure interactions there will be many criteria that can be used and a comprehensive treatment is beyond the scope of this paper. Instead, as a very relevant example, the focus of this particular paper is on the role of the thermal gradients in structural behavior inferring when it is necessary to precisely establish these gradients. This approach, in principle, applies to any structure, but in particular to protective structures, given their critical function.

2. Assessing Thermal Performance

2.1 Fire Dynamics

At the core of a fire there is a flame or a reaction front that is effectively the result of a combustion process, and thus is governed by the mechanisms and variables controlling combustion [1]. The interaction between fire and its surrounding environment determines the behavior of the flame and nature of the combustion processes. An extensive introduction to the topic is provided by Drysdale [2].

As indicated by Drysdale [2], the dynamics of a fire involves a compendium of different sub-processes that start with the initiation of a fire and end with its extinction. The onset of the combustion process, i.e. ignition, in a fire is a complex process that implies not only the initiation of an exothermic reaction but also a degradation process that provides the fuel effectively feeding the fire. During a fire, it is common to have different materials involved in the combustion process, and given the nature of the fire growth many could be involved simultaneously but others sequentially. The sequence of ignitions of items in an enclosure will affect the nature of the combustion processes. Thus, ignition mechanisms set the dynamics of the fire and also are affected by the fire itself, creating a feedback loop [3].

Once a material is ignited, the flame propagates over the condensed fuels by transferring sufficient heat to the fuel until a subsequent ignition occurs. This process is commonly referred to as flame spread and is described in detail by Fernandez-Pello [4]. Flame spread defines the surface area of flammable material that is delivering gaseous fuel into the combustion process. The quantity of fuel produced per unit area is known as the mass burning rate. The mass burning rate multiplied by the surface area determines the total amount of fuel produced. If the total amount of fuel produced is multiplied by the effective heat of combustion (energy produced by combustion per unit mass of fuel burnt), it yields the heat release rate. Generally, the heat release rate is considered the single most important variable to describe fire intensity [5]. Given the nature of the surrounding environment, the oxygen supply might not be enough to consume all the fuel, thus in many cases combustion is incomplete (i.e. under-ventilated) and therefore the heat of combustion is not a material property but a function of the interactions between the environment and the fire. In these cases, it is usually deemed appropriate to calculate the heat release rate as the energy produced per unit mass of oxygen consumed multiplied by the available oxygen supply.

If the fire is within a compartment, smoke will accumulate in the upper regions of the compartment. Hot smoke will radiate and/or convect heat towards all surfaces in the compartment. If the surfaces are flammable, remote ignition of different materials might occur. If remote ignition occurs in the lower (i.e. cold) layer then the fire tends to suddenly fill the entire compartment. This transition is generally known as flashover. Before flashover, the lower layer tends to have enough oxygen to burn the pyrolyzing fuel and the heat release rate is determined by the quantity of fuel generated. This period is termed pre-flashover, fire growth or fuel limited fire. After flash over, fuel production tends to exceed the capability of air to enter the compartment, the compartment becomes oxygen starved and the heat release rate is determined by the supply of oxygen through the various ventilation inlets/outlets of the compartment (e.g. doors, windows, etc.). This period is termed as post-flashover, fully developed fire or oxygen limited fire. The process of fire growth and the definition of the different variables affecting it is provided by Drysdale [2].

For small compartments (approximately 4 m x 4m x 4m) a characteristic time to flashover is of the order of 4-6 minutes while the post-flashover period can reach tens of minutes depending on the compartment size and fuel available [6]. Structures tend to have high thermal inertia, thus the temperature increase, at the surface (or in-depth) of solid elements, to levels where the loss of mechanical properties is significant, takes also in the order of tens of minutes. Thus for purposes of structural assessment, the effects of fires tend to be only considered at the post-flashover stage [7]. The temperature inside the compartment as well as the burning rate can be established simply as a function of the available ventilation, this process can follow different levels of complexity; Drysdale [2] reviews all these. It is important to note, that while the compartment temperature can be established by means of a simple energy balance, the heat being transferred to each structural element does not necessarily correlate with this temperature [6]. These relationships and time scales are of particular importance for protective structures, given that fires can cover a very wide range of characteristics when originating in environments that are different from the conventional compartment. Any analysis involving unusual compartments will have to revisit the evolution of the fire in a very detailed manner because many of the assumptions embedded in current design practices will no longer be valid.

A fire can end when it is extinguished or when oxygen or fuel supplies are depleted; oxygen starvation and burnout, respectably. In all cases, extinction of the combustion process is brought by the interactions of fuel, oxygen supply and the energy balance that fundamentally allows for the combustion reaction to remain self-sustained [8]. Suppression agents affect a fire by reducing fuel and oxygen supply, or by removing heat. At each stage of fire growth, it becomes more or less feasible to have an effect over these three fundamental variables of fire. Thus the effectiveness of a suppression system is dictated by its capability to affect the targeted variables at the moment of deployment. Once again, time scales are of critical importance. If the effect of suppression agents is to be incorporated to the design of protective structures, then it will have to be demonstrated that the time scales of deployment, heat transfer and fuel/oxygen displacement are consistent with the time scales of other solicitations and with those that deliver the desired effect.

A common way to describe the evolution of a fire in a compartment is by means of design temperature-time curves. There are numerous variants of these, from a purely standardized version [9] used to conduct standard furnace tests, to others supposedly more representative of ‘real’ fire conditions [10]. A commonly used expression is that proposed by Lie [11] and simplified here for small fuel loads:

$$T_g = 250(10F)^{\frac{0.1}{F^{0.3}}} \exp(-F^2 t) . [3(1-\exp(-0.6t)) - (1-\exp(-3t)) + 4(1-\exp(-12t))] \quad (1)$$

Where T_g is the temperature of the gas phase inside the compartment and t is time in hours. F is the ventilation factor, which is a commonly used combination of the opening surface area (A_W), the height of the opening (H) and the total area of the compartment (A_T) excluding floor and opening ($F=A_W H^{1/2}/A_T$) [10]. Fig. 1 describes a typical scenario where all stages have been marked.

The thermal boundary conditions at the exposed surface of a solid element (i.e. strutural element) are fundamentally based on conservation of energy [12], and thus typically formulated in terms of heat fluxes. In Fig. 1, the maximum heat flux at the exposed surface of the strucuture has been calculated assuming that the solid surface remains at ambient during heating (based on the assumption that heating of the gases and that of the solid occur in different time scales) and at T_g during cooling. The gas phase temperature is assumed to be 20°C during cooling. The total heat transfer coefficient, $h_T= 45\ W/m^2$, is a value commonly used by fire engineers to account for convection and radiation [2, 8]. While clearly, the surface and gas phase temperatures will vary during heating and cooling, this value of the heat flux is indicative of the conditions that a structure will experience during a fire.

EXAMPLE

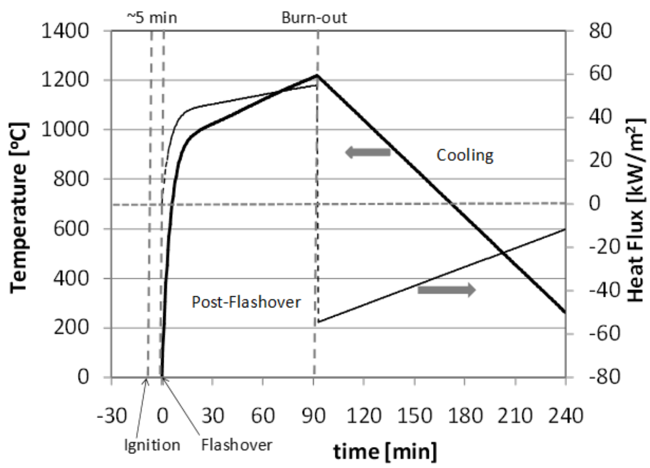

Figure 1 – Typical temperature and heat flux evolution for a small compartment fire. From ignition to flashover there is a period of approximately 5 minutes where temperatures and heat fluxes are negligible. This is followed by a longer period where temperatures and heat fluxes increase. The cooling period starts with burn-out (total fuel consumption). The plots were obtained using a 4 m x 4 m x 4 m compartment with a single opening (2 m x 3 m) a fuel load of 60 kg/m^2 and a total heat transfer coefficient of 45 W/m^2. The temperature vs. Time curve was obtained using the expression by Lie [11], Eq. (1).

In summary, structures exposed to fire will see temperatures close to ambient and negligible heat fluxes during the pre-flashover period (4-6 min for a small compartment). Temperatures and heat fluxes will increase after flashover reaching values of approximately 1200^{o}C and 60 kW/m^2, respectively. These thermal exposures can last for periods in excess of an hour, depending on the fuel load, compartment geometry and ventilation. Finally, fuel will innevitably burn-out and the compartment will cool down. In the cooling period the heat flux will be negative denoting heat losses from the structure surface to the colder gases inside the compartment. The correlation between temperature and heatflux can be linear, but this is only under the assumption that an overall constant heat transfer coeefficient can be established in space and time. It is important to emphasize, that if this procedure is followed, then the thermal boundary condition at the surface is being imposed not calculated.

The thermal boundary conditions at the surface of the solid is defined by means of Equation 2.

$$\dot{q}"_{Tot} = -k \frac{\partial T}{\partial x}\bigg|_{x=0} \qquad (2)$$

Where k is the thermal conductivity of the solid material. As can be seen in Fig. 1, the range of heat fluxes vary between $60\ kW/m^2$ and $-60\ kW/m^2$ therefore the in-depth gradients of temperature of the structural elements will vary significantly, and in a manner that does not necessarily resemble the gas phase temperature evolution inside the compartment. The importance of the gradients and the resemblance of their evolution to that of the temperature are a function of the thermal properties of the material (e.g. thermal conductivty) and of the gas phase conditions. Therefore, when trying to understand the explicit behaviour of structures in fire it is essential to discuss its evolution not only within the context of the characteristic conditions and time scales involved in a fire but also as a function of the thermal properties of its solid phase. The material thermal properties define if the in-depth thermal gradients within the structure will be insensitive or sensitive to the characteristics of the gas phase. For certain materials where the thermal gradients are very insensitive to the gas phase conditions, major simplifications still deliver precise answers, the opposite will happen when temperature gradients are a strong function of the gas phase. In the latter case a detailed description of the gas phase might be necessary to achieve an appropriate thermal boundary condition.

While the analysis presented above is simplistic in nature, and the values presented are only rough estimates, it does provide a clear image of the thermal conditions that a structural element will experience in the event of a fire. Furthermore, it ilustrates the importance of making an *apriori* analysis of the thermal properties of a material before simplifications to the gas phase treatment are proposed. It is current practise to accept certain simplifications without first establishing their validity (ex. Constant heat transfer coefficients, constant emissivities, fire representation by means of a temperature, etc.).

2.2 Heat Transfer

As abovementioned, when analysing the heat transfer from the fire to a structural element the problem needs to be formulated in terms of heat fluxes. While temperature of the solid phase results from solving the energy conservation equations, all quantities to be balanced are energies [13]. In this section some basic heat transfer concepts are reviewed simply to extract the relevant parameters that will be used in later sections for discussion. These concepts are not novel and can be found in any heat transfer book, for details the reader is refrred to refernce [13], nevertheless, its novelty relies on the application as a screening tools for the assesment of the thermal boundary condition that is required for different strctural configurations.

Heat is transferred from gases to solid surfaces via radiation and convection resulting in a total heat flux, $\dot{q}"_{Tot}$, where:

$$\dot{q}"_{Tot} = \dot{q}"_{rad} + \dot{q}"_{con} \tag{3}$$

Where, $\dot{q}"_{rad}$ is the heat transfer via radiation and $\dot{q}"_{con}$ is the heat transferred via convection. For simplicity, within the scope of the work presented herein, the problem will only be examined in the direction of the principal heat flux, hence considered to be a one dimensional proble and with the thermal boundary condition of the solid element (i.e. structural element) defined as:

$$\dot{q}"_{Tot} = -k_i \frac{\partial T}{\partial x}\bigg|_{x=0} \tag{4}$$

Which is a generic version of Eq. 2 and where the thermal conductivity (k_i) is a property of the solid and the gradient of temperature is taken at the surface. In other words all the heat arriving at the surface of the solid is conducted into the solid. If there are multiple layers then at each interface the following boundary condition should apply:

$$-k_i \frac{\partial T}{\partial x} = -k_s \frac{\partial T}{\partial x} \tag{5}$$

Where the gradients correspond to each side of the interface and the sub-index "s" is a generic way to represent the next layer of solid. Once the thermal boundary conditions are defined, the energy equation can be solved for each material involved. In the case where two layers of solid are involved ("i" and "s"), then the energy equations take the following form:

$$\rho_i Cp_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_i \frac{\partial T}{\partial x}\right) \tag{6}$$

and

$$\rho_s Cp_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_s \frac{\partial T}{\partial x}\right) \tag{7}$$

The solution of the energy conservation equations yields the temperature evolution of the material in space and time. Eq. 6 and 7 could be repeated for as many layers as necessary. If the geometry or the fire exposure is complex, then the problem needs to be resolved in two or even three dimensions. If the properties vary with temperature then, as the temperature increases, these properties need to evolve with the local temperature. Variable properties thus require a numerical solution. If a simple analytical solution is to be obtained, then adequate global properties need to be defined. It is important to note that whatever the solution methodology adopted, the temperature of the structure is the result of the resolution of Eq. 6 and 7 using thermal boundary conditions such as those presented in Eq. 4 and 5. To obtain the numerical solution it is necessary to input material properties for the different layers ("i" and "s"). The material properties required are all a function of temperature and are as follows:

ρ_i , Cp_i , k_i

ρ_s , Cp_s , k_s

Where, (ρ_i , ρ_s), (Cp_i , Cp_s), and (k_i , k_s) are the densities, specific heat capacity, and thermal conductivity for each layer, respectively. For some materials such as steel the thermal properties are very well characterized and thus very little difference can be found between the literature [14]. For other materials such as concrete, wood or building construction thermal insulation materials, the scatter is much greater [7]. The uncertainty is associated to the presence and migration of water, degradation, crack formation, etc.

Furnace data (i.e. in-depth temperature measurements of the solid material taken during standard furnace testing) is generally used as a substitute for the uncertainties associated with defining thermal properties. In many cases global thermal properties are extracted by fitting calculated temperatures to the furnace data. These properties are then extrapolated and widely used in equations such as Eq. 4 to 7 for assessing the thermal performance. Nevertheless, this practise has also its unique complexities. First of all, the model needs to include all the physical variables necessary, so if physical processes such as the degradation or water migration within the material are not explicitly included in the model, the thermal properties extrapolated from furnace data become hybrids that implicitly include these physical parameters. Implicitly introducing physical phenomena into constants inevitably narrows the range of application, thus most of these 'calibrated' thermal properties can only be used to re-evaluate furnace data. Based on these grounds, extrapolation to drastically different scenarios, such as a 'real' fire, becomes doubtful. Harmathy discusses in great detail how to formulate the heat transfer problem within a furnace emphasizing its complexity [15].

An important aspect, many times overlooked, is the need to make sure that the thermal boundary conditions are properly represented. The heat exchange between a furnace and a sample is extremely complex and many times simplifications relevant to furnaces are not valid for 'real' fires **Error! Reference source not found.**. It is essential to understand all those simplifications. The differences between furnace behaviour and fire behaviour are all manifested in the boundary condition associated with Eq. 2.

A common misunderstanding is to attempt representing the evolution of the temperature of a material by a single temperature as represented in Fig. 2. Fig. 2 shows the evolution of temperature for unprotected steel subjected to different fires; defined by gas phase temperature inside a compartment. While plots of this nature serve to compare the evolution of the steel temperature they hide numerous assumptions that while relevant to steel, they are not relevant to other materials, for example, concrete.

Establishing the nature of temperature gradients within a solid allows to establish the range of validity of the assumption that a single temperature can represent the heat transfer process. This is an essential component of the thermal assessment of a material in a fire. The nature of the temperature gradients is defined by the *Biot number*:

$$Bi = \frac{h_T d}{k} \qquad (8)$$

The *Biot number* provide a very simple representation of the relationship between the temperature gradients in the gas phase and the temperature gradients in the solid phase. For a very large or very small *Biot number* the solid phase gradients are very insensitve to gas phase gradeients and therefore the gas phase can be treated with very simple approximations. Depending on what extreme of the *Biot number* range the material is, the simplifications will be different. Intermediate range values of the *Biot number* will require precise treatment and most simplifying assumptions will lead to major errors.

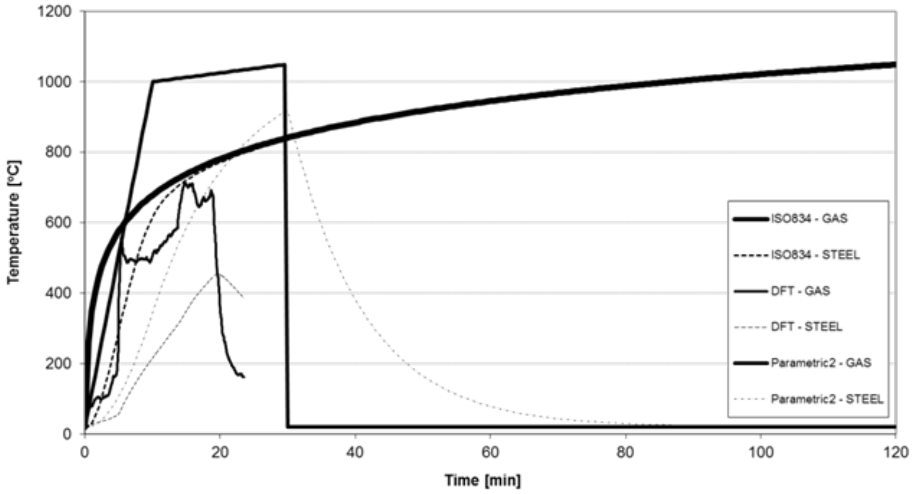

Figure 2 – Temperature evolution of the gas phase of a compartment fire and a small cross section unprotected steel beam. Standard temperature time curve per ISO-834 [9], DFT stands for Dalmarnock Fire Test [17]

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Fig. 3 provides a simple schematic showing the influence of the *Biot number* in a one dimensional heat transfer – evidencing the scope for potential simplifications of the heat transfer problem. If the *Biot number* is close to one (case (b) in Fig. 3) temperature gradients in the gas and solid phases are large and therefore Equations 6 and 7 will need to be fully resolved, hence no simplifications are possible. If the *Biot number* is much greater than one (case (c) in Fig. 3) the temperature differences in the gas phase are much smaller than those in the solid phase and it can be assumed that surface and gas temperatures are almost the same. This simplification is very important when modelling furnace tests because it enables to ignore the complex boundary condition imposed by the furnace and simply imposed the monitored gas temperature at the surface of the solid. Finally, if the *Biot number* is much smaller than one (case (a) in Fig. 3) then the temperature differences in the solid phase are much smaller than those in the gas phase, therefore temperature gradients in the solid phase can be ignored and a single temperature can be assumed for the solid. Heat conduction within the solid can be approximated by the boundary conditions and Eq. 6 and 7 lead to a single temperature solution like the ones shown in Fig. 2.

The representation of a structural element by means of a single temperature is therefore only valid if *Bi <<1*. This simplification is called a "lumped capacitance formulation" and while it does not resolve spatial temperatures distributions it still requires an adequate definition of the heat transfer between the source of heat (e.g. furnace or 'real' fire) and the solid. An important observation is that for materials with *Biot numbers* much smaller than one, the thermal energy is rapidly diffused through the integrity of the material, so if the density was to be high (see Eq. 6 and 7), then the lumped solid will lag significantly the gas phase temperature (Fig. 2). Heat transfer is therefore dominated by the temperature difference between the solid and the gas phase, and errors in the definition of the heat transfer coefficient become less relevant. It is common for studies attempting to understand the behaviour of structures in fire to make use of constant heat transfer coefficients [7], this will be appropriate for materials with a *Bi<<1*. Nevertheless, there is also significant inconsistencies in the numbers quoted and furnace heat transfer coefficients are many times extrapolated to natural fire coefficients. These values are not necessarily the same, in particular if radiation and convection are to be amalgamated into a single heat transfer coefficient [3].

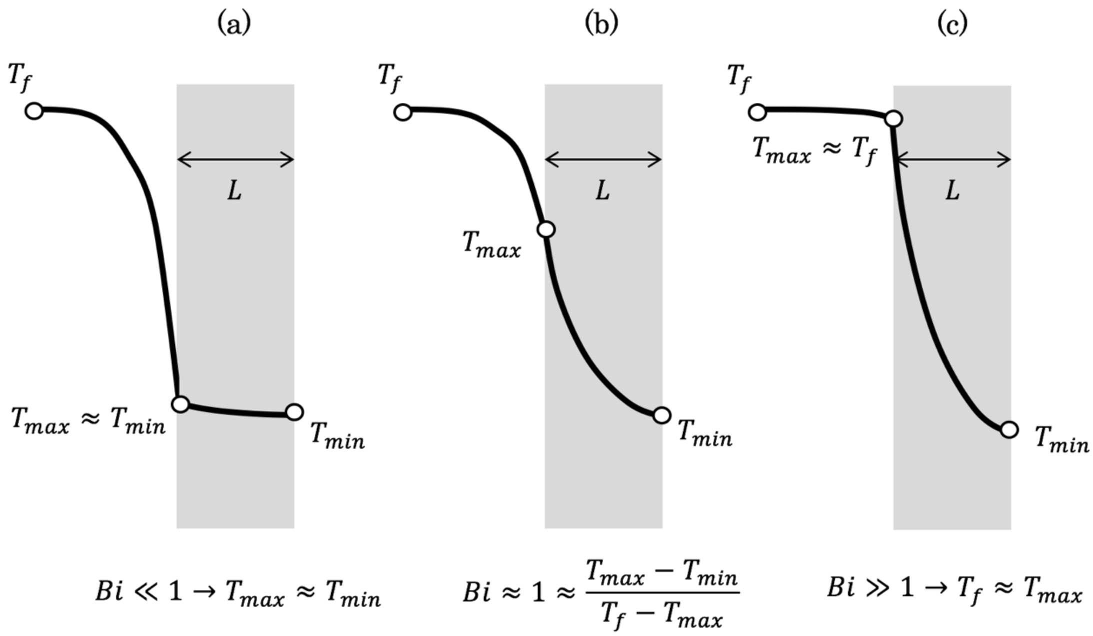

Fig. 3. Schematic of the typical temperature distributions for different extreme values of the *Biot number*.

Given the importance of the *Biot number* in the characteristics of the temperature gradients, it is important to estimate the thickness of a material that leads to a *Bi=1*. Samples that are much thicker will allow approximating the surface temperature to that of the gas phase. Samples that are much thinner will allow to “lump” the solid phase into a single temperature.

Table 1 shows typical thermal properties for different construction materials and the characteristic thickness (L) that will result in a *Biot number* of unity. As can be seen for high thermal conductivity materials like aluminium or steel, sections a few millimetres thick can be lumped without any major error. In a similar manner very low thermal conductivity materials like plasterboard or expanded polystyrene (EPS) will allow to assume that the surface temperature of the solid is that of the gas phase. In contrast, concrete has a Biot of unity for a thickness of 50 mm that is in between typical concrete cover thicknesses and the overall thickness of the sample. Therefore, whether the concrete is used as cover for the reinforcement or analysed as the load bearing material, the full resolution of an equation similar to Eq. 6 is necessary. Furthermore, the boundary condition cannot be simplified because the thermal gradients are fully defined by $\dot{q}''_{Tot}$ as per Equation 4.

Table 1 – Typical thermal properties for different construction materials

Material	Density (ρ,kg/m^3)	Thermal Conductivity (k,W/mK)	Specific Heat (Cp,J/kgK)	Thermal Diffusivity (α,m^2/s)	*“L”* for *Bi=1* (mm)
Aluminium	2,400	237	900	1.10E-04	5,300
Steel	7,800	40	466	1.10E-05	900
Concrete	2,000	2.5	880	1.42E-06	50
Plasterboard	800	0.17	1,100	1.93E-07	4
Expanded polystyrene (EPS)	20	0.003	1,300	1.15E-07	0.1

3. Structural Fire Performance

3.1. Steady State Thermal Gradients

Given that the thermal properties of concrete do not allow for a simplified analysis, the temperature gradients within the structural element needs to be estimated. The resulting gradients can then be incorporated into a structural analysis to define the significance of thermal bowing.

If a slab of thickness L separates a fire of temperature T_f and ambient conditions, T_{amb}, then, at thermal steady state the energy conservation equation leads to the equalities presented in Equation (9). Note that heat exchange at the surfaces has been split in convective and radiative terms.

$$\sigma\varepsilon\left(T_f^4 - T_{max}^4\right) + h_i\left(T_f - T_{max}\right) = \frac{k}{L}(T_{max} - T_{min}) = \sigma\varepsilon\left(T_{min}^4 - T_{amb}^4\right) + h_0(T_{min} - T_{amb}) \tag{9}$$

Where sub-indexes "max" and "min" is a generic way to represent the exposed and unexposed surface of the solid. The explicit solution of Equation 9 yields the following expressions for the minimum and maximum temperature (T_{min} , T_{max}) within the solid:

$$T_{min} = \frac{L}{k}\left(T_{max}\left(\sigma\varepsilon T_{max}^3 + h_f + \frac{k}{L}\right) - T_f\left(\sigma\varepsilon T_f^3 + h_f\right)\right) \tag{10}$$

$$T_{max} = \frac{L}{k}\left(T_{min}\left(\sigma\varepsilon T_{min}^3 + h_b + \frac{k}{L}\right) - T_0(\sigma\varepsilon T_0^3 + h_b)\right) \tag{11}$$

Then, the heat flux between the exposed and unexposed surface of the solid can be formulated as:

$$\sigma\varepsilon\left(T_f^4 - T_{max}^4\right) + h_i\left(T_f - T_{max}\right) = \sigma\varepsilon\left(T_{min}^4 - T_{amb}^4\right) + h_0(T_{min} - T_{amb}) \tag{12}$$

This can be rearranged as follows:

$$\sigma\varepsilon\left(T_f^4 - T_{max}^4\right) + h_i\left(T_f - T_{max}\right) - \sigma\varepsilon\left(T_{min}^4 - T_{amb}^4\right) - h_0(T_{min} - T_{amb}) = 0 \tag{13}$$

When the relevant substitutions are made, this results in a 7^{th} order polynomial which can be solved numerically. Where the assumption of total heat transfer coefficient is adopted, this can be simplified to allow T_{min} and T_{max} to be expressed as a function of Biot. This can be achieved as follows:

$$h_i\left(T_f - T_{max}\right) = \frac{k}{L}(T_{max} - T_{min}) = h_0(T_{min} - T_{amb}) \tag{14}$$

Then:

$$Bi_i\left(T_f - T_{max}\right) = (T_{max} - T_{min}) = Bi_0(T_{min} - T_{amb}) \tag{15}$$

The sub-indexes "i" and "o" represent the exposed and unexposed face, respectively. Therefore, at the exposed surface:

$$T_{max} = \frac{Bi_i(Bi_0+1)T_f+T_{amb}\cdot Bi_0}{(Bi_0+Bi_i\cdot Bi_0+Bi_i)} \tag{16}$$

By assuming that the *Biot number* is the same for the exposed and unexposed surfaces (i.e. $Bi_i \approx Bi_o$), this expression can be simplified to:

$$T_{max} = \frac{(Bi_i+1)T_f+T_{amb}\cdot Bi_i}{(Bi_i+2)} \tag{17}$$

and

$$T_{min} = \left(T_f - T_{max}\right) + T_{amb} \tag{18}$$

This approach or the numerical solution of Eq. 9 may be used for calculating a reference thermal gradient within a structural element. Selection of the method used will depend on the resolution (or accuracy) at which the solution is required. While this approach is not precise it allows to establish the impact of changing the Biot number on structural behavior.

3.2 Steady State Thermal Gradients

The fire response (or behavior) of the structure is defined by thermally-induced changes of the mechanical properties, and the developments of thermal expansion [18]. However, the interaction of these two parameters has a significant impact on the response of a structure. This interaction is a function of the bulk temperature increase within the material and thermal gradients. The temperatures and thermal gradients are a function of the thermal boundary conditions, thermal properties, and material thickness as examined above.

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Where the temperature distribution of an unrestrained structural element is simplified to a one dimensional (through- or in-depth) heat transfer analysis, a linear thermal gradient will result in a member curvature. Where a thermal gradient is non-linear, this will result in the development of internal mechanical strains within the depth of the structural element; these strains (or rather, the force and moment induced by them) must be resolved in order to maintain static equilibrium of the structural element. Assuming that the material remains in the elastic range, the curvature (ϕ) and total axial strain (ε_a) of the structural element can be solved using the following equations:

$$0 = \sum_{i=1}^{i=n}\{(\phi y_i + \varepsilon_a + \alpha T_i)E_i(T_i)y_iA_i\} \tag{19}$$

$$0 = \sum_{i=1}^{i=n}\{(\phi y_i + \varepsilon_a + \alpha T_i)E_i(T_i)A_i\} \quad (20)$$

Where "n" is the number of fibres into which an element is discretized, "y_i" is the distance from the centroid of the section to the centroid of each fibre, "a" is the coefficient of thermal expansion, T_i is the temperature of each fibre, $E_i(T_i)$is the temperature dependent elastic modulus of each fibre, and A_i is the area of each fibre.

For a simply supported beam, the axial elongation then becomes:

$$dL = L - \varepsilon_a L - \frac{\sin\left({}^{L\phi}/_{2}\right)}{{}^{L\phi}/_{2}} \quad (21)$$

and the total deflection due to thermal curvature becomes [18]:

$$d = \frac{1}{\phi}\left(1 - \cos\left({}^{L\phi}/_{2}\right)\right) \quad (22)$$

3. Canonical Example

On the basis of the equations shown in this paper, the full set of heat transfer and structural calculations can be solved. This allows the comprehensive study for the effects of the thermal boundary condition, as a function of the *Biot number* on the mechanical behavior of the structural element. This section executes this analysis.

Equations (17), (18), (21), and (22) represent the terminal state of temperature distributions and mechanical deformations. This allows establishing the general influence of the *Biot number* on the ultimate state of the structure. Nevertheless, this might not represent the critical state of the structure because the coupled effects of bulk expansion and temperature gradient induced curvature might result in worst case conditions before thermal steady state is attained. The thermal properties (and consequentially the *Biot number*) will influence also the transient state. Therefore, a numerical analysis of the transient evolution was performed to establish the role of the *Biot number* on transient deformation.

The equations were solved for a unit length, unit width structural concrete element subject on one side to a constant gas temperature of 1,000°C. It was assumed, for the numerical simulations, that $h_i =$

35 W/m^2, $h_o = 8$ W/m^2, and $\varepsilon = 0.7$; where these represented the internal and external convective heat transfer coefficients and ε the emissivity for the radiative component (equations 9 and 10). The values used as inputs are common values used in the literature. The thermal properties of concrete were as described above, and it was assumed that the degradation of elastic modulus was as illustrated in Fig. 4. Three material thicknesses were analyzed: 28, 50, and 100 mm. This is a convenient way of changing the Biot number without changing thermal properties. These values correspond to a *Biot number* of 0.5, 1, and 2 (assuming and approximate linearized heat transfer coefficient of $h_T = 45$ W/m^2). The analysis was continued until an approximate stead state was achieved after 2 hrs, and the results for a simply supported section in terms of total deflection and total elongation are illustrated in Fig. 5.

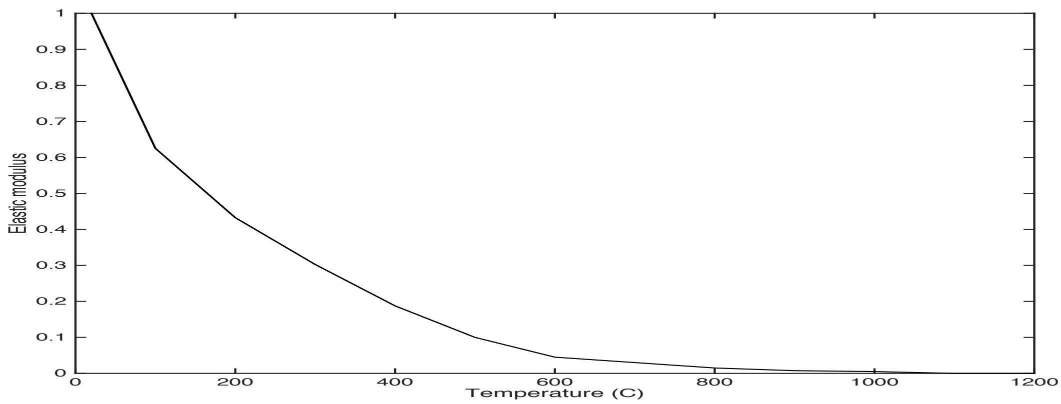

Figure 4. Degradation of elastic modulus (corresponds to the tangent stiffness of concrete at zero strain as per BS EN 1992-1-2)

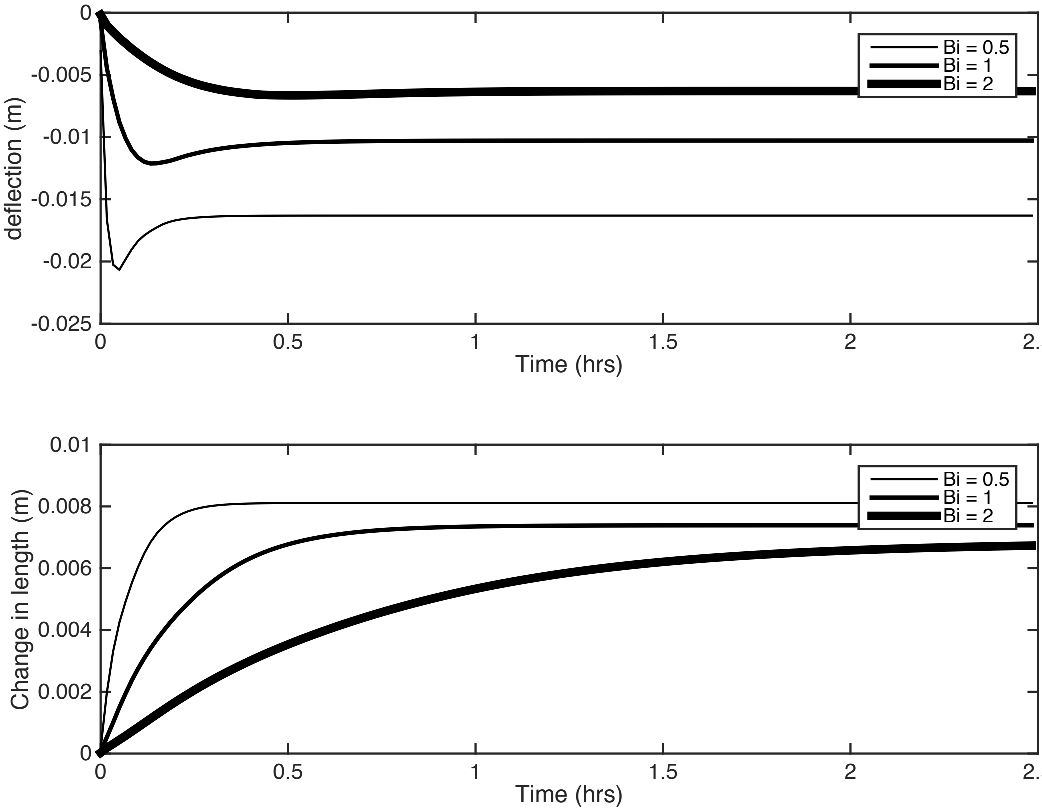

Figure 5. Resulting transient deflections and elongations.

These results demonstrate that, at the steady state, different *Biot numbers* induce different structural behavior. As the *Biot number* increases, the bulk change in length diminishes as well as the deflections showing an overall less significant effect of heat on the structural element. For lower *Biot numbers* the overall expansion of the structural element results in a greater final deflection. However, the results of the numerical model also demonstrate that highest deflections occur during the transient stages of a fire. Indicating that the *Biot number* also has a significant importance on the nature of the transient deformation and potentially early adverse effects. The maximum deflections in the steady state and in the transient analysis were calculated for a wide range of *Biot numbers* and the results presented in Figure 6. The results show that, for the canonical structure studied here, above a *Biot number* of approximately three transient and steady state solutions are almost identical, with a negligible error if only a steady state solution was to be applied (Figure 6). For smaller *Biot numbers* the two solutions diverge and given the worst-case deflections of the transient period, a transient analysis is necessary. This is a very important observation in that it allows not only to establish the precision required in the definition of the thermal boundary conditions but also to determine if a transient analysis is necessary.

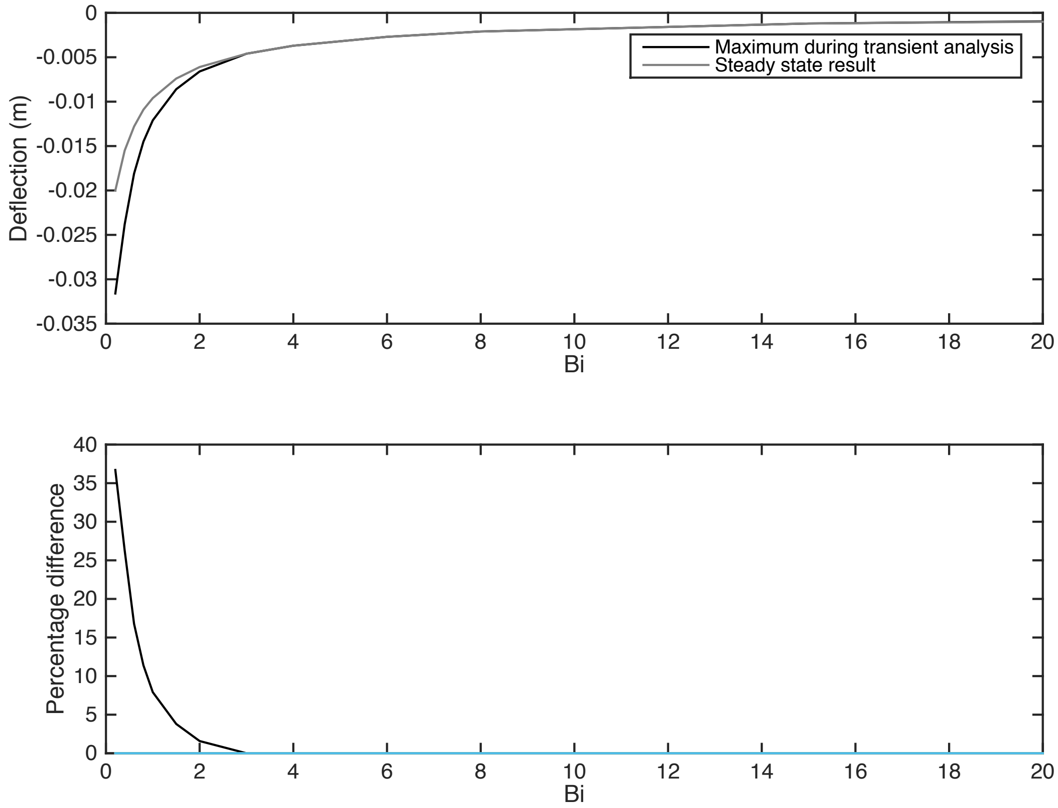

Figure 6. Resulting transient deflections and elongations. Absolute values of deflection obtained from the two analyses (top), and the relative errors associated with the different between the steady state value and the transient analysis (bottom).

4. Conclusions

An assessment of the role of detailed boundary conditions has been made. A simple analysis based on classic principles shows that the temperature gradients within a material are primarily a function of the *Biot number*. A demonstration of the role of the *Biot number* on deflections and elongation was used to illustrate how the *Biot number* can be used to establish if it is necessary to conduct a transient thermo-mechanical analysis as well as to determine the level of precision necessary when treating the thermal boundary conditions. The following conclusions were drawn:

- structural performance is an unavoidable result of the real evolution of the in-depth temperature of a structural element in space and time;

- to define the performance of a structural system in fire it is necessary to establish the correct thermal boundary condition. The evolution of this boundary condition will determine internal temperature distributions and thus structural behavior;
- the *Biot number* is a simple non-dimensional parameter that combines material characteristics and the thermal boundary condition allowing to establish the sensitivity of structural behavior to the precision of the boundary conditions as well as to transient behavior;
- the *Biot number* is an effective method to classify different forms of thermally induced structural behavior. The higher the *Biot number* the lesser transient effects and the more effective steady state modelling of a structure is to define the worst-case conditions. The lower the *Biot number* the more important is to model transient behavior;
- for the particular case studied, the greater the *Biot number* the less significant the effect of a fire on structural deformations;
- defining the thermal boundary conditions in terms of a single temperature (e.g. during the analysis of data from a standard "fire resistance" furnace test) can only be done for structural elements with *Bi<<1*. In this case the sensitivity to the thermal boundary condition is low therefore a global heat transfer coefficient will suffice. Nevertheless, appropriate quantification of the overall heat transfer coefficient is necessary. Extrapolation of furnace coefficients to "real" fire conditions may provide an unrealistic representation of the thermal conditions;
- and the gas phase temperature can be used as a boundary condition only if *Bi>>1*, in this case, furnace or "real" fire are only differentiated by the gas phase temperature differences; and
- constitutive properties of various building construction materials (e.g. concrete) are intimately linked to the formation of in-depth thermal gradients and therefore, current values are at best approximate.

Protective structures for fire safety are complex systems that require a precise and detailed representation of their transient behavior – as different solicitations are considered. In some areas such as explosions, this is done in a very careful way, and while questions remain about the adequacy of the calculations and testing procedures, all these are perfectly geared towards the explicit determination of performance. When it comes to the representation of fire performance, this paper has shown that current methods do not represent the underlying physics behind the definition of the thermal solicitation induced by the fire. This inadequate representation of the boundary conditions has been shown to have significant consequences on the predicted mechanical behavior of structural systems that are not consistent with the common believe that current methods are representing a "worst-case" performance scenario. This is particularly true for concrete structures.

The performance of protective structures has to be addressed in an explicit manner; this will enable not only to establish their reaction to a single hazard but their resilience when it comes to multiple hazards – fire being one of them. In the absence of an explicit performance assessment strategy for fire it is not possible to determine the adequacy of the protection provided by a structure.

Acknowledgements

The author wishes to acknowledge the support of ARC through DP150102354 and LE130100089, and the support of EPSRC through the "Real Fires for the Safe Design of Tall Buildings" project.

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